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Bearing Capacity of Eccentrically Loaded Surface Circular Foundation on Sand of Limited Thickness

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Abstract. Laboratory model tests results for the ultimate bearing capacity of a circular surface foundation supported by a sand layer of limited thickness underlain by a rigid rough base are presented. The circular foundation was subjected to eccentric loading (load eccentricity = e) with e/B varying from zero to 0.15 (B = diameter of foundation). Based on the model test results, a reduction factor has been proposed which can be used to estimate the ultimate load on the foundation on a limited depth of sand from the ultimate load of the foundation on the sand layer extending to a great depth.

Keywords. Bearing capacity, circular foundation, reduction factor, rigid rough base, sand.

1. Introduction

At ultimate load per unit area q_u, the failure surface in soil for a surface shallow foundation supported by a granular soil extending to a large depth will be of the type shown in Figure 1(a). In this case, which is the case of a strip foundation, the failure surface extends to a depth D below the bottom of the foundation. However, if a rough rigid base is located at a depth $H < D$, full development of the failure surface will be restricted. In such a case, the soil failure zone and the slip lines for a strip foundation at ultimate load will be as shown in Figure 1(b). Problems of this type as they relate to centrically loading conditions have been studied theoretically and/or experimentally by Mandel and Salencon [1], Milovic and Turnier [2], Meyerhof [3], Pfeifle and Das [4] and Cerato and Lutenegger [5]. The purpose of this paper is to provide some recent laboratory experimental results related to the ultimate bearing capacity of a circular model surface foundation of diameter B supported by a sand layer of limited thickness and subjected to an eccentric vertical load. The *average* load per unit area of the foundation can thus be given by the notation $q_{u(H/B,e/B)}$ where the load eccentricity is e.

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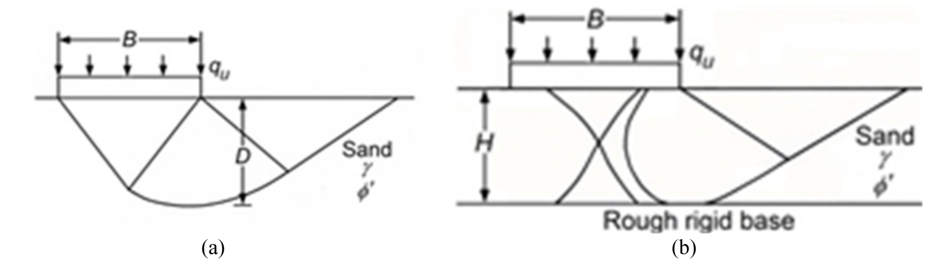

Figure 1. Failure surface under a rough rigid strip foundation: (a) with the sand extending to a large depth; (b) with a rigid rough base located at a shallow depth.

2. Literature review and problem statement

The ultimate bearing capacity of a circular foundation subjected to a centric loading and located on the surface of a sand layer of limited thickness underlain by a rigid rough base can be expressed as

$$q_{u(H/B,e/B=0)} = \frac{1}{2}\gamma B N_{\gamma}^{*} \lambda_{\gamma}^{*} \tag{1}$$

where $q_{u(H/B,e/B=0)}$=ultimate bearing capacity when the thickness of the sand bed is equal to H; B=diameter of the foundation; e=load eccentricity; γ =unit weight of sand; N_{γ}^{*}=bearing capacity factor; λ_{γ}^{*}=shape factor.

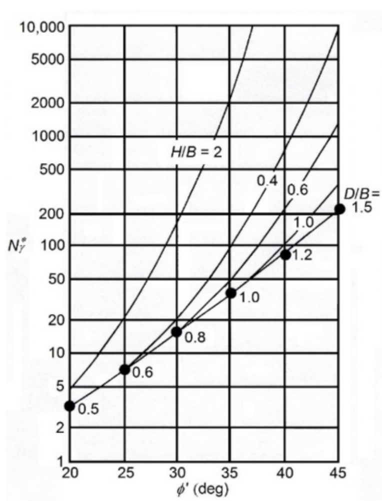

Figure 2. Variation of N_{γ}^{*} with ϕ' and H/B (based on Mandel and Salencon [1]).

The theoretical solution of N_{γ}^{*}, which is a function of H/B and the soil friction angle ϕ', has been provided by Mandel and Salencon [1] and is shown in Figure 2. For a given ϕ', the value of N_{γ}^{*} decreases with the increase in H/B and reaches a minimum value $N_{\gamma}^{*} = N_{\gamma}$ at $H/B \geq D/B$.

Again, a theoretical variation of λ_γ^* with ϕ' for rectangular foundations has been proposed by Meyerhof [3] which can be expressed as

$$\lambda_\gamma^* = 1 - m\left(\frac{B}{L}\right) \tag{2}$$

where L=length of the foundation and $m = f(\phi' \text{ and } H/B)$.

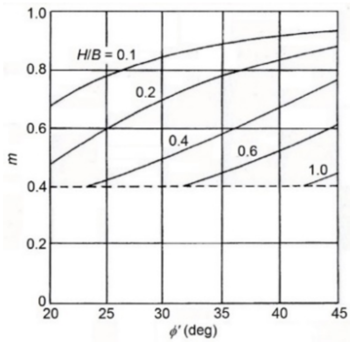

Figure 3. Variation of m with ϕ' and H/B (based on Meyerhof [3]).

The variation of m is shown in Figure 3. For a circular foundation with B=L,

$$\lambda^* = 1 - m \tag{3}$$

It is important to note that, for $H/B \geq D/B$,

$$q_{u(H/B,e/B=0)} = q_u = \frac{1}{2}\gamma B N_\gamma \lambda_\gamma \tag{4}$$

where

$$\lambda_\gamma = 1 - 0.4\left(\frac{B}{L}\right) \tag{5}$$

The bearing capacity factor N_γ has been a subject of study by many investigators over the last sixty plus years and is available in any foundation engineering textbook.

As stated earlier, the present paper relates to the estimation of the ultimate *average load per unit area* for a surface circular foundation with eccentric loading condition for $H/B < D/B$, i.e. $q_{u(H/B,e/B)}$. When H/B becomes equal to or greater than D/B, the ultimate average load per unit area of the foundation will be equal to $q_{u(e/B)}$. Based on laboratory experimental results in this study, a reduction factor R has been proposed. Or,

$$R = f\left(\frac{H}{B}\right) = \frac{q_{u(H/B,e/B)}}{q_{u(e/B)}} \tag{6}$$

The details of the experimental results and the relationship for R are given in the sections that follow.

3. Laboratory model tests

The sand used for the present experimental program was poorly graded with effective size D_{10} = 0.33 mm; C_u = 1.42, and coefficient of gradation C_c = 1.14. For the laboratory model tests, the sand was compacted in the model test box to an average unit weight of 14.36 kN/m^3 by using a raining technique at a relative density D_r of 69%. The angle of internal friction of sand ϕ' as determined from the direct shear test was 40.9°.

Model tests were conducted in a square box with inside dimensions of 0.8m × 0.8m and height of 0.65 m. All four sides of the box were made from mild steel to avoid bulging during testing. The rigid rough base was prepared by placing a 0.8 m × 0.8 m mild steel plate at the bottom of the box. The top surface was made rough by applying glue and sand mixture. Sand was poured into the test tank in layers of 25 mm from a fixed height by a raining technique to achieve the desired unit weight of compaction. The height of fall was fixed by making several trials in the test box to achieve the desired density. The model foundation was placed on the top of the sand layer. The diameter of the model foundation B was 100 mm. The bottom surface of the foundation was made rough by applying a glue-sand mixture. Load to the foundation was applied by a specially designed loading unit. The load could be applied to the model foundation in the range of 0 to 100 kN with an accuracy of 1 N. The settlement along the center line was measured by dial gauges placed on two sides of the model foundation. Two series of tests were conducted, the details of which are:

- Series I – Tests with rigid base at limited depth:
  H/B = 0.3, 0.5, 1.0, 2.0, and 3.0
  e/B = 0, 0.05, 0.1, and 0.15
- Series II – Tests with rigid base at a large depth (H/B = 5.5):
  e/B = 0, 0.05, 0.1, and 0.15

Note that H/B = 5.5 was considered to be the case with $H/B > D/B$.

4. Model test results

The *average* load per unit area applied to the foundation can be calculated as

$$q = \frac{\text{Total load}, Q}{\text{Area of the model foundation}} \tag{7}$$

For each test, plots were drawn for the average load per unit area versus average settlement, s, along the center line of the foundation. The ultimate average load per unit area [i.e. $q_{u(H/B,e/B)}$ and $q_{u(e/B)}$] thus determined from these plots are given in Col. 3 of Table 1 and Col. 2 of Table 2. For any value of H/B, the ultimate bearing capacity decreased with the increase in H/B. Figure 4 shows plots of $q_{u(H/B,e/B)}$ and $q_{u(e/B)}$ versus H/B for e/B = 0 and 0.15. For any given e/B, the magnitude of $q_{u(H/B,e/B)}$ decreased and reached a minimum at $H \approx 3B'$ (B'=effective width=$B - 2e$). For $e/B = 0$ condition, a similar observation was made by Cerato and Lutenegger [5].

Table 1. Model test results – Series I.

H/B **(1)**	***e/B*** **(2)**	$q_{u(H/B,e/B)}$ **(kN/m²) (3)**	***R*** **(*Expt*) (4)**	***R*** **[*Eq.* (*10*)] (5)**
0.3	0	880	7.59	7.69
	0.05	810	7.79	7.66
	0.10	690	7.84	7.63
	0.15	565	7.34	7.61
0.5	0	425	3.66	3.66
	0.05	390	3.75	3.63
	0.10	330	3.75	3.60
	0.15	270	3.51	3.57
1	0	194	1.67	1.74
	0.05	170	1.63	1.71
	0.10	144	1.64	1.68
	0.15	110	1.43	1.66
2	0	128	1.10	1.19
	0.05	120	1.15	1.16
	0.10	104	1.18	1.15
	0.15	92	1.19	1.10
3	0	119	1.03	1.07
	0.05	111	1.07	1.04
	0.10	94	1.07	1.01
	0.15	83	1.08	0.99

Table 2. Model test results—Series II (*H*/*B* = 5.5).

e/B **(1)**	$q_{u(e/B)}$ **(2)**
0	116
0.05	104
0.15	88
0.15	77

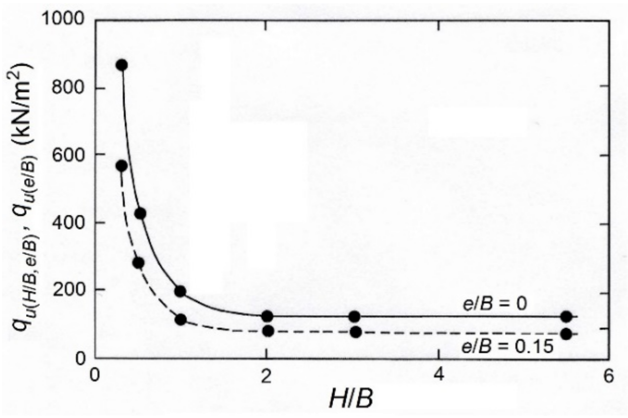

Figure 4. Variation of $q_{u(H/B,e/B=0)}$ and $q_{u(e/B)}$ with *H*/*B* (for *e*/*B* = 0 and 0.15).

4.1. Variation of N_γ^* with H/B

From Eqs. (1) and (3), the bearing capacity factor N_γ^* can be obtained as

$$N_\gamma^* = \frac{q_{u(H/B,e/B=0)}}{0.5\gamma B(1-m)} \tag{8}$$

Using the experimental values of $q_{u(H/B,e/B=0)}$ given in Table 1 and the values of m interpolated from Figure 3 (for $\phi' = 40.9°$), the experimental variations of N_γ^* with H/B have been calculated. This has been shown in Figure 5 along with the theoretical variations obtained from Figure 2. It can be seen that that the experimental N_γ^* plot is higher than that obtained from theory (Figure 2). This observation is not unusual and has been the subject of discussion over the last fifty years. A good discussion on this topic may be found in DeBeer [6]. It is important to note, however, that the theoretical value of N_γ^* reaches a minimum $(= N_\gamma)$ at $H/B = D/B \approx 1.3$; whereas, for the experimental values of N_γ^*, it is at $H/B = D/B \approx 3$.

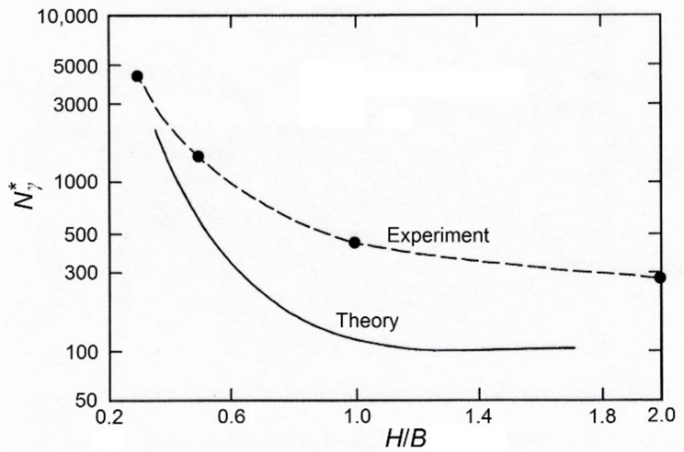

Figure 5. Variation of N_γ^* with *H*/*B*.

4.2. Reduction Factor

A nondimensional relationship for reduction factor R has been defined in Eq. (6). It appears that the general form of R can be expressed as

$$R = \left[a_1 \left(\frac{H}{B} \right)^{a_2} + a_3 \left(\frac{e}{B} \right) + a_4 \right] \tag{9}$$

With the experimental values of R (Col. 4, Table 1), nonlinear regression analyses (NLREG) were performed to obtain the magnitude of a_1, a_2, a_3, and a_4. NLREG performs statistical regression analysis to estimate the values of parameters for linear, multivariate, polynomial, logistic, exponential, and general nonlinear functions. The regression analysis determines the values of the coefficient that cause the function to best fit the observed data that are being provided. The values thus obtained are: $a_1 = 0.78$, $a_2 = -1.79$, $a_3 = -0.56$, and $a_4 = 0.96$.

Thus, substituting the values of a_1, a_2, a_3, and a_4 into Eq. (9),

$$R=\left[0.78\left(\frac{H}{B}\right)^{-1.79}-0.56\left(\frac{e}{B}\right)+0.96\right] \qquad \text{(for } H/B \le 3\text{)} \tag{10}$$

For comparison purposes the predicted values of R obtained using Eq. (11) are shown in Col. 5 of Table 1. There is good agreement between the experimental (Col. 4) and predicted (Col. 5) values of R.

5. General comments

In a practical application, when required to estimate the *ultimate load* of a circular foundation, $Q_{(H/B,e/B)}$, supported by a sand layer of limited thickness and subjected to eccentric loading, one can use the relationship

$$Q_{(H/B,e/B)}=RQ_{(e/B)} \tag{11}$$

where $Q_{(e/B)}$ is the *ultimate load* on the foundation with eccentric load application with $H/B \ge D/B$. This can be done with the procedure available in the existing literature.

There may be some concern about the possible scale effects in this type of study; however, the authors feel that the reduction factor given by Eq. (10) is a ratio of average load per unit area obtained from laboratory model tests. Thus, the scale effects may be minimal in the determination of the ultimate failure load, $Q_{(H/B,e/B)}$.

It is important to point out the following:

- For practical design and construction purposes in the field, this type of study is applicable for shallow foundations that are to be located on a relatively thin *dense sand layer* (considering the relative density and soil friction angle of sand during the tests).
- As in any geotechnical problem, a reasonable estimate of ±10% to 15% variation to Eq. (10) may be taken into consideration for design which should account for the possible variability with soil gradation and compactibility.
- At this time, large-scale field test results are not available in the literature. Future large-scale tests, if and when available, may be used to verify/modify Eq (10).

6. Conclusions

Laboratory model test results for the ultimate load carrying capacity of an eccentrically loaded circular surface foundation resting on a sand layer of limited thickness underlain by a rigid rough base have been presented. Based on the test results, the following conclusions can be drawn.

- The average ultimate load per unit area [$Q_{(H/B,e/B)}$] decreases with the increase in H/B and reaches a minimum value at $H/B \approx 3$.

- Based on the experimental results, a nondimensional reduction factor *R* has been derived which is a function of *H*/*B* (for $H/B \leq 3$).
- The reduction factor can be used to estimate $Q_{(H/B,e/B)}$ from $Q_{(e/B)}$.

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