Geotechnical Engineering in the XXI Century: Lessons learned and future challenges
N.P. López-Acosta et al. (Eds.)

doi:10.3233/STAL190053

Shear Modulus of a Saturated Sand Subjected to Cyclic Loading

Hernán PATIÑO [a,b,1] and Eliana MARTÍNEZ [a]
[a] *Technical University of Madrid, Spain*
[b] *Colombian Institute of Geotechnical Researches*

Abstract. The objective of this paper is to present the results of 120 determinations of the small strain shear modulus (G) of several specimens of saturated sand (20-40 Ottawa sand) for different conditions of relative density (D_r), effective consolidation pressure (σ'_c) and level of torsional excitation (T_e). The equipment used for the execution of the tests was a resonant column apparatus. The tests were performed on reconstituted specimens with relative density (D_r) values ranging from 20 to 80%; effective confining pressures (σ'_c) varying from 50 to 300 kPa. Specimens were subjected to torsional excitations of 0.025, 0.05, 0.1, 0.2 and 0.4 volts (V) producing shear strains (γ) between 0.002% and 0.023%. The results led to very simple empirical expressions for the shear modulus as a function of the angular strain for different effective consolidation pressures and void-ratio values.

Keywords. Resonant column; Shear stiffness modulus; Relative density; Void ratio; Effective consolidation pressure; Torsional excitation.

1. Introduction

The dynamic behavior of granular media has been intensively studied around the world in the course of several decades and new results have been obtained as a consequence of various research programs that are disseminated through proceedings of international conferences and indexed journals related to geotechnical engineering. Since the information available related to the dynamic behavior of granular media is abundant and the topics dealt with are commonplace, in this paper reference will be only made to some references directly related to resonant column tests.

Relevant investigations published in the literature, in the last 25 years, include the following: Development of a field resonant-column test in a gravel deposit [1]. Evaluation of the influence of the strain rate in the determination of the shear modulus of granular soils [2]. Proposal of an empirical function (a potential expression) between the shear modulus and the isotropic consolidation stresses (σ') [3]. Development of a resonant column apparatus that allows testing of specimens up to 15 cm in diameter that was used to study the dynamic properties of sand and gravel [4]. Analysis of the influence of the history of dynamic loading on the properties of dry sands [5]. Research with 27 types of clean sand to evaluate the influence of the coefficient of uniformity and of the grain size distribution, in the determination of the maximum shear stiffness modulus; the results obtained indicate that for equal values of the void ratio and of the effective

[1] Correspondence to: ETSICCP, C/Profesor Aranguren s/n, Madrid 28040, Spain.
E-mail: hernan.patino@upm.es.

consolidation pressure, the maximum shear stiffness modulus decreases as the coefficient of uniformity increases, whereas it becomes independent of the mean particle size [6], [7] and [8]. Study of the influence of the index properties in the determination of the dynamic parameters of a saturated granular medium [9]. Research that found out that, within the range of small deformations, the shear stiffness modulus varies very little in terms of particle size [10]. Evaluation of the Young's modulus of a quartzitic sand derived from flexural resonant column tests [11].

Taking into account the background information referred to before, the objective of this investigation is to study in depth the factors influencing the relative density, the effective consolidation pressure and the magnitude of the torsional excitation for evaluating the shear stiffness modulus of a saturated sand and to develop simple empirical functions to correlate these parameters.

2. Material used

Tests were carried out using 20-40 Ottawa sand (which is standard material of this and many other investigations related to the behavior of granular media [12]); its main characteristics are: very hard uniform particles, fine and rounded grains and quartz-type mineralogy.

The index properties of particles passing sieve No. 20 and retained in sieve No. 40 are as follows: specific gravity G_s=2.669, maximum void ratio $e_{max.}$=0.754 [13], and minimum void ratio $e_{min.}$=0.554 [13]. Based on this information the initial properties of the specimens tested are presented in Table 1.

Table 1. Properties of specimens tested.

Relative Density, D_r, %	Height, H, mm	Diameter, D, mm	Weight, W, g	Dry density, γ_d, g/cm^3	Initial Void ratio, e_i
20	105	49.5	314.59	1.557	0.714
40	105	49.5	322.13	1.594	0.674
60	105	49.5	330.05	1.633	0.634
80	105	49.5	338.36	1.675	0.594

3. Description of the equipment used

The resonant column is an apparatus consisting in a forced oscillation system with a single degree of freedom. It makes the specimen vibrate within a range of frequencies in which its first natural mode can be found. In this particular case, the specimen remained fixed at its base and it is free to vibrate at its upper end.

Testing was performed with the resonant column device manufactured by an English Company; a full description can be found in [14].

4. Experimental program

This investigation was aimed at determining the effect of D_r, σ'_v, and the magnitude of the torsional excitation on determination of G.

A total of 120 resonant column tests were conducted on reconstituted saturated specimens measuring 49.5 mm in diameter and 105 mm in height. The dimensions of the specimens result in a height-to-diameter ratio approximately equal to 2, thus eliminating the uncertainty related to the slenderness of the specimens [15]; the ratio specimen diameter to particle diameter was of about 120, therefore eliminating the scale effect [15].

The total number of tests is a result of the combination of D_r values equal to 20, 40, 60 and 80%, σ'_v values equal to 50, 100, 150, 200, 250 and 300 kPa. and amplitudes of sinusoidal waves equal to 0.025, 0.05, 0.1, 0.2 and 0.4 volts. In all the tests the backpressure used to saturate the specimens testing was equal to 400 kPa.

5. Preparation and setting of specimens

Great attention was given to the need to reproduce specimens complying with a certain relative density and therefore the setting process was very careful and repetitive. The preparation of all the samples was made by the pluviation method [16].

6. Results

The results obtained make it possible to evaluate the effect of D_r, σ'_v and γ on the determination of G using resonant column testing.

6.1. Variation of G_0 with γ_s

Figure 1 shows plots depicting the variation of G, as a function of the γ for σ'_v values varying from 50 to 300 kPa and 20% D_r. It can be observed that in the range of small deformations (0.002–0.023%) the degradation of G (or an increase of the inverse value of G) can be approximated using a linear function. This linear-type degradation recurs within the range of relative densities, from 20 to 80%, and effective confining pressures, from 50 to 300 kPa, studied herein.

As similar trends were obtained with D_r between 20 and 80%, only the plots corresponding to 20% D_r are shown.

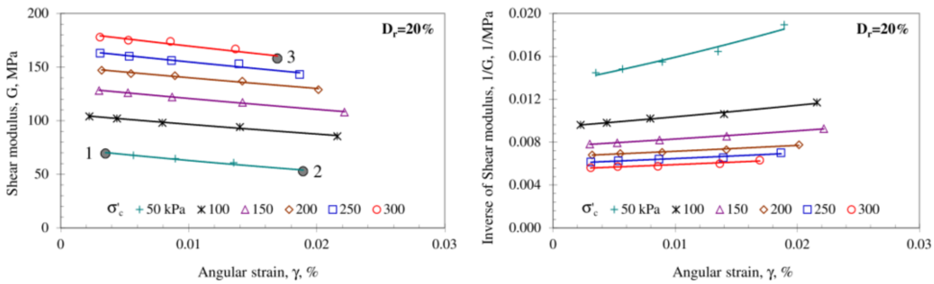

Figure 1. Typical variation trends of the shear modulus G, and its inverse 1/G as a function of the angular strain (γ) for different effective consolidation pressures and relative densities of 20%.

The degradation of the shear modulus (or the inverse) as a function of the angular strain has low values and reaches a maximum of 24% when the relative density is equal to 20%, the effective consolidation pressure amounts to 50 kPa and the angular strain increases from 0.0035% to 0.019% (large dots 1 and 2, Figure 1).

Hardin and Drnevich proposed a mathematical model to simulate the degradation of G with γ,

$$\frac{1}{G} = \frac{1}{G_o}\left(1 + \frac{\gamma}{\gamma_{ref}}\right) \tag{1}$$

where G is the shear modulus, γ is the shear strain and G_o and γ_{ref} are the two parameters of the model [17].

In order to find values of G_o and γ_{ref} of the Hardin and Drnevich model that can contribute to better analyze the results of the investigation, a diagram of 1/G versus γ, Figure 1, has been plotted and the best fit for straight lines was obtained. The resulting G_o and γ_{ref} values are given in Table 2.

Table 2. Values of G_o and γ_{ref} that best fit the test results.

D_r	Values of G_o (MPa) Consolidation pressure σ'_c (kPa)						Values of γ_{ref} (%) Consolidation pressure σ'_c (kPa)					
	50	100	150	200	250	300	50	100	150	200	250	300
20	75.8	107.5	133.3	151.5	166.7	185.2	0.045	0.092	0.100	0.124	0.122	0.118
40	78.1	116.3	140.9	161.3	178.6	192.3	0.068	0.103	0.114	0.124	0.114	0.147
60	78.7	109.9	137.0	158.7	175.4	192.3	0.062	0.104	0.124	0.141	0.118	0.122
80	80.0	119.1	144.9	166.7	185.2	204.1	0.060	0.085	0.106	0.124	0.130	0.107

The low sensitivity of G_o can be attributed to the nature of the sand comprising hard quartz grains (rounded and very uniform in size), which implies only little variation between the maximum and minimum void ratios.

6.2. Variation trend of G_o with σ'_c

The values of the shear modulus for small strains, G_o, obtained as indicated in the previous paragraph, are given in Table 2. It is clear that for each value of the relative density, the value of G_o increases as the consolidation pressure increases. See Figure 2. Usually, the relation among these values (G_o and σ'_c) is thought to be of the type.

$$G_o = M \cdot p_o \left(\frac{\sigma'_c}{p_o}\right)^N \tag{2}$$

where M is a “modulus number”, N is an “exponent number” and p_o is the value of a standard reference pressure. For this investigation a typical value of p_o = 98.1 kPa is used.

In order to investigate whether the expression (2) is applicable to this particular case, values of G_o/p_o were plotted versus the corresponding values of σ'_c/p_o on a log-log diagram. Figure 2 is the plot that corresponds to D_r = 20%.

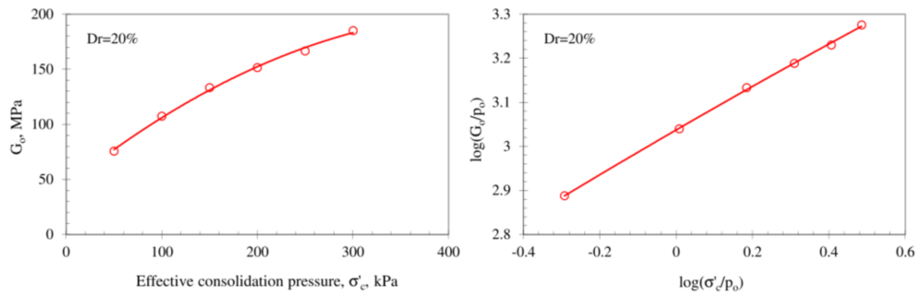

Figure 2. Values of G_o and σ'_c and double log plot of G_o/p_o and σ'_c/p_o, for D_r = 20%.

From this type of plot, the model parameters can be automatically obtained from the data fit, made by minimizing the sum of the squares of the deviations of the test results that correspond to each relative density. The resulting M and N values are given in Table 3.

It is known that the void ratio is a better parameter to analyze the dynamic moduli of sands than the relative density. During the consolidation process, some change in the volume takes place that reduces the initial void ratio (e_i); the values of the final void ratio (e_f), after consolidation, for each relative density are given in Table 3.

Table 3. Dimensionless model parameters M and N and e_f.

Relative density	M	N	e_f
20 %	1084	0.495	0.71
40 %	1143	0.501	0.67
60 %	1122	0.502	0.63
80 %	1175	0.517	0.59

Variations of the parameters M and N are now plotted in Figure 3 as a function of the final void ratio.

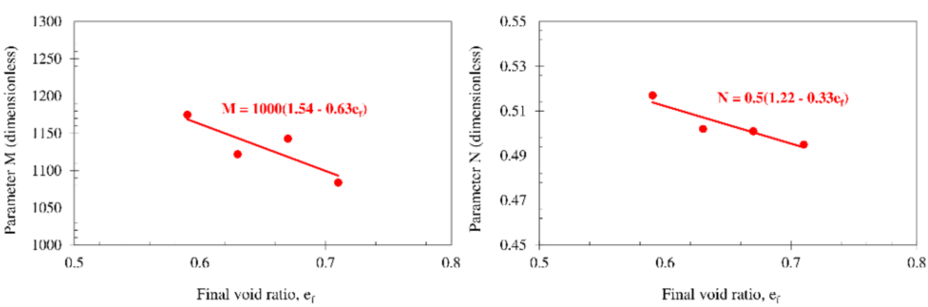

Figure 3. Values of the model modulus number (M) and of the model modulus exponent (N) as a function of the final void ratio (e_f).

6.3. Variation trend of γ_{ref} with σ'_c

From the values of γ_{ref} in Table 3 it seems that this parameter could be considered to be a constant but only for consolidation pressures above 200 kPa; for lower values of the consolidation pressure, the value of γ_{ref} decreases and it can no longer be considered as a constant in an hypothetical mathematical model.

A better option, which would account for the effect of large G degradation rates for lower values of the consolidation pressure, could be based on considering γ_{ref} as a variable that depends on the consolidation pressure.

It is known that G_o increases with the square root of σ'_c and, as a consequence, a value of γ_{ref} increasing with the square root of σ'_c should be expected.

For this reason, a value of γ_{ref} can be found to reasonably fit the data with an expression involving $(\sigma'_c/p_o)^{0.5}$. The best fit is given in Figure 4. As we can see, the relative density also has an effect on γ_{ref}, but it is not easy to draw a clear figure showing the effect of the relative density at present.

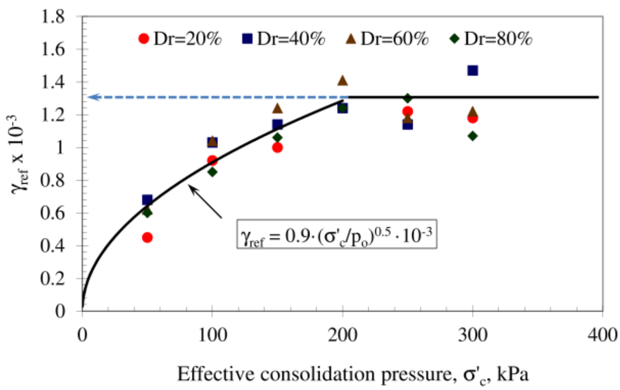

Figure 4. Reference values of the shear deformation.

This should be valid for values of γ with the interval ($2 \times 10^{-5} < \gamma < 23 \times 10^{-5}$) and for the range of densities of this particular investigation and for consolidation pressures lower than 200 kPa.

7. Proposal

The interpretation of the data obtained by testing the 120 samples of 20–40 Ottawa sand leads to proposing the following value for the shear modulus:

$$G = \left(1 + \frac{1000\cdot\gamma}{0.9\left(\frac{\sigma'_c}{p_o}\right)^{0.5}}\right)^{-1} \cdot M\left(\frac{\sigma'_c}{p_o}\right)^{N} \cdot p_o \tag{3}$$

where the main variables are the shear strain (γ) and the consolidation pressure (σ'_c); in this expression, $p_o = 98.1$ kPa is a reference pressure and the values of the dimensionless parameters are $M = 1000\ (1.54 - 0.63e_f)$ and $N = 0.5\ (1.22 - 0.33e_f)$, Figure 3.

A detailed analysis of the empirical function (3) compared with that proposed in [12], can be consulted in [18].

8. Conclusions

A total of 120 saturated specimens of Ottawa sand were tested in a resonant-column apparatus under angular strains (γ) of 0.002 to 0.023%, relative densities ranging from 20 to 80% and effective consolidation pressures (σ'_c) between 50 and 300 kPa. The main conclusions derived are as follows:

- Relative density is the factor with the least influence on the obtained shear modulus. Following in importance are the angular strains.
- The greatest influence in the value of the shear modulus is the effective consolidation pressure.
- A simple empirical expression is proposed for G as a function of γ and σ'_c, for the range of γ tested.
- For the same γ and D_r, the variation trend of G, as a function of σ'_c, can be fit to a potential function with a correlation coefficient practically equal to unity.
- A simple expression (3) is proposed for G, also taking into account the final void ratio (e_f) values. This research has explored a somewhat narrow range of e_f and therefore this proposal could be less precise than those made to consider other effects.
- Although the material used was saturated Ottawa sand, the model defined in this paper might be valid in the case of other uniform granular soils with fine to medium rounded grains and quartz origin.

References

[1] Koono T., Suzuki Y., Tatoishi A., Ishihara K., Akino K. & Satsuo I. (1993). "Gravelly soil properties by field and laboratory tests", Proceedings of the 3rd International Conference on Case Histories in Geotechnical Engineering, St Louis, Missouri, Vol. 1: 575-594.

[2] Lo Presti D.C.F., Jamiolkowski M., Pallara O., Cavallaro A. & Pedroni S. (1997). "Shear modulus and damping of soils", Géotechnique, 37, No. 3: 603-617.

[3] Díaz-Rodríguez J.A. & López-Flores L. (1999). "A study of microstructure using resonant-column tests", Proceeding of the Second International Conference on Earthquake Geotechnical Engineering, Lisbon, Portugal: 89-94.

[4] Menq F. (2003). "Dynamic properties of sandy and gravelly soils", Ph. D. Dissertation, University of Texas, Austin.

[5] Wichtmann T. & Triantafyllidis T. (2004). "Influence of a cyclic and dynamic loading history on dynamic properties of dry sand, part I: Cyclic and dynamic torsional prestraining", Soil Dynamics and Earthquake Engineering, 24: 127–147.

[6] Wichtmann T. & Triantafyllidis T. (2009). "Influence of the Grain-Size Distribution Curve of Quartz Sand on the Small Strain Shear Modulus Gmax", Journal of Geotechnical and Geoenvironmental Engineering, ASCE, Vol. 135 (10): 1404-1418.

[7] Wichtmann T. & Triantafyllidis T. (2013). "Effect of Uniformity Coefficient on G/Gmax and Damping Ratio of Uniform to Well-Graded Quartz Sands", Journal of Geotechnical and Geoenvironmental Engineering, ASCE, Vol. 139(1): 59-72.

[8] Wichtmann T. & Triantafyllidis T. (2014). "Stiffness and Damping of Clean Quartz Sand with Various Grain-Size Distribution Curves", Journal of Geotechnical and Geoenvironmental Engineering, Technical Note, ASCE, Vol. 140: 1-4.

[9] Martínez E. (2012). "Influencia de la densidad relativa, índice de poros, presión de consolidación y amplitud de excitación en los parámetros dinámicos de una arena saturada (Master tesis), Universidad Complutense de Madrid.

[10] Yang J. & Gu X.Q. (2013). "Shear stiffness of granular material at small strains: Does it depend on grain size", Géotechnique 63, 2: 165-179. DOI: http:// dx.doi.org/10.1680/geot.11.P.083

[11] Madhusudhan B.N. & Senetakis K. (2016). "Evaluating use of resonant column in flexural mode for dynamic characterization of Bangalore sand", Soils and Foundations, 56(3):574–580.

[12] Hardin B.O. & Drnevich V.P. (1972). "Shear Modulus and Damping in Soils: Design Equations and Curves. ASCE, Journal of the Soil Mechanics and Foundations Division 98, 7, 667-692.
[13] ASTM Designation D2049-69. "Test Method for Relative Density of Cohesionless Soils".
[14] https://www.controls-group.com/eng/soil-mechanics-testing-equipment//resonant-column.php
[15] ASTM Designation: D4015 – 92 (Reapproved 2000). "Standard Test Methods for Modulus and Damping of Soils by the Resonant-Column.
[16] Fretti C., Lo Presti D. & Pedroni S. (1995). "A Pluvial Deposition Method to Reconstitute Well-Graded Sand Specimens", Goetechnical Testing Journal, Vol. 18, No. 2: 292-298.
[17] Hardin, B.O., Richart, F.E. 1963. Elastic wave velocities in granular soils", ASCE, Journal of the Soil Mechanics and Foundations Division 98, 1: 33-65.
[18] Patiño H., Martínez E., González J. & Soriano A. (2017). "Shear modulus of a saturated granular soil derived from resonant-column tests", Acta Geotechnica Slovenica, 2: 33-45.