

Evaluating Rigidity Index, OCR, and s_u from Dilatometer Data in Soft to Firm Clays

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Abstract. The rigidity index is defined as the ratio of shear modulus to shear strength ($I_R = G/\tau_{max}$) and serves as an important input parameter for geotechnical applications involving bearing capacity, pile driving, porewater pressure generation, and dissipation problems. First, a hybrid formulation of spherical cavity expansion and critical state soil mechanics (SCE-CSSM) for obtaining the operational undrained I_R from piezocone penetration (CPTu) data is introduced, and the approach is extended to evaluate I_R using flat plate dilatometer test (DMT) data based on an established link between the DMT and piezocone (CPTu) data in soft-firm clays. This I_R can be used to estimate profiles of overconsolidation ratio (OCR) and undrained shear strength (s_u) of soft to firm clays using the same SCE-CSSM framework. Two case studies covering well-behaved clays with different geologies from the UK and USA are used to demonstrate the effectiveness of the proposed methodology for obtaining I_R , OCR, and s_u .

Keywords. Clays, flat plate dilatometer, spherical cavity expansion, critical state soil mechanics.

1. Introduction

The flat plate dilatometer test (DMT) is a quick, reliable, repeatable and robust in-situ test to collect information to investigate subsurface characteristics. The data collected also allows for the interpretation of geoparameters (e.g. strength and stiffness) through empirical correlations in an efficient, economic, and expedient manner. The DMT involves pushing a stainless-steel blade, with a 60mm diameter circular flexible steel membrane mounted on one side, into the ground at depth increments varying from 20 to 30 cm (sometimes at 25 cm intervals and one-foot increments are also common). Two pressure measurements, namely p_0 = contact pressure and p_1 = expansion pressure, are recorded at each depth by inflating the membrane with nitrogen. Details concerning the DMT equipment, field test procedures, and interpretations are provided by Marchetti [1].

The soil rigidity index is defined as the ratio of shear modulus (G) to shear strength (τ_{max}), thus: $I_R = G/\tau_{max}$. Its value is difficult to determine since G is strain-dependent varying from small-strains ($10^{-4}\%$) to large strains (e.g. 20%). Yet, I_R is an important input parameter for geotechnical calculations of bearing capacity for shallow and deep foundations, pile driving, and porewater pressure generation. It is also required for evaluating the coefficient of consolidation (c_{vh}) from piezocone dissipation tests using

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solutions based on cavity expansion, strain path method, and finite element method (FEM). Agaiby & Mayne [2] utilized data from piezocone penetration tests (CPTu) to evaluate the undrained I_R of clays for determining the horizontal coefficient of consolidation (c_{vh}) from piezocone dissipation tests at several field sites.

In practice, OCR and s_u of clays are estimated from DMT data with empirical correlations. However, a nexus between CPTu and DMT measurements in soft-firm clays with low OCRs < 2.5 was established using spherical cavity expansion (SCE) theory [3]. This link between the DMT and CPTu in clays is further explored in this paper to develop an approach to obtain undrained I_R from DMT data using a hybrid formulation of spherical cavity expansion and critical state soil mechanics (SCE-CSSM). This allows determination of both OCR and s_u with an analytical-based approach. The methodology is applied to two case studies.

2. SCE-CSSM solution for undrained rigidity index for soft to firm clays

2.1. Original SCE-CSSM Solution

A hybrid formulation based on spherical cavity expansion theory and critical state soil mechanics (SCE-CSSM) defines the cone tip resistance (q_t) and porewater pressure (u_2) with the following Eqs. [4] [5]:

$$q_t = \sigma_{v0} + [(4/3) \cdot (\ln I_R + 1) + \pi/2 + 1] \cdot (M/2) \cdot (OCR/2)^\Lambda \cdot \sigma_{v0}' \tag{1}$$

$$u_2 = u_0 + [(2/3) \cdot (\ln I_R + 1) \cdot M \cdot (OCR/2)^\Lambda \cdot \sigma_{v0}'] + [1 - (OCR/2)^\Lambda] \cdot \sigma_{v0}' \tag{2}$$

where q_t is the cone resistance, σ_{v0} = total overburden stress, u_2 = measured porewater pressure at the shoulder position of the piezocone, u_0 = hydrostatic porewater pressure, $M = (6 \sin \phi') / (3 - \sin \phi')$ = slope of the frictional envelope for triaxial compression in q - p' space, $OCR = (\sigma_p' / \sigma_{v0}')$ = overconsolidation ratio, σ_p' = preconsolidation stress and σ_{v0}' = effective overburden stress. The parameter $\Lambda = (1 - C_s / C_c)$ = plastic volumetric strain potential, where C_s = swelling index, C_c = virgin compression index. Typically, the value of $\Lambda \approx 0.8$ to 1.0 for most clays.

The hybrid SCE-CSSM model can be rearranged to obtain OCR in terms of the net cone resistance ($q_{net} = q_t - \sigma_{v0}$), excess pore pressure ($\Delta u = u_2 - u_0$) and effective cone resistance ($q_E = q_t - u_2$), as follows:

$$OCR = 2 \cdot \left[\frac{(2/M) \cdot (q_{net} / \sigma_{v0}')}{4/3 \cdot (\ln I_R + 1) + \pi/2 + 1} \right]^{1/\Lambda} \tag{3}$$

$$OCR = 2 \cdot \left[\frac{(\Delta u / \sigma_{v0}') - 1}{2/3 \cdot M \cdot (\ln I_R) - 1} \right]^{1/\Lambda} \tag{4}$$

$$OCR = 2 \cdot \left[\frac{1}{1.95 \cdot M + 1} \left(\frac{q_t - u_2}{\sigma_{v0}'} \right) \right]^{1/\Lambda} \tag{5}$$

Combining Eq. (3) and (4), the value of the rigidity index can be obtained in terms of normalized CPTu measurements and friction parameter M :

$$I_R = \exp \left[\frac{1.5 + 2.925M \cdot (U^* - 1)/Q}{M - M(U^* - 1)/Q} \right] \tag{6}$$

where $Q = \text{normalized tip resistance} = q_{net}/\sigma_{vo}'$ and $U^* = \text{normalized porewater pressure parameter} = \Delta u_2/\sigma_{vo}'$. Note that the alternate and more common porewater pressure parameter, $B_q = \Delta u/q_{net}$ [5] is determined simply as $B_q = U^*/Q$.

In addition, an alternative representation for (6) can be obtained in the following format:

$$I_R = \exp \left[\frac{1.5 + 2.925M \cdot a_q}{M - M \cdot a_q} \right] \tag{7}$$

where $a_q = (U^* - 1)/Q$. Hence, a_q can be determined as a single value for any clay layer or deposit by taking the slope of a plot of the parameter $(U^* - 1)$ versus Q , or alternatively taken as the slope of $(u_2 - \sigma_{vo})$ versus $(q_t - \sigma_{vo})$. The method only applies at sites with shallow groundwater level since a requirement for performance is that $u_2 - \sigma_{vo} > 0$ where the water pressure recorded by the CPTu is higher than the total overburden stress.

2.2. Nexus between CPTu and DMT to determine I_R

Ouyang & Mayne [4] derived two links between measurements taken by CPTu and DMT in soft to firm clays using spherical cavity expansion theory (SCE). As such, an equivalent net cone resistance (q_{netDMT}) and excess porewater pressure (Δu) from DMT's soundings can be expressed:

$$q_{netDMT} = 2.93p_1 - 1.93p_0 - u_0 \tag{8}$$

$$\Delta u_{DMT} = p_0 - u_0 \tag{9}$$

where p_0 and p_1 are the contact and expansion pressures from the DMT, respectively.

This nexus provides two links between the two tests for soils that are characterized as normally-consolidated to lightly-overconsolidated clays with OCRs between 1 to 2.5. The above expressions and associated CPTu-DMT relationships for soft to firm clays are summarized in graphical form in Figure 1.

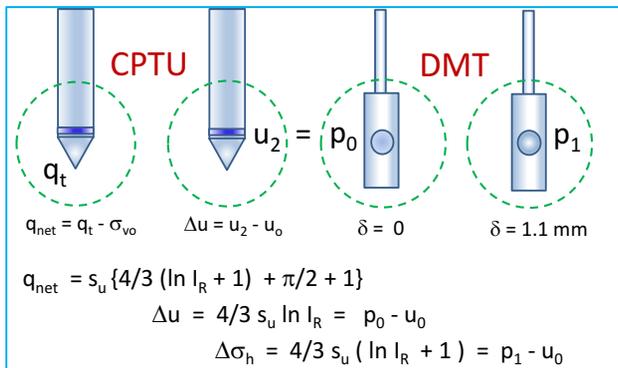


Figure 1. Summary of the nexus between CPTu and DMT readings in soft to firm clays using spherical cavity expansion solutions.

From the relationship, the undrained I_R derived from Eq. (7) can be re-expressed in terms of the DMT readings p_0 and p_1 :

$$I_R = \exp \left[\frac{1.5 + 2.925M \cdot a_{q\text{-DMT}}}{M - M \cdot a_{q\text{-DMT}}} \right] \quad (10)$$

where $a_{q\text{-DMT}} = (U^*_{\text{DMT}} - 1)/Q_{\text{DMT}}$. The equivalent normalized porewater pressure parameter (U^*) and equivalent normalized resistance (Q) from the DMT are found as:

$$U^*_{\text{DMT}} = (p_0 - u_0)/\sigma_{v0}' \quad (11)$$

$$Q_{\text{DMT}} = (2.93p_1 - 1.93p_0 - u_0)/\sigma_{v0}' \quad (12)$$

As noted earlier, $a_{q\text{-DMT}}$ can be determined as a single value for any clay deposit by taking the slope of a plot of the parameter $(U^*_{\text{DMT}} - 1)$ versus Q_{DMT} . Alternatively, the value of $a_{q\text{-DMT}}$ corresponds to the slope of a plot of the DMT porewater pressure difference, $\Delta u_{\text{DMT}} = (p_0 - \sigma_{v0})$ versus the DMT equivalent net cone resistance, $q_{\text{net-DMT}} = (2.93p_1 - 1.93p_0 - u_0)$.

If ϕ' values from triaxial tests are unavailable, the magnitude of ϕ' can be evaluated in soft to firm clays using the DMT-NTH solution [5, 6]. This is an effective stress limit plasticity solution for undrained CPTu penetration developed by Senneset et al. [6]. Because of the SCE link, ϕ' can be evaluated with the following closed-form solution based on DMT measurements:

$$Q_{\text{DMT}} = \frac{\tan^2(45^\circ + \phi'/2) \cdot \exp(\pi \tan \phi') - 1}{1 + 6 \tan \phi' \cdot (1 + \tan \phi') \cdot (U^*_{\text{DMT}}/Q_{\text{DMT}})} \quad (13)$$

For the inversion solution, an approximate expression for DMTs in inorganic and insensitive clays with OCRs < 2.5 allows a direct assessment of ϕ' from:

$$\phi' \approx 29.5 \cdot (U^*_{\text{DMT}}/Q_{\text{DMT}})^{0.121} [0.256 + 0.336 \cdot (U^*_{\text{DMT}}/Q_{\text{DMT}}) + \log Q_{\text{DMT}}] \quad (14)$$

which is valid for the following ranges: $20^\circ \leq \phi' \leq 45^\circ$ and $0 \leq U^*_{\text{DMT}} \leq 4$.

2.3. Undrained shear strength evaluation

The undrained shear strength of clays (s_u) can be evaluated from the equivalent $q_{\text{net-DMT}}$ and a bearing capacity factor (N_{kt}) that depends upon the magnitude of I_R [3]. Therefore, s_u can be obtained from:

$$s_u = q_{\text{net}}/N_{kt} \quad (15)$$

The SCE solution determines N_{kt} as a function of I_R with the following expression [4] that corresponds to a triaxial compression mode:

$$N_{kt} = (4/3) \cdot (\ln I_R + 1) + \pi/2 + 1 \quad (16)$$

3. Case studies validating the I_R evaluation

Two case studies are presented to illustrate the application of the DMT methodology.

3.1. Bothkennar, UK

Bothkennar soft clay is a lightly- to normally-consolidated estuarine soft clay deposit in Scotland that serves as a national test site for geotechnical experimentation in the UK [7]. The clay has OCR values decreasing with depth and ranging from 3 to 1.5 below a 2 m thick crust layer. The water table at the site is relatively shallow, typically on the order of 1 m depth. Profiles of representative DMT readings and the interpreted DMT material index parameter with depth are shown in Figure 2.

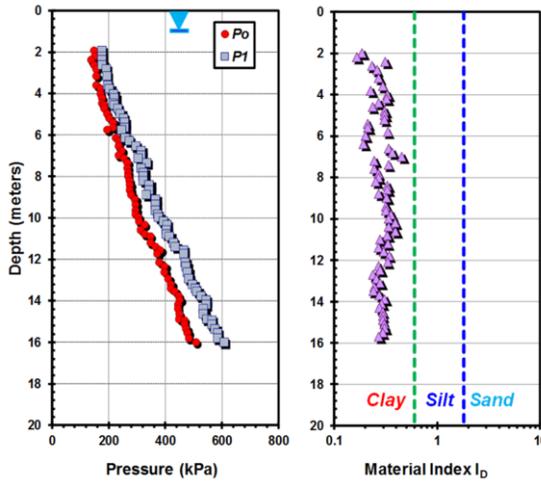


Figure 2. DMT profiles for soft Bothkennar clay.

Using Eq. (14), the NTH solution yields $\phi' = 34^\circ$, hence $M = 1.37$. This matches with results of CAUC triaxial tests on the soft clay. Figure 3 shows the evaluation of the slope parameter a_q for the Bothkennar site where $(p_0 - \sigma_{v0})$ is plotted versus $q_{\text{net-DMT}} = 2.93p_1 - 1.93p_0 - u_0$, giving a parametric value $a_q = 0.38$. This slope value is used with the effective friction angle in (10) to obtain $I_R = 39$.

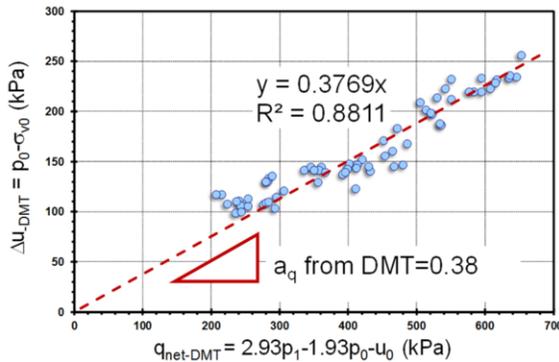


Figure 3. Evaluation of slope parameter a_q to determine I_R for Bothkennar, UK.

Figure 4 shows estimates of OCR using Eqs. (3), (4) and (5) with DMT data and $I_R = 39$, $\phi' = 34^\circ$ and $\Lambda = 1$. Overall, a good agreement is observed between theoretical estimates of σ_p' and OCR and those obtained with results of consolidations tests reported by [10]. The evaluated I_R value of 39 gives $N_{kt} = 8.9$ that provides a DMT profile of s_u in general agreement yet slightly higher than laboratory CAUC triaxial compression tests and field vane shear tests (VST) at the site.

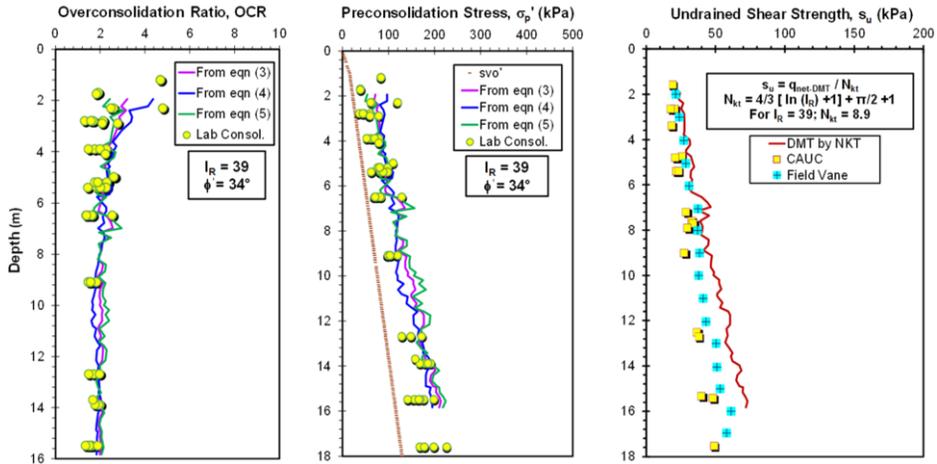


Figure 4. OCR and preconsolidation stress profiles using the SCE-CSSM framework for Bothkennar, UK.

3.2. Northwestern University, IL

The National Geotechnical Experimentation Site (NGES) located at Northwestern University (NWU) in Evanston, Illinois serves as a testing grounds for research. The site is underlain by a 10m thick sand layer over 12 m of soft Chicago clay, which overlies other deeper sedimentary soil layers. The soft clay layer has been a focus of laboratory testing by NWU for several decades [8].

For the soft Chicago clays at Northwestern University, the measured pressures from a representative DMT conducted by Georgia Tech personnel are presented in Figure 5a. The profile of interpreted effective stress friction angle using the approximate DMT-NTH equation is compared with the friction angle determined by a series of laboratory CK₀UC triaxial compression tests reported by [8], as shown in Figure 5b. It can be seen that the field DMT ϕ' is in good agreement with the laboratory value, both giving a friction angle of around $\phi' = 28.8^\circ$ ($M=1.15$).

Figure 6 illustrates the plot of $(U^*_{DMT}-1)$ versus Q_{DMT} , giving a slope value $a_q = 0.43$. Together with the characteristic friction parameter $M = 1.15$, an operational rigidity index $I_R = 103$ is determined for the soft lacustrine clay layer.

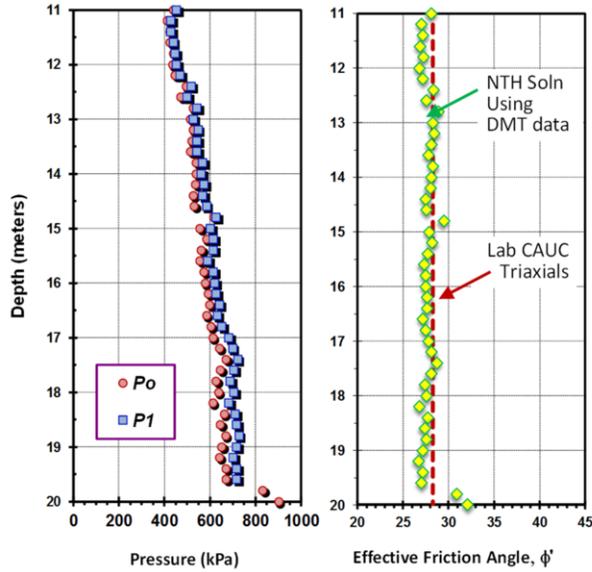


Figure 5. DMT pressures and effective friction angle ϕ' profiles in soft clay at Northwestern University.

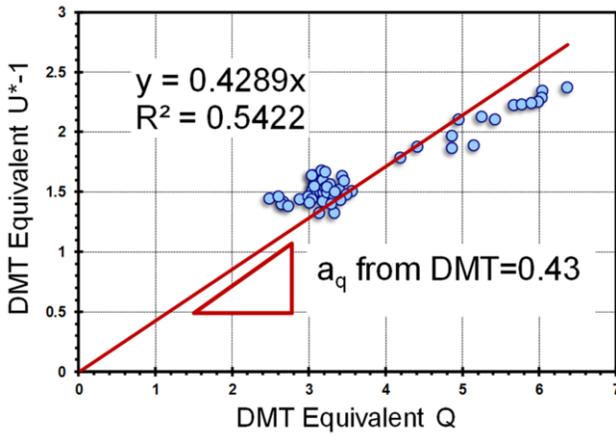


Figure 6. Evaluation of slope parameter a_q to determine I_R in soft Chicago clay at Northwestern University.

Applying the SCE-CSSM equations with $I_R = 103$ and $\phi' = 28.8^\circ$ (adopting $\Lambda = 1$), the three OCR profiles determined by Eqs. (3), (4) and (5) give good agreement with each other at depths between 11 m to 20 m, as presented in Figure 7. Very good agreement is also observed in comparison with laboratory-measured σ'_p and corresponding OCR profiles obtained from consolidation tests. The value $I_R = 103$ gives a corresponding $N_{kt} = 10.2$ which provides an excellent agreement with the CAUC undrained shear strength data reported by NWU [8].

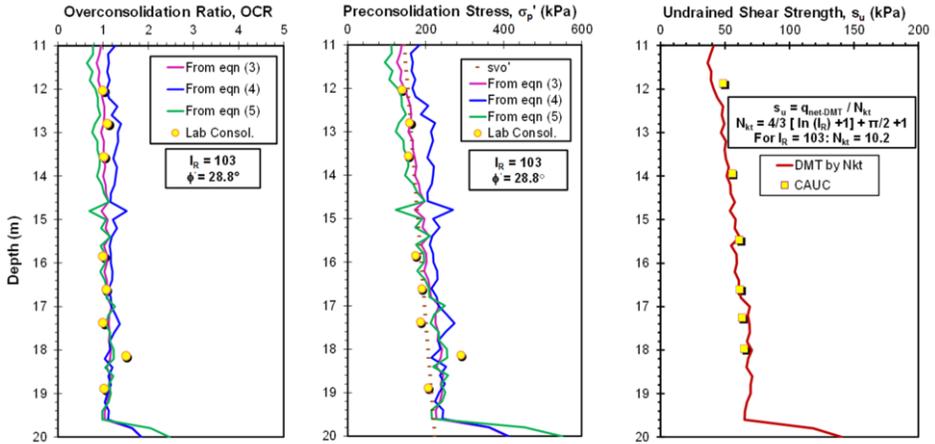


Figure 7. Interpreted profiles of OCR, preconsolidation stress, and undrained shear strength in the soft Chicago clay layer at NWU

4. Conclusions

Using a hybrid spherical cavity expansion – critical state (SCE-CSSM) framework, it is shown that an operational value of rigidity index (I_R) of soft to firm clays can be obtained from the readings from flat plate dilatometer test (DMT), namely, the contact pressure p_0 and the expansion pressure p_1 . The methodology applies to inorganic and insensitive clays with OCRs < 2.5 . The effective stress friction angle of the clay (ϕ') is obtained from a limit plasticity solution developed by NTH for the piezocone test (CPTu) and an established link from SCE permits its extension to the DMT. The two parameters together (I_R and ϕ') are used in the SCE-CSSM algorithms to assess the profiles of OCR and s_u in soft-firm clays in closed-form analytical solutions, where OCRs < 2.5 . Two case studies covering natural clays with different geologies are used to demonstrate the effectiveness of the methodology. Further research on the small-strain shear modulus G based on the I_R calculated from DMT data is warranted to compare lab determined G .

References

- [1] Marchetti, S. (1980). “In-situ tests by flat dilatometer”, *Journal of the Geotechnical Engineering Division ASCE* 106 (GT3): 299-321.
- [2] Agaiby, S.S., & Mayne, P.W. (2018). “Evaluating undrained rigidity index of clays from piezocone data”, *Proc. 4th Intl. Symposium on Cone Penetration Testing*, TU Delft, Taylor & Francis, CRC Press, 65-72.
- [3] Ouyang, Z., Mayne, P.W. (2017). “Effective friction angle of soft to firm clays from flat dilatometer tests”, *Geotechnical Engineering*. Vol. 170 (2) Proc. Institution of Civil Engineers, London: 137-147.
- [4] Mayne, P.W. (1991). “Determination of OCR in clays by piezocone tests using cavity expansion and critical state concepts”, *Soils and Foundations* 31 (2): 65-76.
- [5] Mayne, P.W. (2007). “In-situ test calibrations for evaluating soil parameters”, *Characterization & Engineering Properties of Natural Soils*, Vol. 3, Taylor & Francis, London: 1602-1652.
- [6] Senneset, K., Sandven, R. and Janbu, N. (1989) “Evaluation of soil parameters from piezocone tests”, *Transportation Research Record 1235*, National Academies Press, Washington, DC: 24-37.
- [7] Hight, D.W., Böese, R., Butcher, A.P., Clayton, C.R.I., & Smith, P.R. (1992) “Disturbance of the Bothkennar clay prior to laboratory testing”, *Géotechnique*, 42(2): 199-217.
- [8] Chung, C.K. and Finno, R.J. (1992) “Influence of depositional processes on the geotechnical parameters of Chicago glacial clays”, *Engineering Geology* 32(4):225-242.