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# Managing Alternative Patient Appointments Using P-Graph Methodology

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**Abstract.** Background: Patient appointment scheduling is one of the main challenging tasks in the healthcare administration and is constantly in the focus of theoretical researches. Objectives: The aim of this study was to investigate the applicability of the P-graph (Process graph) methodology to find the *n*-best alternative for patient's scheduling. Methods: The patient appointment scheduling task was formalised as an integer linear programming problem and was considered as a process network synthesis problem. The optimal and *n*-best alternative solutions were determined by an efficient branch and bound algorithm implemented in a decision support system. Results: Experimental results show, that the P-graph methodology can be effectively applied to produce the optimal scheduling for the examinations and to find the alternatives of the best scheduling.

**Keywords.** appointments and schedules, medical process optimisation, process graph, decision support system

## 1. Introduction

Patient appointment scheduling is a complex combinatorial optimisation problem, where a certain number of patients has to be assigned to a number of limited timeslots of physicians. In the simplest case, only one patient should be scheduled at a time, but also in this case, many constraints have to be considered arising from the availability of limited resources (doctors, rooms, test tools), or from the expectations of the patients (e.g. travel restrictions or time preferences in outpatient care). Owing to the NP-hard nature of the problem, there does not exist a general and universal solution.

To solve the problem, many different approaches have been proposed in the literature [1, 2]. Classic solutions come from operations research in which the optimality of schedules is formulated as a simple objective function, for example, as the weighted average of expected idle times for physicians and waiting times for patients, or as the time interval elapsed between the beginning of the patient's first and last examinations. The main advantage of applying operations research is that different objective functions and diverse constraints can be formulated for appointment scheduling. However, it should be noticed that alternative solutions can only be achieved in several steps with iterative adjustments of the parameters.

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To tackle the complex mathematical combinatorial optimisation problem, several metaheuristics approaches have also been published in the last decade. Some of them utilise a one-solution approach (e.g. simulated annealing [3], or hill climbing [4, 5]), while others follow a multi-solution model (e.g. genetic algorithm [6], particle swarm optimisation [7, 8]). The main disadvantage of one-solution approaches is that they tend to stuck in local optima and do not guarantee a global optimal patient-schedule time assignment. Multi-solution methods try to solve the problem of one-solution local search algorithms, as they parallel optimise several possible candidate solutions. Furthermore, while one-solution approaches provide only one possible solution in each run, the resulting set of the solutions of the multi-solution approaches can also be seen as alternative patient-appointment assignments. However, these methods do not guarantee, that the set of the resulting solutions contains the most appropriate patient-schedule time assignments. Besides the artificial intelligence-based methods, other approaches (e.g. stochastic programming [9], or dynamic programming methods [10]) have also been proposed, but these methods are also not able to find the most appropriate set of the alternative schedules.

In this paper, a fundamentally new approach is proposed to find the optimal patient-examination time assignment and its alternatives. The suggested method is based on the P-graph methodology widely applied in different process optimisation problems (e.g. chemical or business processes), but to best to our knowledge, it was earlier not utilised for patient appointment scheduling. The proposed method provides the *n*-best schedules of the required medical examinations or therapeutic treatments as a result of a single run without defining any parameters.

In the following, firstly the mathematical formalisation of the problem is given, then the fundamentals of P-graph methodology are shortly introduced, and the interpretation of the P-graph methodology for the task of patient appointment scheduling is presented. Following this, a simple case study is presented to demonstrate the applicability of the proposed method for patient appointment scheduling resulting in alternative proposals.

## 2. Methods

The appointment optimisation problem can be formalised as a linear integer programming problem where the goal is to give the optimised order of the examinations and determine the times for each appointment.

### *2.1. Problem definition*

Let $N$ be the number of different types of examinations (e.g. laboratory test, ultrasound) represented by the set $E = \{E^{(1)}, E^{(2)}, \dots, E^{(N)}\}$. Each examination type is characterized by its length in minutes denoted as $d^{(i)}$, where $d^{(i)} > 0$, and $d \in \mathbb{N}$ for each $i = 1, 2, \dots N$. The available free time-intervals ($TS$) for the examination type $E^{(i)}$ can be defined as the set of triplets as follows:

$$TS^{(i)} = \{e_{i,1},\ e_{i,2}, \dots , e_{i,m_i}\}, \tag{1}$$

where $m_i$ is the number of the free time-intervals for examination type $E^{(i)}$, and $m_i \geq 1$ for all $i = 1, \dots N$. Furthermore, $e_{i,k}$ is defined as:

$$e_{i,k} = \left(E^{(i)}, st_k^{(i)}, et_k^{(i)}\right). \tag{2}$$

A triplet $e_{i,k} = \left(E^{(i)}, st_k^{(i)}, et_k^{(i)}\right)$ denotes the $k$-th free time-interval of the examination type $E^{(i)}$ with $st_k^{(i)}$ starting time and $et_k^{(i)}$ ending time ($st_k^{(i)} < et_k^{(i)}$). As an example, a simple graphical representation of two examination types ($E^{(1)}$ and $E^{(2)}$) with their available free time-intervals is presented in Figure 1.

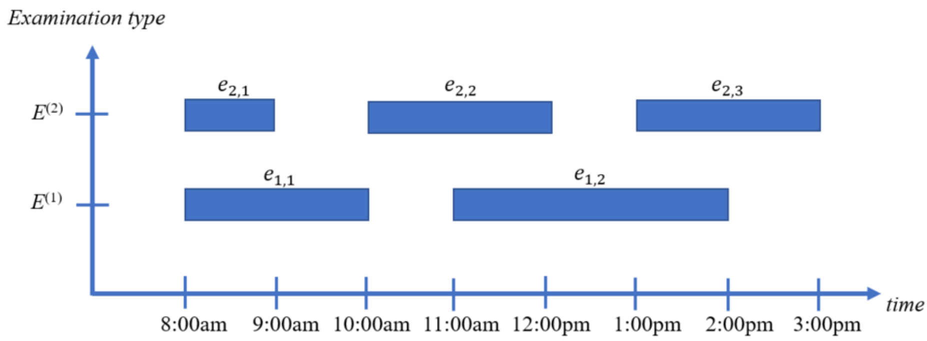


**Figure 1.** Representation of free time-intervals for examination types $E^{(1)}$ and $E^{(2)}$. The free time-intervals for $E^{(1)}$ is given by $TS^{(1)} = \{e_{1,1}, e_{1,2}\}$, where $e_{1,1} = \left(E^{(1)}, 8{:}00am, 10{:}00am\right)$, and $e_{1,2} = \left(E^{(1)}, 11{:}00am, 2{:}00pm\right)$, and for examination type $E^{(2)}$ is given by $TS^{(2)} = \{e_{2,1}, e_{2,2}, e_{2,3}\}$, where $e_{2,1} = \left(E^{(2)}, 8{:}00am, 9{:}00am\right)$, $e_{2,2} = \left(E^{(2)}, 10{:}00am, 12{:}00pm\right)$ and $e_{2,3} = \left(E^{(2)}, 1{:}00pm, 3{:}00pm\right)$.

Assuming, that we want to schedule multiple examinations for a patient at the same time, it is necessary to specify the lengths of timeslots that should be left between successive examinations. A waiting time between two consecutive examinations $E^{(i)}$ and $E^{(j)}$ is denoted by $w_{i,j}$ ($w_{i,j} > 0,\ i,j = 1, \ldots, N$). $w_{i,j}$ means that examination type $E^{(j)}$ can only begin after $w_{i,j}$ waiting time following the end of the examination type $E^{(i)}$. This waiting time is reserved, for example, to ensure the time for the transportation or dressing of patients.

The assignment of a set of examinations to a patient involves the determination of the sequence of the prescribed examinations. Denote $\pi(j)$ that examination type which will be done as the $j$-th examination for the patient in question. Furthermore, denote $t^{(i)}$ the time for examination type $E^{(i)}$, when the patient has to appear. In this way, if the aim is to minimise the time interval elapsed between the start and the completion of an examination series, the goal function of the optimisation problem can be formally given as follows:

$$min_{\pi,t}\left(t^{(\pi(m))} + d^{(\pi(m))} - t^{(\pi(1))}\right), \tag{3}$$

where $m$ ($m \leq N$) denotes the number of the examinations to be scheduled.

During the optimisation, the starting time of the examination $E^{(i)}$ to be scheduled is determined from exactly one element of $TS^{(i)}$. Thus, the optimal solution for scheduling $n$ examinations is arising from the set of the optimal intervals defined as:

$$OptIntervals = \cup_{i=1}^{m}\left\{e_{i,k} \mid e_{i.k} = \left(E^{(i)}, st_k^{(i)}, et_k^{(i)}\right) \in TS^{(i)}\right\}. \tag{4}$$

The starting times of the examinations in the optimal solution must fall within these free time intervals. The duration $d^{(i)}$ for the examination $E^{(i)}$ has to be also taken into account when it comes to the upper bound of variable $t^{(i)}$, since an examination must be finished by the end of the optimal time interval. Accordingly, for the starting time of the examination $E^{(i)}$ the following bounds can be defined:

$$st_k^{(i)} \leq t^{(i)} \leq et_k^{(i)} - d^{(i)}, \tag{5}$$

where $\left(E^{(i)}, st_k^{(i)}, et_k^{(i)}\right) \in OptIntervals$.

Due to the order of the examinations, the ending time and the starting time of two consecutive examinations have to be restricted as follows:

$$t^{(\pi(l))} + d^{(\pi(l))} + w_{\pi(l),\pi(l+1)} \leq t^{(\pi(l+1))}, \qquad l = 1, \dots, m-1 \tag{6}$$

The presented mathematical model defines such a complex NP-hard scheduling problem for which using traditional optimisation tools would not be expedient. In the following section, the suggested graph-based technique is introduced, which not only provides the optimal solution for the problem but also results in the *n*-best examination appointment scheduling.

### *2.2. P-graph methodology*

The P-graph framework was developed in the ’90s [11] for proposing an efficient way to represent chemical processes and solve related optimisation problems. Several other areas of applications have also been published in recent decades, such as business process modelling [12], vehicle assignment problem [13], or risk management [14]. In our study, the patient appointment optimisation problem was transformed into a process network synthesis (PNS) problem, and in this way, the suggested approach can also be seen as a novel application of the P-graph methodology.

The main advantages of the P-graph-based PNS description are the automated model generation based on the input parameters, and its graphical representation. A PNS problem can be visualised by a bipartite graph, where a type of nodes represents the *states* (solid circles), while the rest of the nodes are the *activities* (horizontal bars). Formally, a PNS problem is given by a triplet $(P, R, O)$, where set *P* contains the final targets or the end of the process, set *R* refers to the available resources or to the representation of starting a process, and set *O* includes the activities defined by preconditions and outcomes. Figure 2 illustrates a simple P-graph representation with *P* = {End}, *R* = {Start}, and $O = \{O_1, O_2, O_3\}$, where the entry and exit points of the process are the nodes ‘Start’ and ‘End’, while the elements of set *O* denote any kind of activities related to the problem. Each activity $O_i$ is clearly defined by a pair $(\alpha_i, \beta_i)$, where set $\alpha_i$ represents its preconditions and set $\beta_i$ gives its outcomes, thus, the set *O* in the presented example is specified as follows: $O = \{(\{Start\}, \{s_1, s_2\}), (\{Start\}, \{s_2\}), (\{s_1, s_2\}, \{End\})\}$. It is easy to see that there are two possible paths (i.e., feasible solutions) from the node ‘Start’ to the ‘End’: the path containing activities $O_1$ and $O_3$, as well as, the path with activities $O_1, O_2,$ and $O_3$. Note that, an activity can only be performed if all its preconditions are available.

Scheduling of appointments for a patient can be given by a special P-graph, since mutually exclusive activities representing free time intervals can also appear in the graph. Furthermore, always a single entry point and a single exit point exists which represent the start and the end of the examination (or treatment) process respectively.

The structure of the free time-intervals for an examination can be represented by several sequentially connected sets of parallel operating units. In this way, the structure

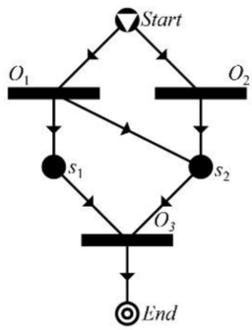


**Figure 2.** A simple illustrative example for the P-graph representation

of the PNS representing an examination scheduling problem is given by the *activities* (marked with horizontal bars) representing the free time-intervals for the examinations or for the opportunity of handling occasions between examinations (waiting for or dressing up before the next examination), and by the intermediate nodes (marked with solid circles) representing the *states* between the examinations. That is, in accordance with the formal description, an activity in the P-graph description corresponds to one element of the set $TS^{(i)}$ $(i = 1, \dots N)$ or to a $w_{i,j}$ waiting time. Furthermore, the possible order of the appointments is specified by the edges among the nodes in the graph, while the optimal order of the examinations represented by vector $\boldsymbol{\pi}$ is given by a path in the solution structure of the graph.

For the analogy of equation (6), a time variable is also assigned to each node in the P-graph to satisfy the time constraints for any scheduling:

$$o_i = (\alpha_i, \beta_i) \in O, \forall s_j \in \alpha_i : t^{(o_i)} \geq t^{(s_j)}, \tag{7}$$

i.e., the starting time of the activity $o_i$ is greater or equal than availability time of any preconditions. Furthermore,

$$o_i = (\alpha_i, \beta_i) \in O, \forall s_j \in \beta_i : t^{(s_j)} \geq t^{(o_i)} + d^{(o_i)} , \tag{8}$$

i.e., the availability time of any outcome states of the $o_i$ in the graph is greater or equal than the finishing time of the given activity.

Several algorithms have been proposed in the literature for solving the PNS problems, i.e. for finding possible solutions. In the patient appointment assignment problem, these solutions correspond to possible schedulings for the patient examinations to be planned. For example, the algorithm SSG [15] generates all the solution structures, i.e. represents every feasible flowsheet of the process of the interest, while algorithm ABB [16] gives the optimal and *n*-best solutions of the mathematical model. As our aim is to offer the *n*-best scheduling for the patients, in this article, the advantages of the last algorithm are utilised.

## 3. Case study

In this section, a simple case study is presented to demonstrate the applicability of the P-graph methodology for the patient appointments scheduling with alternative solutions. In this small example, it is assumed that the number of different types of examinations is *N=3,* where $E = \{E^{(1)} = Blood\ test, E^{(2)} = Ultrasound, E^{(3)} = ECG\}$, and the durations of the examinations are: $d^{(1)} = 4$, $d^{(2)} = 20$, and $d^{(3)} = 10$ minutes. We further assume that $\boldsymbol{\pi} = [1, 2, 3]$, i.e., the order of the examinations is fixed. Although, we have to emphasise, that in more complex cases, this assumption can also be omitted since the mathematical model is able to give proper scheduling for examinations without any order restrictions. The available free time-intervals for the examinations are also given:

$TS^{(1)} = \{e_{1,1}, e_{1,2}, e_{1,3}\}, TS^{(2)} = \{e_{2,1}, e_{2,2}\}$, and $TS^{(3)} = \{e_{3,1}, e_{3,2}\}$, where
$e_{1,1} = (E^{(1)}, \text{8:00am}, \text{8:10am}), e_{1,2} = (E^{(1)}, \text{9:00am}, \text{9:10am})$,
$e_{1,3} = (E^{(1)}, \text{10:00am}, \text{10:10am}), e_{2,1} = (E^{(2)}, \text{9:20am}, \text{10:00am})$,
$e_{2,2} = (E^{(2)}, \text{10:20am}, \text{11:20am}), e_{3,1} = (E^{(3)}, \text{10:00am}, \text{11:00am})$, and
$e_{3,2} = (E^{(3)}, 11{:}30am, 12{:}30pm)$.

The waiting times were chosen as follows: $w_{1,2} = 10$ min, $w_{2,3} = 25$ min, while in all other cases $w_{i,j} = \infty$. The flowchart of the illustrative example is shown in Figure 3, where nodes '×' represent the mutual exclusion relationships among the activities. Figure 4 shows the P-graph corresponding to Figure 3. This P-graph serves the basis of the optimisation.

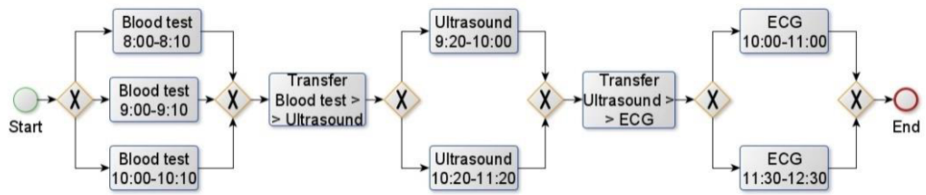


**Figure 3.** Flow chart of the illustrative example represented by Business Process Diagram [17].

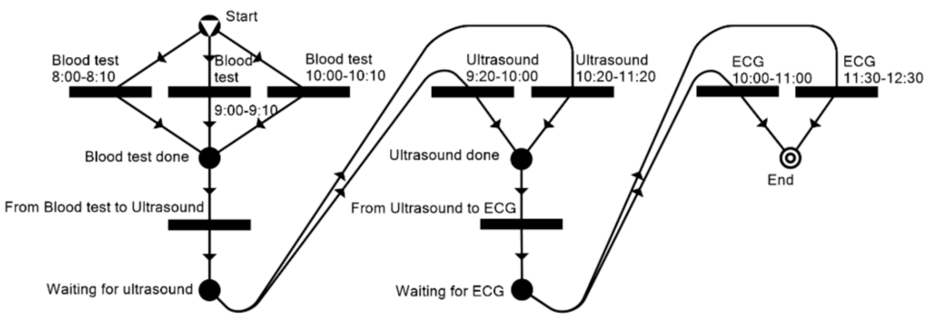


**Figure 4.** P-graph representation of the illustrative example.

The optimal and the *n*-best solutions of the mathematical model were determined by using the P-graph Studio v5.2.3.2 [18]. This decision support software is able to run the ABB algorithm among others, so it can provide not only the best but also the *n*-best solutions to any optimisation problem. Table 1 shows the feasible solutions to the presented example. We can see, that the value of the best goal function is 25 minutes better than the second-best one. Table 1 also shows the six possible alternatives to the best scheduling. The alternative solutions are listed in ascending order with respect to the value of the goal function. Furthermore, Figure 5 presents the graphical representation of the optimal solution.

**Table 1.** The optimal and the $n$-best optimal solutions of the presented example. Solution #1 represents the shortest schedule of examinations with a total duration of 1 hour and 9 minutes. The worst time schedule takes 3 hours and 34 minutes.

| # | Time for examination 1 (blood test), $t^{(1)}$ | Time for examination 2 (ultrasound), $t^{(2)}$ | Time for examination 3 (ECG), $t^{(3)}$ | Total duration of patient appointments |
|---|---|---|---|---|
| 1 | 9:06am | 9:20am | 10:05am | 1 h 9 min |
| 2 | 10:06am | 10:20am | 11:30am | 1 h 34 min |
| 3 | 8:06am | 9:20am | 10:05am | 2 h 9 min |
| 4 | 9:06am | 9:20am | 11:30am | 2 h 34 min |
| 5 | 9:06am | 10:20am | 11:30am | 2 h 34 min |
| 6 | 8:06am | 9:20am | 11:30am | 3 h 34 min |
| 7 | 8:06am | 10:20am | 11:30am | 3 h 34 min |

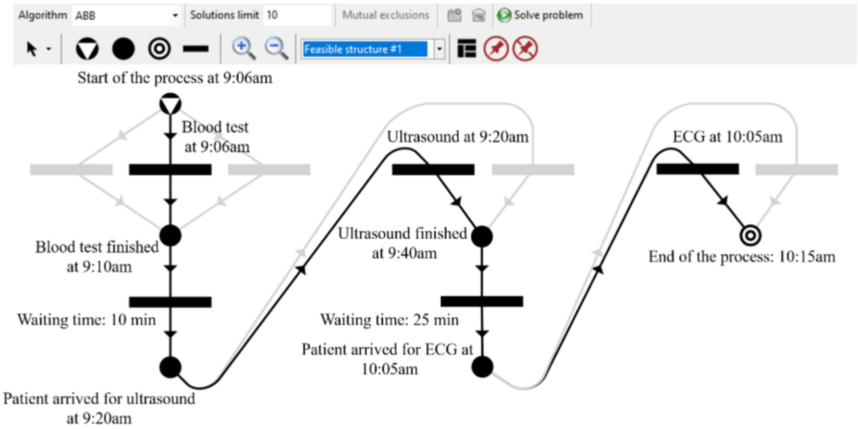


**Figure 5.** The P-graph representation of the optimal solution in P-graph Studio. The total duration of the patient appointment is 1 hour and 9 minutes.

This small example, illustrative present, that the suggested approach can effectively support the presented scheduling tasks. The P-graph-based approach gives an efficient way to solve the patient appointments optimisation problem by using accelerated branch and bound algorithm to find the optimal and near-optimal solutions.

## 4. Discussion

The patient appointment optimisation is one of the most challenging tasks in healthcare administration. In the literature, numerous methods have been proposed to solve the problem, but none of them provides a convenient way to offer the $n$-best solutions for scheduling of examinations without any adjustments of the parameters and running the complete model again. In this paper, the patient appointment scheduling task was formalised as an integer linear programming problem and was considered as a process network synthesis problem. To offer the $n$-best schedulings, the special process network synthesis problem was solved by the ABB algorithm. In contrast to the traditional models, the presented approach is not only able to produce the optimal patient appointment solution, but also offers alternatives. The main advantages of the P-graph-based graphical representation are that it can support the automatic model generation, and it can provide the best schedule and its alternatives for patients and doctors without any user interactions. Furthermore, the mathematical model can be extended by any number of additional constraints, and the goal function, i.e., the focus of the optimisation task can also be changed without limitations. The validation of the model on real data from hospitals will be done in the next phase of our research.

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