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Integrated Optimization of VSS Control Chart and Maintenance for Small-Batch Production Processes

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Abstract. Recent research has increasingly focused on integrated statistical process control (SPC) and maintenance strategies in production systems, highlighting their critical role in quality control. Most integrated modeling of SPC and maintenance currently focus on large-scale production processes. However, production processes are transitioning towards multi-variety and small-batch production mode, which is characterized by insufficient sample sizes. Based on this characteristic, this paper studies the integrated modeling and optimization of Variable Sample Size (VSS) control chart and maintenance. This strategy employs an equal interval maintenance strategy. Within the preventive maintenance cycle, the sampling starts with a small sample size, and when the sampled result falls in the warning zone, the sample size is increased. Also, five maintenance scenarios are defined based on three maintenance types, the state of the process and whether the control chart triggers an alarm. The probabilities, costs, and durations for each scenario are also calculated accordingly. The integrated model then is established based on these scenarios, and genetic algorithms are used for case analysis to solve the optimal objective of the model. Based on the case analysis, it is demonstrated that the proposed integrated model can make effective decisions under various scenarios, thereby validating its applicability and effectiveness in small-batch production processes.

Keywords. Preventive maintenance, variable sample size control chart, integrated modeling of control charts and maintenance

1. Introduction

In the process of controlling product quality, control charts utilize statistical techniques to monitor production processes, infer process states, and issue alerts for abnormal factors that may arise during production. Machine maintenance primarily focuses on improving machine performance, which can influence process states and subsequently impact the strategy for monitoring product quality. Despite the recognized close relationship between machine maintenance and process quality in both industry and academia [1], integrated optimization of machine maintenance and process control has only recently garnered significant attention among scholars [2-3]. Afef et al. [4] highlighted the growing significance of machine influence on process quality with the trend towards intelligent and automated production processes in enterprises. In fact, existing integrated strategies and models of control charts and maintenance are typically for large-scale production process. In designing integrated strategies, we can optimize

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them by using different control charts, optimizing control chart parameters, and adjusting maintenance strategies. For instance, Linderman et al. [5] explored the integrated decision-making problem of preventive maintenance and SPC. They developed a general analytical framework for integrating preventive maintenance and control charts based on scenarios of correct alarms and false alarms and missed alarms, encompassing three distinct situations.

But in modern manufacturing industry, customer demands are shifting towards diversification and personalization. To enhance responsiveness to customer demands, production processes are transitioning towards multi-variety and small-batch production mode.

Therefore, targeting the small-batch production processes, there has been a rise in research on the integrated strategies and optimization of statistical process control and maintenance. In these studies, some integrated strategies employ static control charts [6-7], while others use dynamic control charts [8-9]. For example, Hasan et al. [7] used static control charts, he divided the production process into three states (in-control, out-of-control, and failure), and then presented an integrated model of maintenance and statistical process control. While Xue et al. [9] used dynamic control charts, considering the quality loss, studied the economic design of VSI EWMA control charts based on the quality loss function and preventive maintenance strategy. Compared to static control charts, current research indicates that dynamic control charts are more effective in monitoring the production process and can lead to cost savings compared to static control charts. This is because static control charts require a substantial amount of data to monitor the progress effectively. However, in small-batch production processes, the issue of insufficient sample sizes often arises, which would inevitably lead to inaccurate results of control charts. While in dynamic control charts, the three parameters (sampling intervals, sample sizes, and control limits) can be changed as the process proceeds, so the control chart can adjust dynamically to address the issue of insufficient sample sizes.

Furthermore, with the continuous development of new tools and technologies, an increasing number of studies are integrating optimization with machine learning (ML) and augmented intelligence (AI) when generating maintenance schedules. For example, Wang et al. [10] introduced a Q-learning based solution algorithm, employing well-defined state and action sets to determine appropriate scheduling rules. Meng et al. [11] developed a quality control chart recognition model using MC (Monte Carlo method)-GA (Genetic Algorithm) optimized BP (Back Propagation Neuron Network). This model employs the Monte Carlo method to simulate diverse product quality data characteristics, integrating the data processing capabilities of neural networks with the adaptive global optimization of genetic algorithms.

The major contribution of this paper focuses on the design of maintenance scenarios. In existing research that employs dynamic control chart strategies, the scenario where equipment failure occurs between preventive maintenance and its previous sampling has not been considered. This oversight results in the model that fails to accurately depict the sampling and maintenance activities within a maintenance cycle. So, this is not a perfect integrated strategy. Considering this oversight, this paper redesigns the maintenance scenarios and then an integrated model is established based on these scenarios.

This paper focuses on integrated modeling and optimization of control charts and maintenance for small-batch production processes. It employs an equal interval maintenance strategy, using the VSS EWMA control chart for sampling within the preventive maintenance cycle. The sample size of this control chart can be changed, due to the smaller sample sizes in small-batch production, sampling starts with a small sample size. When the sampled result falls in the warning zone, the sample size is increased. Five maintenance scenarios are then defined based on three maintenance types, the state of the process and whether the control chart triggers an alarm. The probabilities, costs, and durations for each scenario are also calculated accordingly. The integrated model can be established based on these scenarios, and then genetic algorithms are used for case analysis to solve the optimal objective of the model.

The remainder of this paper is organized as follows. Section 2 gives the problem description, including the system description and the design of five maintenance scenarios. Section 3 gives the design of VSS control chart for the small-batch production, which includes parameters of the control chart, such as the design of control lines and probabilities of committing two types of errors. In section 4, the probabilities, costs, and durations for each scenario are calculated, and the integrated model is also proposed. In section 5, a case study is conducted to illustrate the effectiveness of the proposed integrated model. Finally, some concluding remarks are given in Section 6.

Notation

T	preventive maintenance cycle	EC	expected renewal cost
T_1	preventive maintenance time	α	type I error rate
T_2	corrective maintenance time	β	type II error rate
T_3	compensatory maintenance time	α_i	probability of making a type I error with n_1 sampling
C_{m1}	preventive maintenance cost	β_i	probability of making a type II error with n_2 sampling
C_{m2}	corrective maintenance cost	λ	smooth coefficient
C_{m3}	compensatory maintenance cost	h	sampling interval
C_{l1}	quality loss under in-control state	n_i	sample size, n =1,2
C_{l2}	quality loss under out-of-control state	k	control limit coefficient
C_d	production downtime loss	w	warning limit coefficient
C_q	average cost per sampled item	ρ_i	probability of using sample size n_i
C_s	expected cost per sampling $C_s = n_1 \times \rho_1 \times C_q + n_2 \times \rho_2 \times C_q$	l	maximum sampling times in a cycle, $l = [\frac{T}{h}]$
$F(u)$	probability density function of u	μ_0	target value for mean value of quality characteristics
$f(u)$	cumulative distribution function of u	σ_0	target value for standard deviation of quality characteristics
ECT	expected cost per time	$EC(S_k)$	expected renewal cost for S_k $(k = 1,2,3,4,5)$
ET	expected renewal cycle	$ET(S_k)$	expected renewal cycle for S_k $(k = 1,2,3,4,5)$

2. Problem description

This paper studies the production process of a medical device, focusing on the monitoring, maintenance, and control aspects of its quality management system, which adheres to the ISO 13485:2016 standard. Consider the production system which has two states: in-control states and out-of-control states. The system starts operating in a in-control state and is monitored using a VSS EWMA control chart. When the system shifts into the out-of-control state, it would result in the production of defective goods, and further lead to quality loss. But in this situation, the sampled result in the control chart would fall in the out-of-control zone and trigger an alarm. Therefore, the failure can be detected and repaired promptly to minimize losses. However, there may be false alarms or missed alarms in the control chart. To prevent this situation, at the time T, after the l-

th sampling has been taken, if there has been no alarms yet, a preventive maintenance will be carried out.

Maintenance (preventive maintenance, corrective maintenance and compensatory maintenance) can restore the system to a like-new state. Then a same process starts from the beginning again. Based on the above integrated model strategy, one production cycle may end in five different scenarios. The five scenarios are shown in Figure 1.

Scenario 1: The process is always in-control and no alarm occurs in the control chart before the preventive maintenance cycle T. Then, at the time T, the preventive maintenance is arranged.

Scenario 2: The process is always in-control but there is a false alarm in the control chart. After determining the false alarm, the compensatory maintenance is performed.

Scenario 3: The process shifts to an out-of-control state before the preventive maintenance cycle T and an alarm occurs in the control chart. Then the corrective maintenance is performed.

Scenario 4: The process shifts to an out-of-control state before the preventive maintenance cycle T, but no alarm occurs until the preventive maintenance. Since the preventive maintenance cycle may not align with sampling intervals, we further discuss two scenarios based on the failure occurs before or after the last sampling. In this scenario, the failure occurs before the last sampling, and the corrective maintenance is performed at the time T.

Scenario 5: The process shifts to an out-of-control state before the preventive maintenance cycle T, but no alarm occurs until the preventive maintenance. The failure occurs after the last sampling, and the corrective maintenance is performed at the time T.

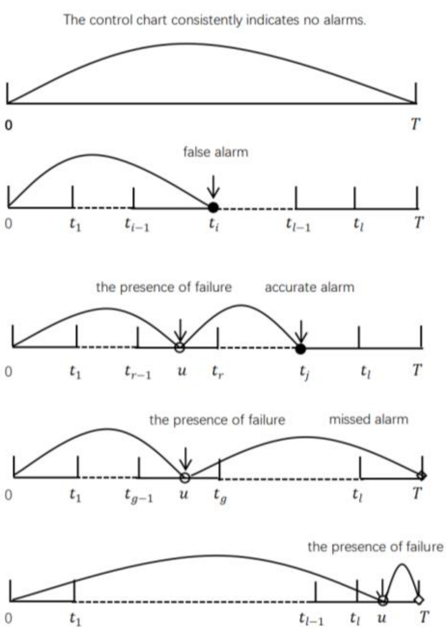

Figure 1. The design of maintenance scenarios.

3. Design of dynamic control chart for the small batch production

Due to the smaller sample sizes in small-batch production, the sampling starts with a small sample size. When the sampled result falls in the warning zone, the sample size is increased.

Given that the quality characteristic X of the system follows a normal distribution, when the system is in control, $\mu = \mu_0$, $\sigma = \sigma_0$. When the system is out of control, σ_0 remains unchanged, while μ shifts from μ_0 to $\mu_0 + \delta$. Define the parameters of the VSS EWMA control chart are $(n_1, n_2, h, \omega, k, \lambda, T)$, where n_1 and n_2 represent two sample sizes (with $n_1 < n_2$), ω is the coefficient for the control chart warning line, and k is the control line coefficient (with $0 < w < k$). Therefore, the statistical quantity of the control chart at the t-th sampling is $Z_t = \lambda X_t + (1-\lambda) Z_{t-1}$, $Z_0 = \mu_0$, $0 < \lambda \leq 1$.

In the control chart design, the control lines include: Center Line (CL); Upper Control Limit (UCL) and Lower Control Limit (LCL); Upper Warning Limit (UWL) and Lower Warning Limit (LWL). These control lines can be defined as follows:

$$\begin{cases} CL = 0 \\ UWL = \mu_0 + \omega\sigma_z \\ LWL = \mu_0 - \omega\sigma_z \\ UCL = \mu_0 + k\sigma_z \\ LCL = \mu_0 - k\sigma_z \end{cases} \tag{1}$$

Monitoring rules: If the sampled result falls within the warning zone $(LCL, LWL] \cup [UWL, UCL)$, increase the sample size to n_2 for the next sampling. If the sampled result falls within the central zone (LWL, UWL), decrease the sample size to n_1 for the next sampling. Otherwise, trigger an alarm.

The probability of using the small sample size n_1 for each sampling is denoted as ρ_1, and the probability of using the large sample size is $\rho_2 = 1 - \rho_1$. The control limits and warning limits remain the same for both parameter combinations, so ρ_1 can be calculated accordingly.

$$\rho_1 = \frac{2\phi(\omega)-1}{2\phi(k)-1} \tag{2}$$

The probabilities of committing two types of errors in a control chart are as follows:

$$\alpha = \rho_1 \times \alpha_1 + (1-\rho_1) \times \alpha_2 \tag{3}$$

$$\beta = \rho_1 \times \beta_1 + (1-\rho_1) \times \beta_2 \tag{4}$$

4. Integrated model of VSS control chart and maintenance

Based on the control chart design and maintenance scenario classification from the previous sections, this section focuses on designing an integrated model of VSS EWMA control charts and preventive maintenance strategies for small-batch production systems. It also analyzes the occurrence probabilities of various maintenance scenarios and establishes a comprehensive cost decision model.

The preceding text has already divided the production process into five scenarios. However, the probabilities, durations, and costs for each scenario are different. Therefore, an integrated model is designed considering the probabilities, durations, and costs of each scenario.

4.1. Assumptions

The model is based on the assumption that the destiny and distribution functions of the time until system failure are known. Relevant parameters are shown in the notation, involving different maintenance durations, maintenance costs, parameters of control charts and so on.

4.2. Probability of occurrence, ET and EC for each scenario

(1) $ET(S_1)$ and $EC(S_1)$

In Scenario 1, the process is always in-control and no alarm occurs in the control chart before the preventive maintenance cycle T. Then, at the time T, the preventive maintenance is arranged. The probability of this event occurring is defined as $P(S_1) = [1 - F(T)] \times (1 - \alpha)^l$

Here, $T(S_1) =$ System Operating Time $T +$ Preventive Maintenance Time T_1. The system maintenance costs consist of production quality losses, inspection costs, preventive maintenance costs, and production downtime losses. So $C(S_1) = T \cdot C_{l1} + l \cdot C_s + C_{m1} + T_1 \cdot Cd$

Then $ET(S_1)$ and $EC(S_1)$ can be defined as:

$$ET(S_1) = (T + T_1) \times P(S_1) \tag{5}$$

$$EC(S_1) = (T \cdot C_{l1} + l \cdot C_s + C_{m1} + T_1 \cdot Cd) \times P(S_1) \tag{6}$$

(2) $ET(S_2)$ and $EC(S_2)$

In Scenario 2, the process is always in-control but there is a false alarm in the control chart. After determining the false alarm, the compensatory maintenance is performed. The probability of this event occurring is defined as $P(S_2, t_i) = [1 - F(t_i)] \times (1 - \alpha)^{i-1} \times \alpha$

Here, $T(S_2, t_i) =$ System Operating Time t_i + Compensatory Maintenance Time T_3. The system maintenance costs consist of production quality losses, inspection costs, compensatory maintenance costs, and production downtime losses. So $C(S_2, t_i) = t_i \cdot C_{l1} + i \cdot C_s + C_{m3} + T_3 \cdot Cd$. Considering all possible values of i (where i=1,2, …l), we obtain the following results:

$$P(S_2) = \sum_{i=1}^{l} P(S_2, t_i) \tag{7}$$

$$ET(S_2) = \sum_{i=1}^{l} (t_i + T_3) \times P(S_2, t_i) \tag{8}$$

$$EC(S_2) = \sum_{i=1}^{l} (t_i \cdot C_{l1} + i \cdot C_s + C_{m2} + T_3 \cdot Cd) \times P(S_2, t_i) \tag{9}$$

(3) $ET(S_3)$ and $EC(S_3)$

In Scenario 3, the process shifts to an out-of-control state before the preventive maintenance cycle T and an alarm occurs in the control chart. Then the corrective maintenance is performed. The probability of this event occurring is defined as $P(S_3, t_{r-1,r}, t_j) = \int_{t_{r-1}}^{t_r} f(u)du \times (1 - \alpha)^{r-1} \times \beta^{j-r} \times (1 - \beta)$

Here, $T(S_3, t_{r-1,r}, t_j) =$ System Operating Time $t_j +$ Corrective Maintenance Time T_2. The system maintenance costs consist of production quality losses, inspection costs, corrective maintenance costs, and production downtime losses. So $C(S_3, t_{r-1,r}, t_j) = u \cdot C_{l1} + (t_j - u) \cdot C_{l2} + j \cdot C_s + C_{m2} + T_2 \cdot Cd$

Considering that u could occur during any sampling interval and correct alarms could be issued at any sampling check following the anomaly, we integrate over all possible values of r (where $r=1,2, \dots l$) and j (where $j=r, r+1, \dots l$) to obtain the following results:

$$P(S_3) = \sum_{r=1}^{l} \sum_{j=r}^{l} P(S_3, t_{r-1,r}, t_j) \tag{10}$$

$$ET(S_3) = \sum_{r=1}^{l} \sum_{j=r}^{l} (t_j + T_2) \times P(S_3, t_{r-1,r}, t_j) \tag{11}$$

$$EC(S_3) = \sum_{r=1}^{l} \sum_{j=r}^{l} \left(u \cdot C_{l1} + (t_j - u) \cdot C_{l2} + j \cdot C_s + C_{m2} + T_2 \cdot Cd\right) \times P(S_3, t_{r-1,r}, t_j) \tag{12}$$

(4) $ET(S_4)$ and $EC(S_4)$

In Scenario 4, the process shifts to an out-of-control state before the preventive maintenance cycle T, but no alarm occurs until the preventive maintenance. The failure occurs before the last sampling, and the corrective maintenance is performed at the time T. The probability of this event occurring is defined as $P(S_4, t_{g-1}, T) = \int_{t_{g-1}}^{t_g} f(u)du \times (1-\alpha)^{g-1} \times \beta^{l-g+1}$

Here, $T(S_4, t_{g-1}, T)$ = System Operating Time T + Corrective Maintenance Time T_2. The system maintenance costs consist of production quality losses, inspection costs, corrective maintenance costs, and production downtime losses. So $C(S_4, t_{g-1}, T) = u \cdot C_{l1} + (T-u) \cdot C_{l2} + l \cdot C_s + C_{m2} + T_2 \cdot Cd$

Considering that u could occur during any sampling interval and correct alarms could be issued at any sampling check following the anomaly, we integrate over all possible values of r (where $r=1,2, \dots l$) to obtain the following results:

$$ET(S_4) = \sum_{r=1}^{l} (T + T_2) \times P(S_4, t_{g-1}, T) \tag{13}$$

$$EC(S_4) = \sum_{r=1}^{l} (u \cdot C_{l1} + (T-u) \cdot C_{l2} + j \cdot C_s + C_{m2} + T_2 \cdot Cd) \times P(S_4, t_{g-1}, T) \tag{14}$$

(5) $ET(S_5)$ and $EC(S_5)$

In Scenario 5, the process shifts to an out-of-control state before the preventive maintenance cycle T, but no alarm occurs until the preventive maintenance. The failure occurs after the last sampling, and the corrective maintenance is performed at the time T. The probability of this event occurring is defined as $P(S_5) = \int_{t_l}^{T} f(u)du \times (1-\alpha)^l$.

Here, $T(S_5)$ =System Operating Time T + Corrective Maintenance Time T_2. The system maintenance costs consist of production quality losses, inspection costs, corrective maintenance costs, and production downtime losses. So $C(S_5) = u \cdot C_{l1} + (T-u) \cdot C_{l2} + l \cdot C_s + C_{m2} + T_2 \cdot Cd$

Then $ET(S_1)$ and $C(S_1)$ can be defined as follows:

$$ET(S_5) = P(S_5) \times T(S_5) \tag{15}$$

$$EC(S_5) = P(S_5) \times C(S_5) \tag{16}$$

4.3. Integrated optimization of VSS control chart and maintenance

According to the renewal process theory, the long-term expected cost per time ECT equals the ECT within one single maintenance cycle. Meanwhile, the unit time within a cycle equals the sum of durations in each scenario within that cycle. While the unit cost within a cycle equals the sum of costs in each scenario within that cycle. So, the cost decision model can be established as follows:

$$ECT = \frac{EC}{ET} = \frac{\sum_{k=1}^{5} EC(S_k)}{\sum_{k=1}^{5} ET(S_k)} \tag{17}$$

In other words, for the integrated economic design of preventive maintenance and VSS EWMA control chart, the objective is to find an appropriate parameter set $(n_1, n_1, h, \omega, k, \lambda, T)$ that minimizes the ECT. Due to the large number of decision variables, a genetic algorithm is employed for optimization.

5. Case study

5.1. Case description

The data is sourced from a medical device company's bone screw diameter analysis. When the system is in control, the output quality Q follows N $(307,11^2)$. When the system shifts to an out-of-control state, it leads to a shift in the mean of Q to 357. Component failure durations follow a Weibull distribution (shape=2.00, scale=1000.00). Maintenance durations and costs are as follows: T_1=5.60 h, T_2=8.39 h, T_3=2.78 h, C_{m1}=1248 yuan, C_{m2}=2556 yuan, C_{m3}=257 yuan, C_{l1}=1.86 yuan, C_{l2}=46,4 yuan, C_d=3980 yuan, C_q=2.75 yuan. Sampling time is negligible. Current preventive maintenance cycle: T=1350 h.

5.2. Integrated optimization

Using MATLAB with a genetic algorithm, we aim to optimize the decision parameters of our model $(n_1, n_2, h, \omega, k, \lambda, T)$ and ECT. The decision results of the model are shown in Table 1.

Table 1. Results of decision parameters and ECT.

Decision Parameters							***ECT***
n_1	n_2	h	ω	k	λ	T	
5	9	11.4411	1.9899	2.6081	0.2968	864.8883	24.5374

Based on the results, the proposed integrated strategy involves using a VSS EWMA control chart for sampling. The sampling would start with a small sample size of 5, then if the sampled result falls in the warning zone, the sample size changes into 9. And sampling is conducted every 14.4411h. Other control chart parameters are defined as follows: a warning limit coefficient ω of 1.9899, a control limit coefficient k of 2.6081, and a smooth coefficient λ of 0.2968. Additionally, the preventive maintenance cycle T is set at 864.8883h, which means if there have been no alarms at this time, a preventive maintenance will be carried out to avoid missed alarms. Under the above configuration, it would cost the process 24.5374 yuan per hour.

5.3. Sensitivity analysis

Integrating experimental design and regression analysis methods, we investigate the impact of four cost parameters $(C_{m1}, C_{m2}, C_{l2}, C_q)$ on our model. We assign three levels to each of these four parameters and then by using MATLAB with a genetic algorithm program, we compute the decision results. The results are as follows:

Table 2. Results of decision parameters and ECT under different C_{m1}.

C_{m1}	n_1	n_2	h	ω	k	λ	T	ECT
1500	5	9	11.4503	1.9805	2.7674	0.2968	872.1138	25.5296
2000	6	9	12.2883	2.0394	3.0148	0.2983	969.3165	30.9368
4000	10	15	15.9999	2.2295	3.1097	0.3029	1443.8413	39.1064

Table 3. Results of decision parameters and ECT under different C_{m2}.

C_{m2}	n_1	n_2	h	ω	k	λ	T	ECT
2500	5	9	11.4595	1.9899	2.6018	0.2969	865.2928	24.5364
4000	7	11	10.5532	1.9927	2.7647	0.2985	844.0353	29.4782
6000	9	14	10.0603	2.0805	3.0298	0.3015	809.4950	35.3086

Table 4. Results of decision parameters and ECT under different C_{l2}.

C_{l2}	n_1	n_2	h	ω	k	λ	T	ECT
50	5	9	10.9375	1.9994	2.6958	0.2988	863.3165	24.7577
100	6	10	10.0268	2.0333	2.9630	0.3011	850.4000	27.7589
500	10	13	8.3020	2.6394	3.1097	0.3030	832.2298	34.0474

Table 5. Results of decision parameters and ECT under different C_q.

C_q	n_1	n_2	h	ω	k	λ	T	ECT
2.5	5	9	11.4397	1.9875	2.6034	0.2968	864.8528	24.5370
4	5	8	12.2861	2.0892	2.8965	0.2976	879.3651	26.0052
10	3	6	14.0000	2.3671	3.2637	0.3002	933.3333	29.0840

Based on the results shown in Table 2, it can be observed that as C_{m1} increases, the system prioritizes reducing the frequency of preventive maintenance to lower the high costs associated with it. Consequently, the preventive maintenance cycle T is extended.

Based on the results shown in Table 3, it can be observed that as C_{m2} increases, the system focuses more on reducing the occurrence of failures to minimize the high costs of corrective maintenance. Therefore, the system needs to sample more frequently to detect potential failures and shorten the sampling interval h for more frequent monitoring of the process. Additionally, the preventive maintenance cycle T is shortened, increasing the frequency of preventive maintenance.

Based on the results shown in Table 4, it can be seen that as C_{l2} increases, the system places greater emphasis on process monitoring to reduce the high costs associated with being out of control. Therefore, the system samples more frequently to detect and correct out-of-control states early, thereby reducing high-quality loss costs. Additionally, the sampling interval h is shortened for more frequent process monitoring. The system also focuses more on preventive maintenance to avoid high-quality loss costs.

Based on the results shown in Table 5, it can be seen that as C_q increases, the system aims to reduce the sampling frequency to cut down on the high costs of sampling. Consequently, the system reduces the sampling frequency and extends the sampling interval h.

Overall, decision parameters $(n_1, n_2, h, \omega, k, \lambda, T)$ and ECT are sensitive to these four cost parameters $(C_{m1}, C_{m2}, C_{l2}, C_q)$, and these four cost parameters have a positive impact on the ECT, indicating the effectiveness of the model.

5.4. Comparison of integrated models

To validate the superiority of the integrated modeling of VSS control chart and maintenance, we compare it with the integrated modeling of static control chart and maintenance. Let $n = 6$, and compare it with the former strategy. The results are as shown in the Table 6.

Table 6. Comparison before and after changing sample size.

		Decision Parameters					ECT
		h	ω	k	λ	T	
Integrated model of VSS control chart and maintenance	$n_1 = 6$ $n_2 = 10$	11.4411	1.9899	2.6081	0.2968	864.8883	24.5374
Integrated model of static control chart and maintenance	$n = 6$	9.9999	2.0476	2.7529	0.3122	858.6846	29.3268

When using the integrated model of static control chart and maintenance strategy, the sample size is fixed. While on the contrary, in the integrated model of VSS control chart and maintenance strategy, the sample size can be adjusted dynamically. So, in our model, the system uses a small sample size for sampling at first. Then if the sampled result falls in the warning zone, the sample size can be increased. This changing parameter makes it easier to detect failures, thereby minimizing losses.

6. Conclusions and future work

This paper addresses the characteristics of small-batch production processes by establishing an integrated model of VSS EWMA control charts and preventive maintenance strategies. Because of using the VSS control chart, the sample size can be changed when the sampled result falls in the warning zone, which can help detect failures and minimize losses. Based on the equal interval maintenance strategy, five maintenance scenarios are designed based on three maintenance types, the control state of the process and whether the control chart triggers an alarm. Also, the probabilities, costs, and durations associated with each scenario as well as the final ECT are also calculated. The integrated model then is established based on these scenarios. The results from case studies demonstrate the effectiveness of this model, showing it can make effective decisions under various scenarios. Therefore, this model can be used in small-batch production to improve product quality and reduce costs.

Although this paper uses the dynamic control chart to improve the performance of control charts, a higher quality standard can be achieved by further improving the sensitivity of control charts. So, in the future work, we can further research on adding more limits to the control charts to further improve sensitivity.

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