Proceedings of the 20th International Ship and Offshore Structures Congress (ISSC 2018) Volume III – M.L. Kaminski and P. Rigo (Eds.)

doi:10.3233/PMST200006

COMMITTEE III.1
ULTIMATE STRENGTH

COMMITTEE MANDATE

Concern for the ductile behaviour of ships and offshore structures and their structural components under ultimate conditions. Attention shall be given to the influence of fabrication imperfections and in-service damage and degradation on reserve strength. Uncertainties in strength models for design shall be highlighted. Consideration shall be given to the practical application of methods.

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1. DISCUSSION

1.1 Official Discussion by Preben Terndrup Pedersen (DTU)

1.1.1 Introduction

It is an honour to have been given the opportunity to serve as Official Discusser for ISSC 2018. Committee III.1 was the first ISSC committee I joined in Hamburg in 1973. Since then I have continuously been involved in and paying interest into the efforts of ISSC committees in their critical evaluation of new research results and in presenting future directions for research and development of our field.

According to the mandate, Committee III.1 shall specifically deal with ductile behaviour of ships and offshore structures and their components under ultimate conditions, and consideration shall be given to the practical application of methods. With this mandate the Committee becomes a very central committee for ISSC since it deals with the fundamental tools for assessment of structural safety and evaluation of safety margins for maritime structures.

The Committee has presented a comprehensive summary and evaluation of current publications related to the mandate and performed two benchmark tests. The report has 107 pages of which 14 consist of the reference list.

According to the statutes of ISSC the Official Discusser shall critically assess the Committee's report as to its success in complying with its mandate, and confirm that all significantly relevant progress in the subjects concerned have been addressed in the committee report. With this in mind I shall in the following give my evaluation of the core of the committee report and present additional comments together with suggestions for the future committee work.

1.1.2 Assessment of the ultimate strength.

As described in Chapter 2 of the report the foundation for rational based design is ultimate strength for ship structures and in the maritime industry the ultimate limit state is now widely applied as the basis of structural design and strength assessment. For this reason ultimate strength calculation is a mandatory task and of paramount importance in order to ensure the safety of marine structures. Most structures can fail in several different modes and at the same time the tools available for ultimate strength analyses are still under development and results from non-linear analyses can have large scatter. Thus, the role of this committee is of outmost importance.

In Chapter 3 of the report an excellent review is presented on available methods for assessment of the ultimate hull girder strength. I really like that the report includes the background and some of the history of the development of the most important procedures for estimation of the ultimate ship hull girder collapse loads.

The procedures have been classified as on one hand *analytical methods* which include closed form methods and progressive collapse methods based on beam-column assemblies (Smith's incremental method) and on the other hand *numerical methods* which include Idealized Structural Unit Methods (ISUM) and Non-linear Finite Element Methods. For inter-frame collapse analyses this discusser finds that there is much similarity between the beam-column approach and the idealized structural unit method. Both methods are fast procedures and experience shows the overall performance of these methods are comparable for collapse analyses of hull girders subjected to vertical bending.

1.1.3 Smith incremental method

The report rightly stresses that the Smith incremental method plays a major role for practical evaluation of the ultimate hull girder collapse load for intact as well as damaged conditions. It is important to note that the partial safety factors used by the IACS Harmonized Common Structural Rules for ships longer than 150m have been calibrated to the Smith method. For this reason I find it important to discuss the statement in Section 3.3.2 that this method can only consider pure longitudinal bending of a hull girder. The Committee points out that when shear is present the assumption that plane sections remain plane is not strictly valid.

It is true that sectional shear forces impose two different challenges for the ISUM and the Smith methods. Firstly, for cross sections such as tankers with wide decks and for container ships with large double bottom breadth the shear force will result in a shear lag effect where the elastic longitudinal stress distribution varies across the width of the deck and/or double bottom. I expect that the variation of the longitudinal stress field in transverse hull girder cross sections due to shear lag introduces only a negligible effect for the ultimate bending moment for realistic ductile hull girder cross sections. This hypothesis will be discussed further in connection with a discussion on the influence of warping stresses. Secondly, in the presence of a sectional shear force the sectional bending moment will not be constant in the longitudinal direction along the considered structural element. As long as the collapse is assumed to take place between two frames I find it difficult to imagine that this effect shall play a very important role for the results.

The current IACS hull girder analyses are based on the assumption that collapse takes place at cross-sections where the bending moment has an extremum and therefore the sectional shear force is zero. It is possible to imagine that for certain hull structures a combination of bending moments and shear forces could pose a larger threat for hull girder collapse than determined by the currently applied extreme bending moment.

A preliminary analysis of this probability was tested some years ago for a double hull tanker. Here an expanded Smith procedure was developed which allows for load displacement response of the individual beam-columns exposed to: an axial force, an end rotation, a constant shear stress level acting in the plate part only, and a uniform distributed pressure load acting on the stiffener perpendicular to the plate field (Nielsen, 1998). Fig. 1 shows the cross- section of an ultra large crude oil carrier with double hull and Fig. 2 the calculated ultimate moment interaction collapse load calculations for this ship in a ballast condition subjected to different levels of shear sectional loads and at the same time sea and ballast water pressure loads, i.e. taking into account the double bottom deformation. The applied procedure takes into account the

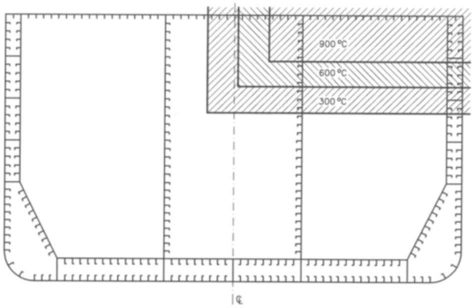

Figure 1. *Structural layout of ULCC midship section and assumed temperature distribution by a fire in the starboard wing cargo tank.*

position and rotational angle of the neutral axis plane as the bending moment increments are applied. The results show that for this double hull tanker moderate shear forces have a limited effect on the ultimate hull girder bending moments and that even for very large shear forces the hull girder can sustain some bending moments due to the fact that the stiffeners are not subjected to shear stresses due to transverse loading of the hull girder.
Besides for the analysis of the design requirements stipulated in the IACS Harmonized Structural Rules an efficient tool is also needed for emergency response analyses. Furthermore, such analysis tools are often needed for salvage operations. One such example was the analyses prior to the removal of the damaged Costa Concordia wreck in 2012.

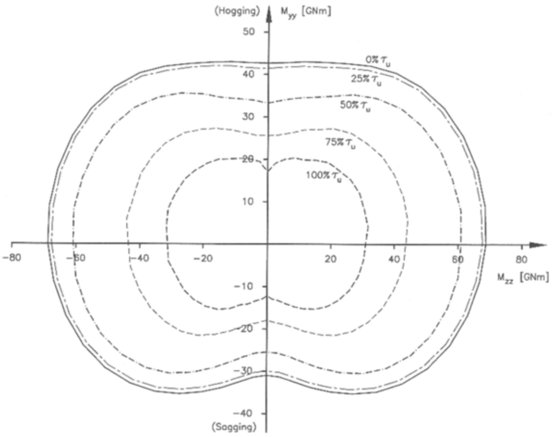

Figure 2. *Ultimate moment interaction chart for intact ULCC in ballast condition subjected to different levels of vertical shear loading (Nielsen 1998)*

The committee report presents a discussion of the applicability of the Smith procedure to estimate the residual strength of ships in conditions with damages due to grounding events and collisions. This is an important practical application of ultimate strength analysis tools and also an application where the effect of shear may be important.

For residual strength assessment of vessels with grounding and collision damages it is especially important to have procedures which can handle asymmetric structures subjected to combinations of vertical and horizontal bending as well as shear and torsional loading. With flooded compartments the shear forces can become significant. Besides the direct structural damages due to grounding events and collisions procedures should also be developed to handle cases where a collision leads to fire and explosions. Recent accidents such as the Sanchi tanker fire in the Chinese Sea and the containers ship Maersk Honan fire in the Arabian Sea, both in the spring of 2018, show that for tankers and for container ships onset of fire happens quite frequently. During a fire there is a strong need for a fast assessment of the probability that the ship may break up and sink.

To test the applicability of the Smith method for hull structures subjected to elevated temperatures a fictitious damage scenario involving fire in a tanker was analysed by Nielsen (1998). Here the resulting temperature distribution in the immediate vicinity of the fire was assumed as sketched in Fig. 1. For inter frame collapse modes the resulting asymmetry of the cross section and the strength degradation due to the elevated temperatures affects the ultimate capacity is estimated as shown in Fig. 3.

Of course, in the case of a damaged cross section it may be less likely that the dominant collapse mode will be associated with inter frame collapse and that the Smith incremental procedure is a reasonable analysis tool.

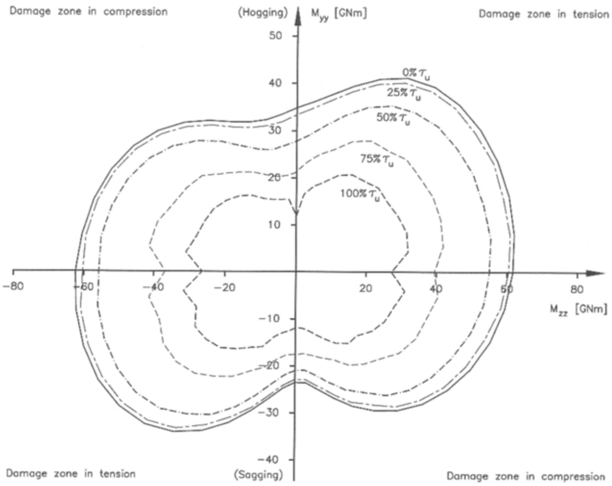

Figure 3. *Ultimate moment interaction chart for the assumed fire damage condition, see Fig. 1, with combined shear loading. (Nielsen 1998)*

This discusser believes that it is within reach to develop the Smith or ISUM procedures to include progressive failure analysis methods to more general purpose assessment of global hull girder strength which can handle longitudinal and horizontal strength, transverse shear strength, and torsional strength for inter frame collapse of intact cross sections as well as of damaged cross sections in a reasonable robust manner.

The review of previously performed benchmark studies in Section 3.3.7 of the performance of various simplified methods for hull girder collapse loads is valuable. However, it could have been interesting if the Committee had related the reported coefficients of variation with the partial safety factor values for the vertical hull girder ultimate bending capacity adopted in the Classification recommended practices discussed in Chapter 2 and in Section 5.2.1.

1.1.4 Non-linear Finite Element Methods

The Committee also reviews applications of Non-linear Finite Element Methods to the ultimate hull girder strength analyses, and the committee must be commended for extracting best practices from these analyses and listing the resulting recommendations for analyses related to steel mono-hulls in Section 3.4.2 of the report. This will be helpful for the future analysts. Ship structures are complex structures because of the many interacting structural-mechanical systems. For direct, first principle, quasi-static structural response calculations of maritime structures the finite element methods have reached a high stage of development. However, the application still poses several difficulties related to modelling of the dynamic loads, degradation of strength, and accuracy. Most often the size and complexity precludes direct modelling of the entire structure with sufficient small mesh sizes. Therefore, the guidance to procedures suggested by the Committee is essential and helpful for practical design.

Even if significant advances have been made in Non-linear Finite Element Analyses of hull girder strength then I find that more work needs to be done. In general it is less likely that the not so experienced analysts can obtain the same accuracy and consistency by using the Non-linear Finite

Element Procedure as he or she can achieve by applying the much simpler Smith or ISUM methods.

1.1.5 Experimental methods

Large scale experiments are in much need to calibrate the performance of the different numerical procedures. In Section 3.5 the Committee reviews some of the experimental methods which were carried out in the past starting with full scale tests performed around hundred years ago and includes also more recent reduced-scale testing of simple box girders.

For the full scale tests I believe that the Committee could have included the breaking into two of the VLCC Energy Concentration during discharge of oil in Rotterdam harbour in 1980. This accident can be considered a full scale test where the loading is very well described by the still water load condition. This hull girder failure was described and analysed by Rutherford and Caldwell (1990).

The Committee reviews experimental torsion tests performed on box girder models with large deck openings and on a relatively large three-hold section of a post-Panamax container vessel subjected to combined vertical bending, vertical shear, and torsion. With background in the research related to the hull girder failure of two container ships, MSC Napoli and MOL COMFORT the Committee in Section 5.3 discusses the special challenges related to the ultimate hull girder strength of container vessels. Besides the influence of lateral loading of the double bottom and the influence of the short duration slamming induced whipping loads on the ultimate load a discussion is presented on the influence of the low torsional stiffness.

The review of the experimental analyses described in Section 3.5 and also the numerical analyses described in Section 5.3.1 on the effect of torsion on the ultimate hull girder collapse loads comments on the special challenges for the complicated stress state for ships with open hulls such as container vessels and some bulk carriers. Here I find it important to stress that the distribution of torsion induced warping stresses cannot in a consistent way be modelled by considering just a segment of the hull. The complete and full hull structure needs to be analysed in order to get the actual distribution of warping induced longitudinal stresses. In addition to this challenge then the warping stress response acts like an in-determinate structural system, as will be described below.

1.1.6 Open hull cross section subjected to torsion

When studying the reduction of the ultimate bending moment capacity of container ships under the influence of a sectional torsional moment M_T I find that it is important to note that the torsional moment is carried partly by a St. Venant torsion part M_s and partly by a warping part M_w.

$$M_T = M_s + M_w$$

The ratio between St. Venant torsion and warping torsion varies along the hull girder and warping dominates at transitions between open and closed cross sections. In these areas a proper analysis of the warping induced normal stresses is important for fatigue life assessments. Most of the literature referenced by the Committee on ultimate hull girder strength does not discuss the relation between St. Venant and warping torsion and in those rare publications where warping is considered specifically the distribution is often calculated for the elastic cross section and then simply introduced as initial normal stresses which a priori reduce the load carrying capacity of the strength elements before executing the progressive bending collapse analysis.

I find that this procedure is questionable and should be improved for the following reason. The warping normal stresses σ_w are orthogonal to the distribution of axial stresses and to the distribution of vertical and horizontal hull girder bending induced stresses:

$$\int_A \sigma_w \, dA = \int_A \sigma_w \cdot y \, dA = \int_A \sigma_w \cdot z \, dA = 0$$

That is, in this respect the distribution of warping stresses resembles other self-equilibrating stress fields such as thermal stresses, shear lag effects, and residual stresses induced by welding. Current simplified procedures for calculating the ultimate hull girder strength do not take into account shear lag effects and thermal stresses. Of course, many such procedures do take into account residual stresses due to welding even if sensitivity analyses show that the effect of welding induced residual stresses on the ultimate hull girder strength can be quite small for actual ship structures.

To give a qualitative description of the effect of the warping induced stresses on the collapse load of ductile structures we can consider the case where the ship hull cross section is subjected to a given torsional hull girder moment and on top of that an increasing, say, vertical bending moment. For containerships the dominant still water loads are hogging loads. Warping induced self-equilibrating, longitudinal stress fields in the ship hull are in this case expected to have negligible effect on the ultimate hull girder bending moment, since once yielding has commenced in the most highly stressed regions further bending loading will produce a redistribution of forces to the lower stressed zones. Provided there is sufficient ductility, this process will continue until the entire tension flange has yielded.

The compressed part of the hull girder may also have the capacity to redistribute self-equilibrating forces across the whole width of the deck or bottom, provided that plate buckling precedes stiffener buckling and thereby allows some redistribution of the longitudinal stresses before the ultimate strength of the component is reached.

The same behaviour is also predicted by the upper bound theorem of the classical theory of plasticity, which says that for sturdy sections where buckling is not the main failure mode, the collapse load will not be affected by self-equilibrating stress fields.

Therefore, if the scantlings of the open hull girder cross sections are such that elastic stiffener buckling does not occur before yielding, then the ultimate hull girder bending moment cannot be expected to be influenced significantly by direct warping stresses associated with a given torsional hull girder moment.

Of course, the moment–curvature relation will be affected by direct stresses due to warping since the stiffness of the hull will be reduced. In the linear elastic regime, the effect of warping stresses will only be to reduce the bending moments that bound this linear elastic regime. In the post-elastic regime, the principal effect of normal warping stresses will be to reduce the stiffness as yielding or buckling progresses from the most highly stressed regions across the width of the deck or bottom. A result of this spread of yielding at a relatively early stage will be a permanent deflection of the hull girder upon unloading and an increase of the St. Venant torsional shear stresses. The permanent deflection will depend on the magnitude of warping induced stresses.

My expectation is that for well-designed container ship structures the warping induced stresses are not expected to have a significant influence of the ultimate hull girder bending load. Provided severe unstable buckling and fracture can be excluded, the main reduction in the ultimate bending moment carrying capability will be caused by the effect of the St. Venant torsion induced shear stresses on the tensile yield stresses of the plate elements. However, warping stresses will affect the moment versus curvature relations.

1.1.7 Ship shaped and marine structures

As an area for future development the Committee discusses in Sections 5.5 and 6.3.3 several investigations of the effect of low temperature on the ultimate strength for ships in an arctic environment. Related to this I would like to point out that the numbers of ships which are fuelled by LNG are increasing in these years. The LNG tanks on such vessels are quite large and the consequences of a spill from these tanks could be an important area of future research.

At present brittle fracture is an unusual event for plate thicknesses smaller than 50 mm at normal temperatures. The committee suggests that interaction of thermal effects and ultimate strength should be examined. I wonder if the committee finds that for such analyses it becomes important to consider the possibility of fracture before ultimate ductile collapse due to welding or fatigue cracks.

Similarly, in addition to the challenges mentioned in the committee report related to strength reductions due to welding of aluminium structures I can also envision that the much lower ductility of aluminium and composite structures will make the existing methods for ultimate strength analysis of steel structures less suitable for aluminium and composite vessels.

I agree on the recommendation to incorporate weld-induced distortions and residual stresses in complex finite element models of intact structures. As indicated in the discussion on the effect of self-equilibrating warping stresses above I do not expect significant effect of the residual stresses for sturdy sections. For this reason recommendations on such effects should be related to the slenderness of the considered structures.

The Committee suggests further research on improved methods for real-world effects such as the effect of weld-induced imperfections. Here it would also be interesting to see what the effect is of gross welding errors on the joints of ship sections due to bad workmanship. In the past we have seen brittle hull girder fracture of containerships, see Fig. 4, so the question is whether brittle fracture could become a problem also today?

Figure 4. *Complete brittle hull girder failure of the container vessel MSC Carla*

Besides a review of recent literature Chapter 6 on Marine Structures gives a good introduction to the existing standards and rules for ultimate strength assessments of offshore structures.
Section 6.3 reviews publications on assessment of ultimate strength of offshore structures. From this literature review it seems that the number of publications on the global ultimate strength of offshore structures is quite small compared to the number of publications related to hull girder ultimate loads.

As a general remark to the Chapters 6-8 then I could wish that the committee had focused more on critical evaluation of the present knowledge situation and given more recommendations and guidance to future research directions.

1.1.8 Benchmark studies

The Committee must be commended for devoting a lot of effort in the two comprehensive benchmark tests reported in Chapter 9.

The first benchmark test consists of a relatively simple part of an offshore topside structure subjected to fire load. The results of this test are described in detail such that other analysts who want to test their own procedures can get good guidance. Temperatures of the structure due to fire loads compare well, but the predicted joint strengths differ significantly. This reviewer is not an expert on calculations of structural fire resistance but wonders whether the scatter of the predicted strength of the joints can be attributed to the way beam stiffness degradation is determined for the joints at elevated temperatures. No information is given on the applied calculation procedures for the strength evaluation of these beam joints.

The purpose of the second benchmark test is to validate the non-linear finite element method to predict the ultimate strength of box girders subjected to pure bending moments. The Committee has here chosen to re-visit the box girders tests performed by Gordo and Guedes Soares in 2009 and used in a benchmark study by Committee III.1 in ISSC 2015.

All the analysts knew the outcome of the experiments, i.e. it must be expected that possible gross modelling errors have been detected by the participants before the results have been handed in. In spite of this fact then the results of the non- linear finite element analyses show considerable scatter. It is unfortunate that the participants have not given details about how welding imperfections have been modelled and which values the analysts have assumed for the material properties. Table 20 only indicates the guaranteed minimum yield stresses. The chosen actual variance of this value and strain hardening coefficients could probably explain some of the significant differences in the calculated ultimate load values.

It is interesting and at the same time disappointing to note that the variance of the results and the correlation with the experiments using these state of the art Non-linear Finite Element Analyses are no better than what was achieved in a benchmark test performed by Committee III.1 24 years ago and reported in ISSC 1994. See Figure 5. Here different computer codes were used for evaluation of the ultimate and post-ultimate capacity of a 1/3 scale model of a steel frigate representing a typical warship hull structure subjected to bending loads (Dow 1991). All the three methods discussed in the present Committee III.1 report were applied, i.e. the Non-linear Finite Element Method, the ISUM method and the Smith beam-column method.

From these results it seems that for inter-frame collapse modes the Non-linear Finite Element Method is no more accurate than the much simpler ISUM and Smith methods. Accurate modelling of the boundary conditions, material properties, and initial imperfections seems more important for this quite thin-walled structure than the choice of method.

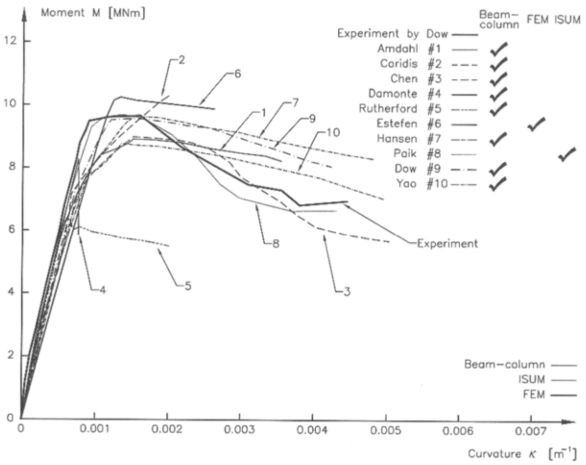

Figure 5. *Calculated and measured moment-curvature relations for 1/3 scale frigate (Dow 1991) based on ISSC 1994 Technical Committee III.1.*

1.1.9 Conclusions and recommendations

In the final chapter the Committee gives a well-balanced set of conclusions and recommendations for future research work. Besides the comments given to the specific parts of the report throughout my discussion, I have a few additional comments:

Evaluations of the ultimate hull girder strength using nonlinear finite element analysis are highly expensive, firstly because of the computational time requirements but also because of the modelling time and the expertise required to develop the nonlinear model and to evaluate the results. The discussions and the benchmark tests on the ultimate strength of box girders subjected to pure bending in the committee report indicate that at present the Non-linear Finite Element Method can hardly be considered as part of a reliable normal ship design process. Even though the Idealized Structural Unit method (ISUM) is much faster than the standard non-linear finite element method, then Smith's beam column method will still in general be the simplest and fastest of the three methods discussed in the report for inter-frame collapse mode analyses. Therefore, the continued development and use of improved simplified methods which can account for different load combinations is still essential. Incorporating torsion, shear loads and transverse pressure into a Smith type analysis method would give the approach much more scope for application in design - not only for the ultimate strength assessment but also for use in scantling optimization, for structural reliability estimates and as part of an emergency response system.

For some container ships it has been shown that the bottom can have relatively low compressive strength under a one-bay-empty condition and without ballasting water in the double bottom. The classification societies have included this effect in the ultimate hull girder collapse analysis by introduction of a double bottom factor. Instead of taking lateral pressure loads into account by using a simple, but relatively large, universal double bottom factor γ_{BD} then the effect of the lateral loading and bi-axial pressure should in a more rational way be included in the stress –strain relations for the individual beam-columns in the Smith simplified procedures. There is a need for a more work towards a precise evaluation of the lateral double bottom loading.

Ultimate strength calculations are needed for evaluation of the reliability of ship structures, and the Committee recommends that more effort is devoted to determine the statistical variation of the effect of material properties on the strength. IACS rules suggest a partial safety factor of 1.1 to account for both modelling and physical uncertainties in the ultimate hull girder calculations. Besides the effect of material factors the physical factors include elements such as: Component scantlings, Initial imperfections, Load types (static or dynamic loads), Thermal loads, Corrosion, etc. In view of the observed scatter in the calculated response in the Committee's benchmark tests, which are characterized by a relatively strict problem definition, known experimental results, small simple box test specimens, experienced analysts, and very detailed finite element modelling, then I wonder whether the Committee still finds that the presently applied partial safety factor γ_C in Eq. (2) is satisfactory? It could be a worthy task for a coming Committee III.1 to make an effort to suggest a proposal for quantification of the modelling uncertainties associated with the three different calculation methodologies and combine this with similar quantified physical uncertainties. I cannot imagine a better set-up for such a task than Committee III.1 which is with fifteen active experts and researchers working for a period of three years and with access to all the previous work performed by the Ultimate Load committees.

A substantial part of the report has been devoted to comparison and discussion of procedures for calculations of the ultimate hull girder collapse loads. A similar comparison of available methods for non-linear static push-over analyses for offshore structures could also be an interesting task for the coming Committee III.1. The committee gives some references to global ultimate strength assessments of offshore structures so work is going on in this area.

The report contains a section on aluminium structures. Investigations of the influence of alloy types, welding procedures and locations, HAZ characteristics and the onset of brittle fractures on the ultimate strength of aluminium components could be a very valuable research topic which could improve the foundation for design of advanced high speed crafts.

1.1.10 Closing Remarks

The committee must be commended for having produced a report with a logical structure, a very large number of recent references, some good discussions of the IACS Harmonized Common Structural Rules for ultimate strength assessments, and for having embarked on two ambitious benchmark tests. The final chapter on conclusions and recommendations is very well balanced and gives identification of important areas requiring future research.

The aim of ISSC is to facilitate evaluation and dissemination of results from recent investigations, to make recommendations for standard design procedures and criteria, to discuss research in progress, to identify areas requiring further research and to encourage international collaboration in promoting these aims. The Committee report certainly lives up to this goal and the report is a good source for critical expert guidance on the status of our existing knowledge on ultimate strength analyses and the report gives recommendations and directions for future research on ductile ultimate strength analyses.

I would again like to congratulate the chairman and the committee members for their comprehensive and insightful report and welcome their comments to my discussion contribution.

1.2 Written and Floor Discussions

1.2.1 Question by Leimart Josefson (written):

Thank you for an excellent report and presentation. Being a participant in the III.1 2015 benchmark, I have some questions on the second benchmark. Can you show what elements (and mesh size) were used? Was the supporting structures now included in the FE-analyses? In 2015 we discovered that some of the stiffeners were made of mild steel. Was that included now in the models? To further improve the FE-analyses it would be good to know in particular an answer to the first question in order to get a better understanding of the outcome of the FE-analyses.

1.2.2 Question by Michael Andersen

A comment about the benchmarks. ISO 19902 has an approach for NL FEA, which should reduce the bias due to the analyst. When doing the benchmarks, the full material curve is important in order to get close to the experiments. This could be added to Volume III.

1.2.3 Question by Robert Sielski

Simplified methods of analysis are based on inter-frame collapse, which is an issue of design. Has the committee found any research as to the required stiffness of transverse members to cause inter-frame collapse rather than compartment collapse?

1.2.4 Question by Torgeir Moan

Have you identified results for ultimate strength, which can be implemented in practice? Such as direct analysis as discussed in the benchmark. In normal design it's the ultimate strength of components which matters. Take an example of stiffened panels, there is a large number of design codes that are different. What is the state of art according to the committee on this issue? We should agree on the best way to design stiffened panels, and thus the current research should give us a better position to decide how to do it.

1.2.5 Question by Philippe Rigo

To Professor Pedersen: A PhD looked at the interaction between the two loads (shear and axial). Anyone getting started on this topic should look into that.

In the second benchmark there is some scatter in the result. But the results are conservative, so it is not bad. Some of the results are not conservative. It seems like participant number 2 ran a number of simulations. It would be interesting in Volume III to show the differences between these results. What are the differences by the same participant getting different results? By that we can find out which hypothesis might give the right results.

1.2.6 Question by George Wang

Heat problem: I have to say I was surprised to see the wide spread observed in the NL FEA. I was more surprised with Preben's data, some were in 1.4 range, other in the 1.6 range. There is a lot of room to grow in the research in this field.

I was also surprised by Preben's talk about the Smith's method. We are as researcher happy to see the method still used. Over the years we realized Smith's method has its limit. It hinges on the assumption that plane sections remain plane. A few years back I was attending a NTNU

PhD defence at Torgeir, since then the hull girder requirement changed. A magic number 1.15 was added to hogging condition. You cannot treat a hull girder as a one dimensional problem anywhere. My suggestion for hull girders is to focus not only on Smiths method, but also on NL FEA. We need a lot of effort on both fronts. I have to give my compliments to Prof. Fujikubo for his torsional analysis. It changed our view to 3D.

1.2.7 Question by Mirek Kaminski

I miss in your results that the people who do the numerical and experimental analysis should be in the same project. That is why e.g. Preben, Torgeir and Carlos are important. Four parts should be included in every research project: a clear statement of a research question, experimental observations, numerical simulations and simplified modelling.

1.2.8 Question by Jeom Kee Paik

I trust the participants on their capability of performing the benchmark. One reason why the strength is underestimated, is that the material properties are based on minimum requirements. The real material properties are much higher. The main objective of benchmark studies should be to compare modelling techniques, hence also material properties should also be changed. For instance, by looking at real material data, for instance supplied by Carlos.

2. REPLY BY COMMITTEE

2.1 Reply to Official Discusser

2.1.1 Introduction

The Committee members wish to thank Professor Terndrup Pedersen for his efforts reviewing the Report, his comments and remarks.

Despite being well represented since the beginning of ISSC, the subject of Ultimate Strength of marine and ship structures is very challenging in many respects. On the one hand, the research carried out under the umbrella of ultimate strength is under continuous and extensive development. On the other hand, the concept of ultimate strength interfaces with many other design disciplines.

In his review of the ISSC 2018 Report "Ultimate Strength", Prof. Terndrup Pedersen recognizes that there is still a significant number of topics and issues regarding ultimate strength design that remain unresolved and open for discussion.

The Committee's main focus was on the ductile behaviour of ships and marine structures and their components under ultimate conditions. Special consideration was given to the practical application of methods. The focus of the Report is on the safe design of marine and ship structures, with emphasis on the application of relevant international design standards such as ISO and NORSOK as well as a number of class society rules.

The objective of the Committee III.1 Technical Report is to outline how the ultimate strength principles can consistently be implemented and applied in design projects. In this sense all necessary definitions, procedures and methods are revisited and explained.

2.1.2 Smith incremental method

With respect to the effect of shear, torsion and self-equilibrating stresses, the Committee agrees with the Discusser that sectional shear should not significantly affect the distributions of longitudinal bending stress, and that plane-sections-remain-plane is valid for conventional ship designs. Professor Pedersen persuasively argues that shear lag and longitudinal variation in sectional shear, as well as bending moment between frames, will have a very little effect on ultimate bending moment. Besides the Nielsen's results, other studies of the interaction between ultimate bending moment and shear (e.g. Paik, 1994) also show that the ultimate bending strength is not significantly reduced by shear until shear force reaches a large percentage of the ultimate shear force.

Professor Pedersen also discusses the effect of torsion, maintaining that the influence on ultimate bending moment is small given that longitudinal warping stresses resemble other self-equilibrating stress distributions such as weld-induced residual stresses. While such stresses contribute to longitudinal stresses in the elastic stress, the question is whether they have any significant effect on ultimate bending moment. The Discusser correctly points out that for sturdily built structure self-equilibrating stresses are not that important. For slender structures, such as the decks of naval vessels, residual stress reduces compressive ultimate strength. Faulkner (1975) discussed the effect of residual stresses on the strength of plating extensively, and Smith (1975) considered its effect on the compressive strength of grillages. When compressive stress due to warping is added to the compression side of longitudinal bending and panel is not stock as in deck part, the effect of warping is significant. Such examples are shown in Tanaka et al (2015) and Tanaka et al (2016).

A limitation of Smith's method is that it assesses the bending strength using structure from one frame spacing. The Committee's report has tried to emphasize that this is inadequate for the study of load combination effects such as bending combined with lateral loads, or the study of general (non-interframe) collapse modes. Hence the current interest in nonlinear FEM, and in the development and extension of Smith's method to these more complex scenarios. The analysis of torsion and combined bending and torsion is similar. There exist already some attempts to extend Smith's method to the problem accompanied by torsion (Tanaka et al (2015) and Tanaka et al (2016)), and by even bottom lateral loads (Tatsumi and Fujikubo (2016) and FJK and TTM (2016)). The Discusser is correct in that at least a full compartment between watertight bulkheads must be considered for realistic modelling of torsion.

Prof. Terndrup Pedersen also suggests that "*it could have been interesting if the Committee had related the reported coefficients of variation with the partial safety factor values for the vertical hull girder ultimate bending capacity adopted in the Classification recommended practices discussed in Chapter 2 and in Section 5.2.1*".

In the Committee's opinion it would be erroneous to relate the CoVs with the partial safety factor values for the vertical hull girder ultimate bending capacity from the Classification practices. That would be wrong in the sense that the factors found in the Classification practices relate to specific types of vessels. How confident could someone be to apply a CoV to entire group of vessels without having analysed a large enough sample of structures? The reliability analysis would need an expression to relate the random variables, although it is common practice to consider random variables unrelated to each other.

In another sense, one could relate the two, but then again it is very easy for someone to discredit this attempt on the basis that there are differences in vessel types, especially in FPSOs, where many of these vessels are modified oil tankers.

Luckily for the analyst, or rather the researcher, they can simply express their opinion and expand knowledge based on the feedback they get. Reliability studies always return a solution and many solutions may arise from various input. It is very difficult here to discern between wrong and right answers.

The Discusser rightly points out that the effect of elevated temperature on residual strength of fire- and/or explosion-damaged structure has not been adequately investigated up until now. Realistic fire/damage scenarios are needed representing the distribution of temperature in the structure and its function of time during fire events to properly assess the risk of hull collapse during a fire for a given load/ballast condition.

2.1.3 Non-linear Finite Element Methods

The Committee does not agree with the Discusser that NLFEM is “expensive” because of computation time and modelling effort in the economic sense. Although the computation time is often high and the modelling effort requires skilled analysts, the total economic cost is still a very small part of the total project investment for a complex asset such as a ship or offshore structure. Perhaps what is more important is the timescale and reliability of NLFEM compared to other methods. The use of NLFEM is still not considered a part of a normal ship design process, and may never need to be if simplified methods provide sufficient reliability. The Smith method, such as described in IACS CSR, continues to be the de-facto standard for ultimate strength assessment in standard design. IACS CSR gives provision for alternative methods such as NLFEM, but this approach is seldom expected to be taken.

However, NLFEM still holds great value for understanding more complex load conditions and verifying the findings of simplified methods. It will be essential in the future development of multiple load combinations within simplified methods such as torsion, shear and lateral pressure. In this respect, the Committee agrees that the further development of Smith type methods to quantify the effects of these load combinations is a priority for future ultimate strength research, using NLFEM to verify and validate these approaches appropriately. It is discussed in the ISSC report and it is a worthy outcome to be highlighted in the conclusions.

2.1.4 Experimental methods

Energy Concentration remains the most important case study and benchmark for ultimate strength bending moment prediction methods. It was discussed extensively in previous Committee III reports. It was omitted in this section only because it is not strictly an “experiment”.

2.1.5 Open hull cross section subjected to torsion

The Committee agrees that container ship double bottom strength is an important issue, especially in light of recent accidents. The development of more direct methods to quantify this effect is important and is linked to the discussion above with regard to multiple load combinations. It is hoped that extended progressive collapse methods will be able to better quantify the complex load pattern of biaxial in-plane and lateral loading.

2.1.6 Ship shaped and marine structures

LNG tanks are quite large and the Committee agrees that the consequences of a spill from these tanks could be an important area of future research.

The consequences of an LNG spill should indeed be investigated to answer questions such as (1) how extensively could a hull structure be affected by a spill; (2) how much embrittlement of the hull material can be expected from a spill; (3) what is the likelihood of localized brittle fracture and its effect on overall hull strength. Whether an LNG spill could result in brittle fracture of entire hull would probably depend on a number of factors.

The Committee is aware of the potential fracture failure before the ultimate strength of the structures or structural components is reached, as revealed by previous experimental or numerical investigations (Qian, X., and Zhang, Y., 2015; Ahmed, A. and Qian, X., 2016; and Li, T. at al, 2017). These studies indicate that the fracture failure may occur at a deformation level earlier than the ultimate limit state. Engineering design guidelines, e.g. the failure assessment diagram outlined in BS7910, have incorporated a procedure to assess the competing failure of the fracture and plastic collapse. The assessment of multiple failure modes (plastic collapse, fracture, fatigue, etc.) is currently a very important topic for the research community as a part of the effort to deliver comprehensive integrity assessment of the ship shaped and marine structures with resilient solutions.

On the other hand, the Committee is also aware that there is a separate committee (III.2 in ISSC) considering the fatigue and fracture failure. The Committee hence recommends a more close interaction between III.1 and III.2, in the form of joint committee meetings for future ISSC committees.

The Committee fully agrees with the Discusser that the much lower ductility of aluminium and composite structures will make the existing methods for ultimate strength analysis of steel structures less suitable for aluminium and composite vessels. There is a strong need to examine the competing failure between the fracture and plastic collapse for the aluminium structures and composite structures. The existing approaches without explicit consideration of the fracture/fatigue failure will lead to un-conservative estimations on the ultimate strength of these structures.

The effect of residual stress on thin sections will indeed induce distortions to the section, and therefore initial geometric imperfections. This will lead to subsequent changes in the slenderness of the section. Some of the recent works (Chen, B. Q., and Soares, C. G., 2016) address this topic in detail.

In relation to the previous comments raised by the Discusser, the residual stress may impose a strong effect on the fatigue strength and fracture resistance of the structural component. In this regard, joint works between committee III.1 and III.2 are again necessary.

Bad workmanship has been a critical source causing brittle fracture in the welded connections.

With improvement on the workmanship requirements, the brittle fracture incidents caused by inadequate workmanship tend to decrease. Nevertheless, brittle fracture failure has remained an important failure mode, as it occurs with little prior indications (plastic deformations). For ships navigating around the Arctic or marine structures operating near the Arctic, or welded connections made of high strength steels, the brittle failure caused by a low ambient temperature still remains a concern to the practicing engineers. Some researchers have performed investigations of this topic (Rondon, A., and Guzey, S., 2017; Chen, S. et al. 2016).

For a better coverage of this topic, collaborative efforts between committee III.1 and III.2 are necessary.

The Committee agrees with the Discusser’s comment on the need of critical evaluation of the present knowledge situation and attempts to give more recommendations and guidance to future research directions.

The Committee is probably a bit over-constrained by the conventional definition of "Ultimate Strength". With the increasing importance of multi-disciplinary research, the future focus of "Ultimate Strength" committee should include coverage of multiple failure mechanisms and research efforts towards the resilient marine and ship structures.

Regarding ultimate strength of marine structures, the main challenge was related to the existing literature; explicitly on the topic of the ultimate strength of marine structures, which seems to be quite limited. ISSC 2015 claimed the same and ISSC 2012 did not have any reference on this subject.

Significant activity was observed with respect to ALS design, with the use of NL-FE to assess ship impact, dropped objects, fire loads and explosions. For these cases, the focus is on the large and inelastic deformation.

As an example it should be noted that in the last release of NORSOK N003, the design energy for boat impact to offshore installations in the North Sea was increased significantly. Also, the type of bow to be considered was discussed reflecting the size of OSV operating in the North Sea, (Kvitrud). In this new context, the traditional methods to assess ship impact (as described in NORSOK) may not be sufficient to assess the response to ship impact, thus NLFE has become popular to better document the response to such events. The increased use of FE can also be explained with the increased computational capacity of today’s computers.

It is to be highlighted that, for some topics, there has been an advance on the methods, but this was not found to be the case for all the items related to global ultimate strength assessment of offshore structures.

In this setting, it was difficult to identify gaps between regulations (which have not suffered major updates) and state of the art and therefore give recommendations. DNV GL RP C208 include some guidelines on material model, FE model etc. for this type of calculations.

It is acknowledged that a comparison of available methods for non-linear static pushover analysis could be an interesting topic. As far as the Committee can see, the most common tool used for assessment of frame structure is USFOS, which includes features to calibrate member capacity per different codes. Unfortunately, the Committee is not aware if other tools are commonly used.

2.1.7 Benchmark studies

Two benchmark studies were performed by the Committee members. The objective of the first Benchmark was to predict the strength of structural joints of topside structures subjected to fires in order to compare different techniques (and solvers) in assessing the strength of these structures. The objective of the second Benchmark study was to validate the ultimate strength of box girders under the pure bending moment using various finite element solvers through comparison with the box girder buckling tests. This is a revisiting of Benchmark of ISSC 2015, performed this time to identify possible sources of uncertainties and errors with reference to correct modelling and assessment of buckling tests.

Joint under fire loads

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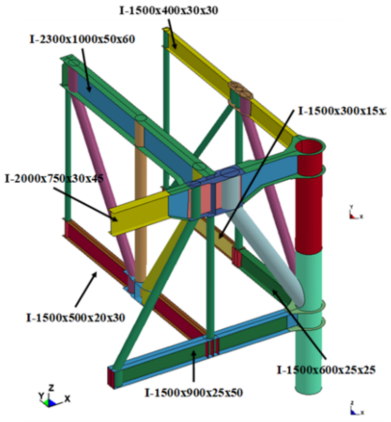
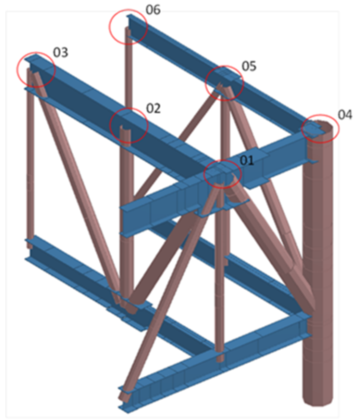
<tr><td>
</td><td>
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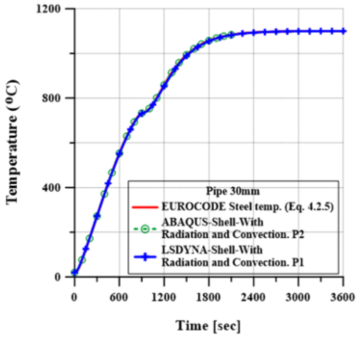
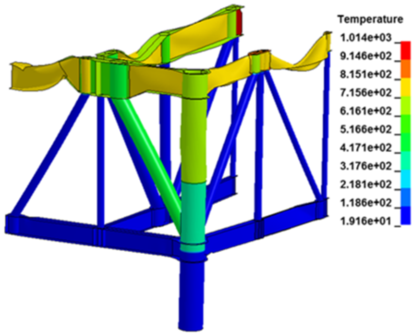
<tr><td>
</td><td>
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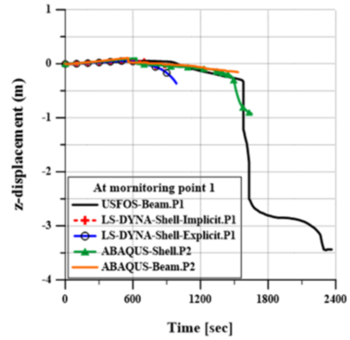
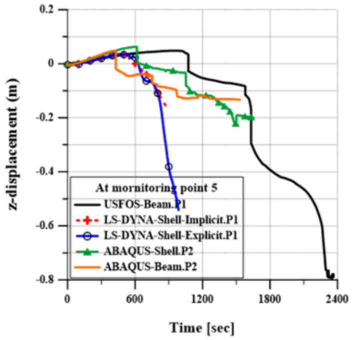
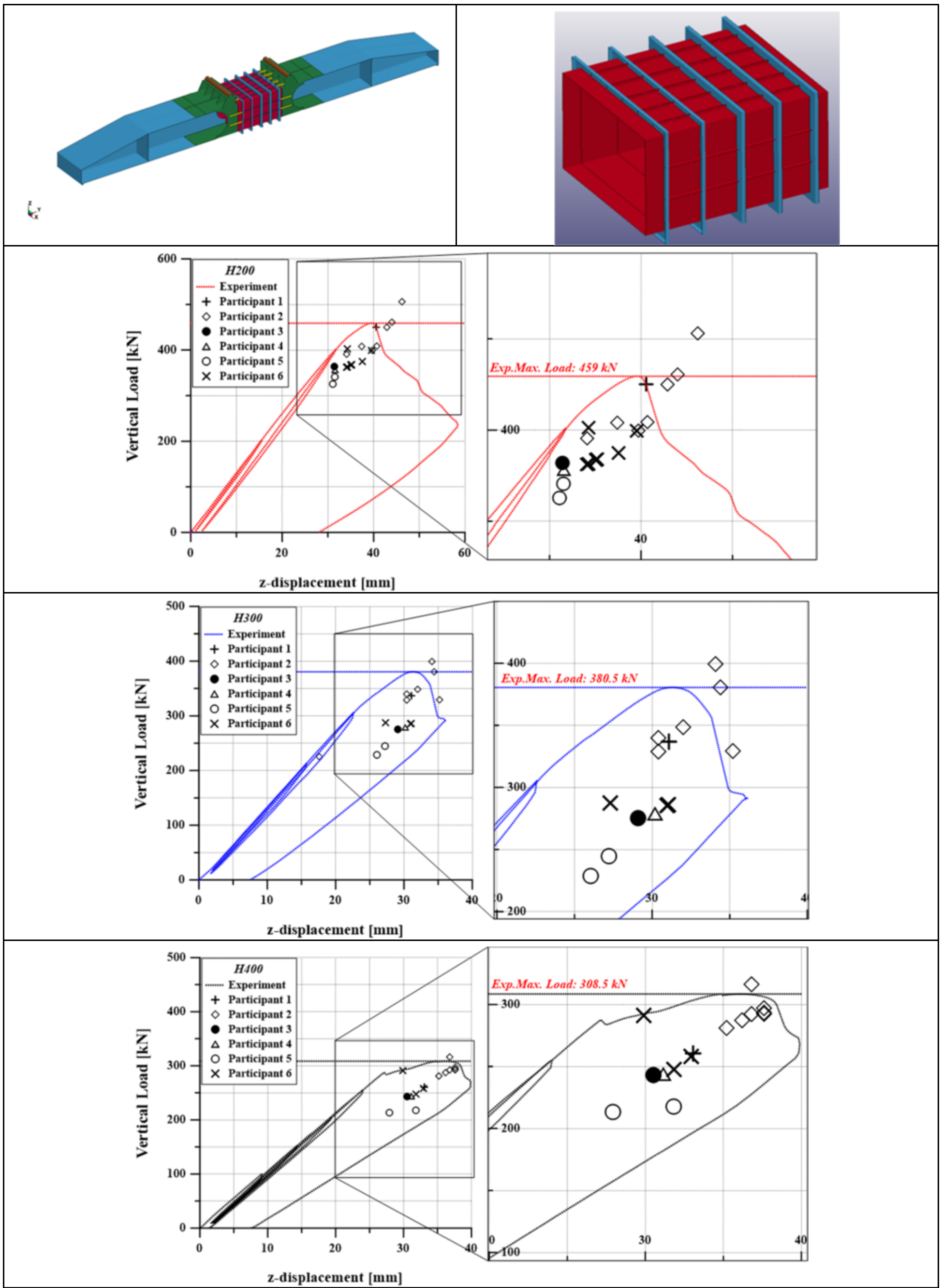
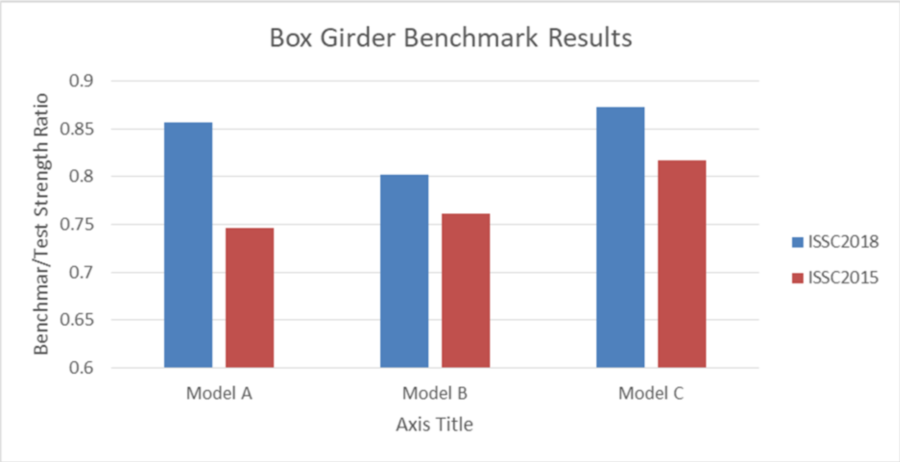
<tr><td>
</td><td>
</td></tr>
<tr><td>Ultimate strength calculation, MP1</td><td>Ultimate strength calculation, MP5</td></tr>
<tr><td colspan="2">Conclusions: Good agreement for temperature calculation. Significant scatter in the results attributed to the underlying simulation assumptions made by the analysts.</td></tr>
</table>

Box girders subjected to bending



Conclusions: The FEM analyses generally predict a lower ultimate strength compared to the equivalent experimental results. The box girder benchmark completed in the 2015 and 2018 ISSC reports show consistent NLFEM results which compare poorly to a set of physical tests. This does not indicate gross modelling errors, but rather the reliability of the experimental tests.

The results of box girder benchmarks ISSC2015 and ISSC2018 are compared on figure below.



The Discusser notes "*All the analysts knew the outcome of the experiments, i.e. it must be expected that possible gross modelling errors have been detected by the participants before the results have been handed in. In spite of this fact then the results of the non- linear finite element analyses show considerable scatter*." This is based on the assumption that "good job gives good results". In other words if there is a discrepancy between tests and benchmark results it's mainly due to "gross errors in modelling". The FE models used by the analysts are based on the same geometries and subjected to detailed quality assurance, and basic material and load parameters used in modelling are the same for all benchmark participants. However, in order to have a good independent benchmark, procedures of NLFE analyses and analysis parameters were freely chosen by the analysts, which probably may be one of the main reasons for differences in results.

The Committee does not agree with the Discusser's comments stating that the relative accuracy of NLFEM is no better than similar benchmark tests conducted in ISSC 1994. The box girder benchmark completed in the 2015 and 2018 ISSC reports show consistent NLFEM results which compare poorly to a set of physical tests. This does not indicate gross modelling errors, but rather the uncertainties in the experimental tests. Furthermore, the current development of NLFEM software enables modelling of much larger and more complex structures. This leads to revised objectives for benchmark tests, whereby the modelling uncertainties derived by the analyst are perhaps more relevant than the accuracy of the solution scheme itself.

The Committee therefore believes that quantifying modelling uncertainties and deriving a more intelligent partial safety factor in a limit state ultimate strength approach is a challenging but worthy task for future committees. The ISSC committees in 2015 and 2018 attempted to quantify the modelling uncertainty on small scale box girders using NLFEM. The findings showed that there was relatively significant uncertainty in the ultimate strength value when different analysts used different software and calculation choices. The results from the benchmark studies in the current ISSC report show similar uncertainty. Placing a number on this is challenging because there are human factors related to the skills and experience of the analysts, as well as calculation uncertainties inherent in the use of different methods.

2.1.8 Conclusions and recommendations

The Committee agrees with the Discusser's proposal about performing a comparison of available methods for non-linear static pushover analyses for offshore structures as being an interesting task for the coming Committee III.1.

2.1.9 Closing remarks

The Committee fully supports Professor Terndrup Pedersen's view that the work needs to be continued, with the mandate including development of a standard FEM modelling and analysis procedure for the assessment of ultimate strength of marine and ship structures.

There is still a lot of work to be done in the field of ultimate strength design.

2.2 Reply to Written and Floor Discussion

2.2.1 Reply to Leimart Josefson

The type of elements and mesh is well presented in the report.Yes, the whole supporting structure was included in the analysis. This findings of 2015 benchmark was not taken into account. Instead the exact geometry and material data as presented by the Carlos team was applied. In our opinion, the element type and mesh design is only one of a large number of parameters to be investigated under the benchmark. The key target is to develop and follow in benchmarks well-established analysis procedures where parameters affecting NL FE solvers should be outlined and clarified.

2.2.2 Reply to Michael Andersen

I agree with the comments. Material data is a key issue and should be better documented and based on experimental teamwork. When buying aluminum or steel the yield strength has a large variation. Considering blast wall design or automotive industry, for instance, the designer sends the material to an independent lab to obtain the material data. However, for our benchmark the exact material data was not available.

2.2.3 Reply to Robert Sielski

No, these were not found.

2.2.4 Reply to Torgeir Moan

What you are asking is continuously important. A PhD or other type of study will be necessary to develop a comparison between different equations and methods. This kind of work should be continuously updated. We should have some kind of research bible under continuous update. We should also have guidelines how to use FEA, such as the ones by DNVGL and Lloyds. I think to compare FEA and design codes should be at the heart of our work. But it is hard task because there are many other important tasks associated with committee work. Maybe later we could talk about a research platform in which this could be done.

2.2.5 Reply to Philippe Rigo

This is already presented in the report. Maybe the non-conservative results are due to dynamic effects, so some dynamic phenomena make it not conservative. I am not sure. But it is very important to discuss when it happens. It is important to realize that when you do analysis for the design, you have no reference to compare with. Here we are lucky, even with the poor reference, we have one. The question is further: how to do analysis in the design projects?

2.2.6 Reply to George Wang

Regarding your disappointment in the first point: in the NL FEA, when we perform analysis you have 20 or 30 analysis parameters to choose and decide. For 90% of the time you do not touch them, by default, but a curious analyst will change them. This changes the analysis results. The guidelines of different codes are similar, probably we should make a parameter study in order to show what leads to over- or underestimation of ultimate strength. Then we can educate it to students, so it becomes common knowledge. Now it is a free choice, so the results are different. We should not be disappointed, this is the way it is. Further I agree with your other comment.

2.2.7 Reply to Mirek Kaminski

I want to take this one step further. You should first do the benchmark, then the experiment. Then adapt the experiment.

2.2.8 Reply to Jeom Kee Paik

My ad-hoc comment is that the committee has investigated the facts of material yield and strength. The study including variation of the material data has been carried out and show that such variation has minor effects on the results. Even with a larger spectrum of material characteristics, we did not come to the level of strength we have from experiments.

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