Computational Models of Argument
C. Reed et al. (Eds.)

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doi:10.3233/FAIA240314

Complexity of Semi-Stable Semantics in Abstract Dialectical Frameworks

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Abstract. Abstract dialectical frameworks (ADFs) have been introduced as a formalism for modeling and evaluating argumentation, allowing for general logical acceptance conditions of arguments. Different criteria used to settle the acceptance of arguments are called semantics. Two-valued semantics of ADFs reflect the 'black-and-white' character of classical logic in non-monotonic frameworks. Stable semantics of ADFs were introduced to exclude cycles of self-justification of arguments among two-valued models. The stable semantics faces the challenge of potential non-existence of stable models. However, one might still want to draw conclusions even in case that an ADF has no two-valued models or stable models. Recently, the notions of semi-two-valued semantics and semi-stable semantics were introduced for ADFs. In the current work, we study the computational complexity of these two novel semantics. We show that the complexity of the semi-stable semantics is in general one level up in the polynomial hierarchy, compared to the stable semantics. We study the prominent reasoning tasks of credulous and skeptical reasoning, as well as the verification problem.

Keywords. Abstract dialectical frameworks, computational complexity, semi-two-valued semantics, semi-stable semantics

1. Introduction

Interest in argumentation theory is growing among artificial intelligence researchers due to its diverse applications and formalisms for evaluating arguments [1,2]. Central to evaluation of arguments is the sub area of abstract argumentation, at the heart of which lie Dung's argumentation frameworks (AFs) [3]. While AFs model individual attack relations among arguments, a number of generalizations have been proposed [4], with abstract dialectical frameworks (ADFs) [5] being an expressive generalization of AFs in which logical relations among arguments can be represented. The higher expressivity of ADFs [6], leading to increased computational complexity in almost all reasoning tasks compared to AFs, has been analyzed in the literature [7,8,9].

In ADFs, acceptance of arguments (truth-value of arguments) is indicated under principles governed by several types of semantics, primarily defined based on three-valued interpretations, which are a form of labeling [10]. The notion of a stable model in ADFs is grounded on the concept of a two-valued model. Stable semantics for ADFs draw inspiration from the concept of stable models in logic programming and answer set programming [11]. In logic programming, certain minimal models exclude self-justifying cycles of atoms. Similarly, stable models in ADFs avoid argument support cycles where each argument justifies itself without any external justification in a two-valued model.

Similarly as in logic programming, stable semantics is a prominent semantics in the realm of argumentation. However, it is worth noting that an argumentation formalism may not always have a stable model. To address situations where an AF lacks a stable model, the concept of semi-stable semantics for AFs was introduced [12] (initially under a different name) and further explored in subsequent works [13], including algorithmic approaches to semi-stable semantics for AFs [14].

For ADFs, which generalize AFs, the concept of semi-stable semantics has been developed based on semi-two-valued models [15]. This approach approximates stable semantics when an ADF lacks a stable model. On one hand, if a given ADF lacks any two-valued model, it also lacks any stable model. For instance, if an ADF has an argument a with a self-attack, denoted in ADFs via a Boolean formula for the acceptance condition of a, e.g., by $\varphi_a : \neg a$, such a statement leads to the absence of two-valued models and stable models as a consequence. On the other hand, an ADF may possess two-valued models, but none of them may be a stable model because of the presence of support cycles, where arguments justify each other in a loop.

The semi-two-valued semantics and semi-stable semantics were introduced [15] as a remedy for cases where an ADF does not have any stable model due to the absence of a two-valued model. Semi-two-valued semantics for ADFs are more robust in the sense that each ADF has at least one semi-two-valued interpretation. Furthermore, if an ADF has a two-valued model, then the sets of semi-two-valued interpretations and two-valued models are equal. Additionally, the sets of stable models and semi-stable interpretations coincide [16].

Whereas several fundamental properties of semi-two-valued and semi-stable semantics for ADFs have been established [15,16], the computational complexity under these semantics has not been studied. This work closes this gap by studying the complexity of the central reasoning tasks under the semi-two-valued and semi-stable semantics.

We show complexity results for deciding credulous and skeptical acceptance, i.e., whether a given argument is assigned to true in at least one or all models under the semantics, respectively. We prove that the credulous acceptance problem and the skeptical acceptance problem for both semi-two-valued semantics and semi-stable semantics reside on the third level of the polynomial hierarchy, i.e., are Σ_3^{P}-complete and Π_3^{P}-complete, respectively. The verification problem is an important cornerstone to show these results and is viable in its own right. The verification problem asks whether a given interpretation is an interpretation under the chosen semantics. The verification problem is in Π_2^{P} for both semi-two-valued and semi-stable semantics, and being hard for the former. In summary, we show that ADFs have higher computational complexity under these semantics when compared to AFs [17,18] and that semi-two-valued semantics and semi-stable semantics experience a complexity jump, compared to stable semantics of ADFs.

2. Abstract Dialectical Frameworks

We summarize key concepts of abstract dialectical frameworks [19,5].

Definition 1. An abstract dialectical framework (ADF) is a tuple $D = (A, L, C)$ where: 1. A is a finite set of arguments (statements, positions); 2. $L \subseteq A \times A$ is a set of links among arguments; 3. $C = \{\varphi_a\}_{a \in A}$ is a collection of propositional formulas over arguments, called acceptance conditions.

An ADF can be represented by a graph in which nodes indicate arguments and links show the relations between arguments. Each argument a in an ADF is labelled by a propositional formula, called acceptance condition, φ_a over $par(a)$, where $par(a) = \{b \mid (b, a) \in L\}$. The acceptance condition of each argument clarifies under which condition the argument can be accepted.

A *three-valued interpretation* v (for D) is a function $v : A \mapsto \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$ that maps arguments to one of the three truth values true ($\mathbf{t}$), false ($\mathbf{f}$), or undecided ($\mathbf{u}$). For reasons of brevity, we will sometimes shorten the notation of three-valued interpretations $v = \{a_1 \mapsto t_1, \ldots, a_m \mapsto t_m\}$ as follows: $v = \{a_i \mid v(a_i) = \mathbf{t}\} \cup \{\neg a_i \mid v(a_i) = \mathbf{f}\}$. For instance, $v = \{a \mapsto \mathbf{f}, b \mapsto \mathbf{t}\} = \{\neg a, b\}$ (arguments assigned undecided are not explicated). An interpretation v is called *trivial*, and v is denoted by $v_{\mathbf{u}}$, if $v(a) = \mathbf{u}$ for each $a \in A$. Furthermore, v is called a *two-valued interpretation* if for each $a \in A$, $v(a) \in \{\mathbf{t}, \mathbf{f}\}$.

Truth values can be ordered via the information ordering relation $<_i$, given by $\mathbf{u} <_i \mathbf{t}$ and $\mathbf{u} <_i \mathbf{f}$, with no other pair of truth values being related by $<_i$. The relation $\leq_i$ is the reflexive closure of $<_i$. Interpretations can be ordered via $\leq_i$ w.r.t. their information content, i.e., $w \leq_i v$ if $w(a) \leq_i v(a)$ for each $a \in A$.

Given an interpretation v (for D), the partial valuation of φ_a by v is $v(\varphi_a) = \varphi_a^v = \varphi_a[b/\top : v(b) = \mathbf{t}][b/\bot : v(b) = \mathbf{f}]$, for $b \in par(a)$. Note that in this work we assume that $D = (A, L, C)$ is a finite ADF and v is an interpretation of D. Given an argument $a \in A$, a is called *acceptable* w.r.t. v if φ_a^v is irrefutable (a tautology) and a is called *deniable* w.r.t. v if φ_a^v is unsatisfiable. Semantics for ADFs can be defined via the *characteristic operator* Γ_D, presented in Definition 2.

Definition 2. Let D be an ADF and let v be an interpretation of D. Applying Γ_D on v leads to v' such that for each $a \in A$, $v'(a) = \mathbf{t}$ if φ_a^v is irrefutable, $v'(a) = \mathbf{f}$ if φ_a^v is unsatisfiable, and $v'(a) = \mathbf{u}$, otherwise.

Most types of semantics for ADFs are based on the concept of admissibility. An interpretation v for a given ADF D is called *admissible* iff $v \leq_i \Gamma_D(v)$; it is *preferred* iff v is $\leq_i$-maximal admissible; it is *complete* iff $v = \Gamma_D(v)$; it is the *grounded* interpretation of D iff v is the least fixed point of Γ_D; it is a *(two-valued) model* iff v is two-valued and $\Gamma_D(v) = v$; it is *stable* iff v is a two-valued model of D and $v^{\mathbf{t}} = w^{\mathbf{t}}$, where w is the grounded interpretation of the *stb*-reduct $D^v = (A^v, L^v, C^v)$, where $A^v = v^{\mathbf{t}}$, $L^v = L \cap (A^v \times A^v)$, and $\varphi_a[p/\bot : v(p) = \mathbf{f}]$ for each $a \in A^v$. The set of all σ interpretations for an ADF D is denoted by $\sigma(D)$, where $\sigma \in \{adm, prf, com, grd, mod, stb\}$ abbreviates the different semantics in the obvious manner.

Example 1. *An example of an ADF $D = (A, L, C)$ is shown in Figure 1. To each argument a propositional formula is associated, the acceptance condition of the argument. For instance, the acceptance condition of c, namely, $\varphi_c : \neg a \vee \neg b$ states that c can be accepted in an interpretation where either a is denied or b is denied.*

In D the interpretation $v_1 = \{a \mapsto \mathbf{u}, b \mapsto \mathbf{t}, c \mapsto \mathbf{u}\} = \{b\}$ is an admissible interpretation. Since $\Gamma_D(v_1) = \{a \mapsto \mathbf{f}, b \mapsto \mathbf{t}, c \mapsto \mathbf{u}\}$ and $v_1 \leq_i \Gamma_D(v_1)$. Furthermore,

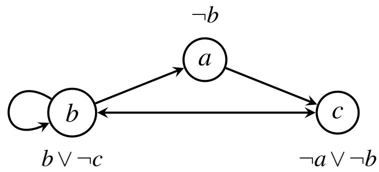

Figure 1. The ADF of Example 1

$prf(D) = \{v_2, v_3\}$, where $v_2 = \{a, \neg b, c\}$ and $v_3 = \{\neg a, b, c\}$. In addition, both v_2 and v_3 are two-valued models of D and they are also complete interpretations of D. While $grd(D) = v_{\mathbf{u}}$. Thus, $prf(D) = mod(D) = \{v_2, v_3\}$, and $com(D) = \{v_2, v_3, v_{\mathbf{u}}\}$.

To investigate whether v_2 is a stable model, we evaluate the stb-reduct $D^{v_2} = (A^{v_2}, L^{v_2}, C^{v_2})$. Here, $A^{v_2} = \{a, c\}$, $L^{v_2} = \{(a, c)\}$, and $\varphi_a : \neg\bot \equiv \top$ and $\varphi_c : \neg a \vee \neg\bot \equiv \top$. Since $grd(D^{v_2}) = \{a, c\}$ and $w^{\mathbf{t}} = v_2^{\mathbf{t}}$, v_2 is a stable model of D. However, v_3 is not a stable model of D. Since $w^{\mathbf{t}} \neq v_3^{\mathbf{t}}$, where $w = grd(D^{v_3}) = \{c\}$. Intuitively, model v_3 is not a stable model of D since in v_3 the acceptance of b depends on b itself, resulting in a cyclic justification. Thus, v_3 violates the main condition of stable semantics, that a stable model should have no self-justifying cycles of atoms. We find that $stb(D) = \{v_2\}$.

An ADF D may not have any stable model, intuitively due to two reasons. The first reason being 1. the existence of support cycles in two-valued models of D. That is, when there is no constructive proof for arguments that are assigned to **t** in a two-valued model, then the two-valued model is not a stable model (see Example 1 for an instance). On the other hand, 2. if $mod(D) = \emptyset$ then there are no stable models, i.e., D does not have any two-valued model to pick a stable model.

As an instance of an ADF that does not have any two-valued model consider ADF $D = (\{a, b\}, \{\varphi_a : \top, \varphi_b : \neg a \vee \neg b\})$. ADF D has a preferred interpretation $v = \{a\}$, which is also the grounded interpretation of D. Although there is no doubt about the acceptance of a in D due to the constructive proof for the acceptance of a, D does not have any two-valued model, and consequently, D does not have any stable model.

2.1. Semi-stable Semantics for ADFs

In this section, we rephrase the concepts of semi-two-valued and semi-stable semantics for ADFs from [15]. In ADFs, to define the notion of stable semantics as it is in [5], we first introduce the notion of two-valued semantics. Then, a two-valued model is called a stable model if it does not contain any support cycles. Due to this distinction between two-valued models and stable models in ADFs, different levels of semi-stable semantics can be considered. A similar method to the one presented in [5] for stable semantics is followed in [15] to introduce the concept of semi-stable semantics. In Definition 3, the notion of semi-two-valued model is introduced as presented in Definition 9 in [15].

Definition 3. Let D be an ADF and let v be an interpretation of D. An interpretation v is a semi-two-valued interpretation of D if the following conditions hold: $v \in com(D)$, and $v^{\mathbf{u}}$ is $\subseteq$-minimal among all $w^{\mathbf{u}}$ s.t. w is a complete interpretation of D.

The set of semi-two-valued interpretations of D is denoted by $semi\text{-}mod(D)$. Note that when an ADF has a two-valued model, then the set of semi-two-valued interpreta-

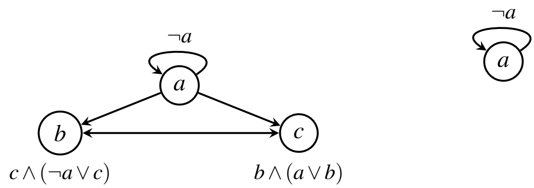

Figure 2. ADF D of Example 2 (left), sub-reduct D^{v_2} of D (right)

tions and the set of two-valued models coincide. Note that according to Definition 3, $v \notin \mathit{semi\text{-}mod}(D)$ if either $v \notin \mathit{adm}(D)$ or there exists an admissible interpretation w such that $v^* \subset w^*$, where $v^* = v^{\mathbf{t}} \cup v^{\mathbf{f}}$. We introduce the concept of semi-stable models over the notion of semi-two-valued interpretations in Definition 4.

Definition 4. Let D be an ADF and let v be a semi-two-valued interpretation of D. An interpretation v is a semi-stable interpretation of D if $v^{\mathbf{t}} = w^{\mathbf{t}}$ s.t. w is the grounded interpretation of sub-reduct $D^v = (A^v, L^v, C^v)$, where $A^v = v^{\mathbf{t}} \cup v^{\mathbf{u}}$, $L^v = L \cap (A^v \times A^v)$, and $\varphi_a[p/\bot : v(p) = \mathbf{f}]$ for each $a \in A^v$.

The set of semi-stable interpretations of D is denoted by $\mathit{semi\text{-}stb}(D)$. Note that in Definition 4, in sub-reduct D^v we assume that v is a semi-two-valued interpretation (complete interpretation) of D, thus D^v may contain an argument that is assigned to $\mathbf{u}$. Intuitively, a complete interpretation v is a semi-stable interpretation if $v^{\mathbf{u}}$ is $\subseteq$-minimal among complete interpretations of D and there exists a constructive proof for arguments which are assigned to $\mathbf{t}$ in v, in case all arguments which are assigned to false in v are actually false.

Since the notion of semi-stable semantics are introduced over the concept of semi-two-valued semantics for ADFs, each semi-stable interpretation of D is a semi-two-valued interpretation. Example 2 illustrates the notion of semi-stable semantics of ADFs.

Example 2. *Let $D = (\{a,b,c\}, \{\varphi_a : \neg a, \varphi_b : c \wedge (\neg a \vee c), \varphi_c : b \wedge (a \vee b)\})$ be an ADF, depicted in Figure 2 (on the left). It holds that $\mathit{prf}(D) = \{v_1, v_2\}$ with $v_1 = \{b,c\}$ and $v_2 = \{\neg b, \neg c\}$. In addition, both v_1 and v_2 are complete interpretations of D. However, none of them are a two-valued model. Thus, D does not have any stable model. Furthermore, both v_1 and v_2 are semi-two-valued interpretations of D, since $v_1^{\mathbf{u}} = v_2^{\mathbf{u}} = \{a\}$. However, we show that only v_2 is a semi-stable interpretation of D. To this end, we first evaluate sub-reduct D^{v_2}, where $D^{v_2} = (\{a\}, \{\varphi_a : \neg a\})$, depicted in Figure 2 (on the right). Since $w^{\mathbf{t}} = v_2^{\mathbf{t}} = \emptyset$, it holds that v_2 is a semi-stable interpretation of D.*

On the other hand, v_1 is not a semi-stable interpretation of D. We have $D^{v_1} = D$ and $\mathit{grd}(D^{v_1}) = \emptyset$, while $v_1^{\mathbf{t}} = \{b,c\}$, i.e., $w^{\mathbf{t}} \neq v_1^{\mathbf{t}}$. Thus, $v_1 \notin \mathit{semi\text{-}mod}(D)$.

3. Computational Complexity

We analyse the complexity under semi-two-valued semantics and semi-stable semantics for the standard reasoning tasks of ADFs [8].

We make use of standard complexity classes of the polynomial hierarchy [20]. For an introduction to complexity theory, we refer the reader to standard text books in this field [21]. In particular, P is the class of decision problems decidable in polynomial time, NP is the class of decision problems decidable in polynomial time with a non-

Table 1. Novel complexity results for ADFs.

semantics	verification	credulous	skeptical
semi-two-valued semantics	Π_2^{P}-complete	Σ_3^{P}-complete	Π_3^{P}-complete
semi-stable semantics	in Π_2^{P}	Σ_3^{P}-complete	Π_3^{P}-complete

deterministic algorithm. The class Σ_i^{P} contains all decision problems decidable in non-deterministic polynomial time with access to an oracle in $\Sigma_{i-1}^{\mathsf{P}}$ (i.e., a problem in $\Sigma_{i-1}^{\mathsf{P}}$ can be decided in constant time), for $i > 0$, $\Sigma_0^{\mathsf{P}} = \mathsf{P}$ and $\Sigma_1^{\mathsf{P}} = \mathsf{NP}$. Class Π_i^{P} is the complementary class of Σ_i^{P}.

We recall main decision problems of ADFs.

Definition 5. Let $D = (A, L, C)$ be an ADF and $\sigma \in \{\mathit{semi\text{-}mod}, \mathit{semi\text{-}stb}\}$. We define the following decision problems.

- Given a three-valued interpretation v the *verification problem*, denoted by $Ver_\sigma(v, D)$, asks whether $v \in \sigma(D)$.
- Given $x \in \{\mathbf{t}, \mathbf{f}\}$, and $a \in A$,
  - * the *credulous (acceptance) problem*, denoted by $Cred_\sigma(a, x, D)$, asks if there exists $v \in \sigma(D)$ s.t. $v(a) = x$, and
  - * the *skeptical (acceptance) problem*, denoted by $Skept_\sigma(a, x, D)$, asks if for each $v \in \sigma(D)$ we find that $v(a) = x$.

We summarize our results in Table 1. In words, complexity of reasoning under semi-two-valued semantics and semi-stable semantics is one level up in the polynomial hierarchy, compared to the stable semantics of ADFs.

We first investigate the complexity of $Ver_\sigma(v, D)$, for $\sigma \in \{\mathit{semi\text{-}mod}, \mathit{semi\text{-}stb}\}$. In the following, let $v^* = v^{\mathbf{t}} \cup v^{\mathbf{f}}$.

Theorem 1. *It holds that $Ver_{\mathit{semi\text{-}mod}}$ is Π_2^{P}-complete.*

Proof. Let us first consider membership in Π_2^{P}. Consider an arbitrary instance of the problem, that is, a given ADF $D = (A, L, C)$ and a three-valued interpretation v. To show membership, we consider the complementary problem, i.e., whether v is not a semi-two-valued interpretation of D. Consider the following non-deterministic algorithm with access to an NP oracle. First, check whether v is admissible in D (this problem is in coNP [7]). Next, non-deterministically construct a three-valued interpretation v' and verify whether $v^* \subset v'^*$ (can be done in polynomial time) and that v' is admissible in D. If either $v \notin adm(D)$, or $v \in adm(D)$ and there exists $v' \in adm(D)$ s.t. $v^* \subset v'^*$, then v is not a semi-two-valued interpretation of D. Thus, the complementary problem $Ver_{\mathit{semi\text{-}mod}}(v, D)$ is in Σ_2^{P} for ADFs. From this it follows that the verification problem is in Π_2^{P}.

For the hardness, it was shown that checking whether there exists a non-trivial interpretation in a given ADF D (assigning at least one argument to true or false) is a Σ_2^{P}-complete problem in ADFs [7]. For any ADF D, it holds that D has a non-trivial admissible interpretation if and only if the trivial interpretation is not a semi-two-valued interpretation. Thus, the problem of deciding whether there is a non-trivial interpretation can be reduced in polynomial time to the problem of verifying whether the trivial interpretation is not a semi-two-valued interpretation. Therefore, the problem of deciding whether a given three-valued interpretation is not a semi-two-valued interpretation is Σ_2^{P}-hard, and, in turn, the verification problem is Π_2^{P}-hard. $\square$

Next, we show membership of $Ver_{semi\text{-}stb}(v,D)$ in Π_2^{P}. This problem aks whether a given interpretation v is a semi-stable interpretation.

Theorem 2. *It holds that $Ver_{semi\text{-}stb}$ is in Π_2^{P}.*

Proof. Given an arbitrary ADF $D = (A,L,C)$ and an interpretation v, by Definition 4, first, we check if v is a semi-two-valued interpretation of D. According to Theorem 1, verifying if v is a semi-two-valued interpretation of D is a Π_2^{P}-complete problem. If $Ver_{semi\text{-}mod}(v,D)$ returns yes, for the given interpretation v, construct the sub-reduct D^v in polynomial time. Next, check if $v^{\mathbf{t}} = w^{\mathbf{t}}$, where w is the grounded interpretation D^v. The grounded interpretation can be computed via a polynomial number of coNP oracle calls [7]. Taken together, there is a non-deterministic polynomial-time algorithm, with access to a coNP oracle, that first checks whether $v \in semi\text{-}stb(D)$ and subsequently computed the grounded interpretation of the constructed sub-reduct. If all computation paths succeed, the answer is yes. Thus, $Ver_{semi\text{-}stb}$ is in Π_2^{P}. □

We conjecture that verifying whether an interpretation is a semi-stable interpretation is Π_2^{P}-hard (due to the verification complexity of semi-two-valued models). Note that for the subsequent results, membership is sufficient.

Next, we turn our attention to the complexity of $Cred_\sigma(a,x,D)$. We first show membership for both semantics and, seperately, hardness. Theorem 3 shows that the credulous problem under both semantics is in Σ_3^{P} and Theorem 4 shows the hardness results below.

Theorem 3. *Let $\sigma \in \{semi\text{-}mod, semi\text{-}stb\}$. It holds that $Cred_\sigma$ is in Σ_3^{P} for ADFs.*

Proof. Let $D = (A,L,C)$ be an arbitrary ADF, $a \in A$, and $x \in \{\mathbf{t},\mathbf{f}\}$. Moreover, let $\sigma \in \{semi\text{-}mod, semi\text{-}stb\}$. To check if there exists a σ interpretation v satisfying $v(a) = x$, guess a three-valued interpretation v with $v(a) = x$, and then verify whether $v \in \sigma(D)$. According to Theorem 1 and Theorem 2, verifying whether $v \in \sigma(D)$ is in Π_2^{P}. Thus, the combined guessing and checking process results in $\mathsf{NP}^{\Pi_2^{\mathsf{P}}} = \Sigma_3^{\mathsf{P}}$. □

To investigate hardness results of credulous and skeptical acceptance problems, several of our results employ a similar reduction technique, wherein a reduction is a function that takes a quantified Boolean formula (QBF) and maps it to an ADF.

For background on QBFs, we refer the reader to chapters of the recent second volume of the Handbook of Satisfiability [22]. For the sake of this paper, we recall QBFs of the form $\Theta = \exists X \forall Y \exists Z\, \theta(X,Y,Z)$, with X, Y, and Z being disjoint set of propositional variables and $\theta(X,Y,Z)$ a non-quantified Boolean formula over these variables (i.e., all variables occuring in the formula are quantified, the formula is closed). The closed QBF Θ is valid (true) if it holds that there is an assignment on the variables in X such that for all extensions of this assignment to the variables in Y there is a completion of these assignments on the set Z such that θ evaluates to true. This problem is Σ_3^{P}-complete [20].

We introduce our main reduction in Reduction 1 and prove properties that we will later utilize in hardness proofs.

Reduction 1. *Let $\Theta = \exists X \forall Y \exists Z\, \theta(X,Y,Z)$ be a QBF with θ in a negation normal form. Define $X^* = \{x^* : x \in X\}$, $X' = \{x' : x \in X\}$, and $X'' = \{x'' : x \in X\}$. Let $\bar{Y} = \{\bar{y} : y \in Y\}$. Construct ADF $RED_1(\Theta) = D_\Theta = (A,L,C)$, s.t.*

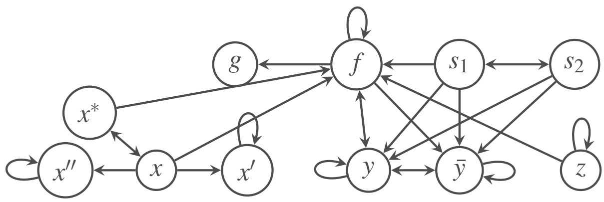

Figure 3. Illustration of Reduction 1

$$
\begin{aligned}
A &= X \cup X^* \cup X' \cup X'' \cup Y \cup \bar{Y} \cup Z \cup \{f, g, s_1, s_2\} \\
C &= \{\varphi_x : \neg x^* \mid x \in X\} \cup \{\varphi_{x^*} : \neg x \mid x^* \in X^*\} \\
&\quad \cup \{\varphi_{x'} : (x \vee \neg x') \mid x' \in X'\} \cup \{\varphi_{x''} : (x \wedge \neg x'') \mid x'' \in X''\} \\
&\quad \cup \{\varphi_y : ((\neg \bar{y} \vee \neg f) \wedge s_2) \vee (s_1 \wedge \neg y) \mid y \in Y\} \\
&\quad \cup \{\varphi_{\bar{y}} : ((\neg y \vee \neg f) \wedge s_2) \vee (s_1 \wedge \neg \bar{y}) \mid \bar{y} \in \bar{Y}\} \\
&\quad \cup \{\varphi_z : \neg z \mid z \in Z\} \cup \{\varphi_f : (\neg f \wedge \theta[\neg x/x^*]) \vee s_1\} \\
&\quad \cup \{\varphi_g : \neg f\} \cup \{\varphi_{s_1} : \neg s_2\} \cup \{\varphi_{s_2} : \neg s_1\}
\end{aligned}
$$

We say that ADF D_Θ is an encoding of Θ, denoted by $RED(\Theta) = D_\Theta$. Figure 3 shows an illustration of the reduction. Table 2 illustrates the two "types" of semi-two-valued interpretations present in ADFs that result from Reduction 1. One property of this encoding is that f may be assigned to $\mathbf{f}$ in an admissible interpretation of D_Θ if Θ is not valid.

Proposition 1. *Given a QBF $\Theta = \exists X \forall Y \exists Z\, \theta(X,Y,Z)$ let $RED(\Theta) = D_\Theta$.*

- *For any two valued interpretation u on X, D_Θ has a preferred interpretation v s.t. $u(x) = v(x)$, for all $x \in X$, and $v(f) = \mathbf{t}$.*
- *If Θ is not valid, then there is a preferred interpretation in D_Θ in which f is assigned to $\mathbf{f}$.*

Proof. We show that for any arbitrary two valued truth values over X, D_Θ has a preferred interpretation with the same truth values over X in which f is assigned to $\mathbf{t}$. Let u be an arbitrary interpretation s.t. for each $x \in X$, $u(x) \in \{\mathbf{t}, \mathbf{f}\}$. We construct interpretation

Table 2. In the first line, the symbol '$[X]_2$, arb' denotes an arbitrary two-valued interpretation over a set X. The symbol $[X]_2^-$ represents that $v(x^*)$ is the reverse of $v(x)$, for each $x^* \in X^*$. The symbol $[X]^{\mathbf{t}}$ under X' represents that for each $x' \in X'$, $v(x') = v(x)$ if $v(x) = \mathbf{t}$; otherwise, $v(x') = \mathbf{u}$. The symbol $[X]^{\mathbf{f}}$ under X'' represents that for each $x'' \in X''$, $v(x'') = v(x)$ if $v(x) = \mathbf{f}$; otherwise, $v(x'') = \mathbf{u}$. The second line indicates the truth value of all arguments under w. In this line, the symbol '$\theta[M]$ unsat' denotes that the formula θ is unsatisfiable for the values assigned to arguments X, Y, and Z ("interpretation" M on the vocabulary of θ). The symbol M_X under X represents that $w(x) = \mathbf{t}$ if $x \in M$; otherwise, it is $\mathbf{f}$. Similarly for M_Y under Y. The symbol $\bar{M}_Y$ under $\bar{Y}$ represents that $w(\bar{y}) = \neg w(y)$.

	s_1	s_2	f	g	θ	X	X^*	X'	X''	Y	$\bar{Y}$	Z
v	**t**	**f**	**t**	**f**	arb	$[X]_2$, arb	$[X]_2^-$	$[X]^{\mathbf{t}}$	$[X]^{\mathbf{f}}$	**u**	**u**	**u**
w	**f**	**t**	**f**	**t**	$\theta[M]$ unsat	M_X	$[X]_2^-$	$[X]^{\mathbf{t}}$	$[X]^{\mathbf{f}}$	M_Y	$\bar{M}_Y$	**u**

v based on the truth value of u over X as follows. Let $v(x) = u(x)$, for each $x \in X$. Then, let $v(x^*) = \mathbf{t}$ iff $v(x) = \mathbf{f}$, and $v(x^*) = \mathbf{f}$ iff $v(x) = \mathbf{t}$. In addition, let $v(x') = v(x)$ if $v(x) = \mathbf{t}$; otherwise, $v(x') = \mathbf{u}$, denoted by $v(X') = v^{\mathbf{t}}(X) = [X]^{\mathbf{t}}$. Let $v(x'') = v(x)$ if $v(x) = \mathbf{f}$; otherwise, $v(x'') = \mathbf{u}$, denoted by $v(X'') = v^{\mathbf{f}}(X) = [X]^{\mathbf{f}}$. Furthermore, let $v(f) = \mathbf{t}$, $v(s_1) = \mathbf{t}$, $v(s_2) = \mathbf{f}$, $v(g) = \mathbf{f}$, and $v(Y) = v(\bar{Y}) = v(Z) = \mathbf{u}$. Since $v = \Gamma_{D_\Theta}(v)$, and it is maximal w.r.t. $\leq_i$, v is a preferred interpretation of D_Θ. That is, for any arbitrary two valued truth values over X, there is $v \in prf(D_\Theta)$ s.t. $v(f) = \mathbf{t}$. Thus, $v \in prf(D_\Theta)$.

Now assume that Θ is invalid. We show that D_Θ has a preferred interpretation that assigns f to $\mathbf{f}$. Since Θ is invalid, it holds that for all truth value assignments on X, there is an extension of these assignments to Y s.t. for all extensions to Z we find that θ evaluates to false. Let I_X be an arbitrary such assignment on X, and I_Y be an assignment such that for all extensions to Z the formula evaluates to false. We construct preferred interpretation w for D_Θ as follows. For each $x \in X$, if $x \in I_X$, then let $w(x) = \mathbf{t}$ and $w(x^*) = \mathbf{f}$; otherwise, let $w(x) = \mathbf{f}$ and $w(x^*) = \mathbf{t}$. For each $x' \in X'$, if $w(x) = \mathbf{t}$, then let $w(x') = \mathbf{t}$; otherwise, let $w(x') = \mathbf{u}$. For each $x'' \in X''$, if $w(x) = \mathbf{f}$, then let $w(x'') = \mathbf{f}$; otherwise, let $w(x'') = \mathbf{u}$. For each $y \in Y$, if $y \in I_Y$, then let $w(y) = \mathbf{t}$ and $w(y') = \mathbf{f}$; otherwise, let $w(y) = \mathbf{f}$ and $w(y') = \mathbf{t}$. Let $w(s_1) = \mathbf{f}$, $w(s_2) = \mathbf{t}$, $w(f) = \mathbf{f}$, $w(g) = \mathbf{t}$, and $w(z) = \mathbf{u}$ for each $z \in Z$. Since $I_X \cup I_Y \cup I_Z \models \neg\theta$ for any I_Z and $w(s_1) = \mathbf{f}$, it follows that $\Gamma_{D_\Theta}(w)(f) = \mathbf{f}$. It is straightforward to verify that $w = \Gamma_{D_\Theta}(w)$ and w is $\leq_i$-maximal. Thus, $w \in prf(D_\Theta)$. □

We prove that for two preferred interpretations in D_Θ that assign argument f to true, they are "incomparable" w.r.t. arguments assigned undecided. Thus, different truth-value assignments to X give rise to different semi-two-valued interpretations.

Corollary 1. *Given a QBF $\Theta = \exists X \forall Y \exists Z\, \theta(X,Y,Z)$, let $RED(\Theta) = D_\Theta$. Moreover, let $v_1, v_2 \in prf(D_\Theta)$ s.t. both of them assign f to* $\mathbf{t}$. *It holds that $v_1^* \not\subset v_2^*$, where $v^* = v^{\mathbf{t}} \cup v^{\mathbf{f}}$.*

Proof. Toward a contradiction, assume there exist $v_1, v_2 \in prf(D_\Theta)$ s.t. $v_1(f) = v_2(f) = \mathbf{t}$, but $v_1^* \subset v_2^*$. By the acceptance condition of f in D_Θ, if $v_1, v_2 \in prf(D_\Theta)$ and $v_1(f) = v_2(f) = \mathbf{t}$, then $v_1(s_1) = v_2(s_1) = \mathbf{t}$, $v_1(s_2) = v_2(s_2) = v_1(g) = v_2(g) = \mathbf{f}$. Since both interpretations assign each $x \in X$ to either $\mathbf{t}$ or $\mathbf{f}$, and by the assumption $v_1^* \subset v_2^*$, $v_1(X) = v_2(X)$, $v_1(X') = v_2(X')$ and $v_1(X'') = v_2(X'')$. As $v_1(f) = v_2(f) = \mathbf{t}$, by the assumption, $v_1(Y) = v_2(Y) = v_1(\bar{Y}) = v_2(\bar{Y}) = v_1(Z) = v_2(\bar{Z}) = \mathbf{u}$. Thus, $v_1^* = v_2^*$. This contradicts the assumption of the existence of $v_1, v_2 \in prf(D_\Theta)$ s.t. $v_1(f) = v_2(f) = \mathbf{t}$ and $v_1^* \subset v_2^*$. Therefore, for any $v_1, v_2 \in prf(D_\Theta)$, if $v_1(f) = v_2(f) = \mathbf{t}$, then $v_1^* \not\subset v_2^*$. □

The main part of correctness of the reduction is then proved in the next result.

Proposition 2. *Given a QBF $\Theta = \exists X \forall Y \exists Z, \theta(X,Y,Z)$, let $RED(\Theta) = D_\Theta$.*

- *If Θ is valid, then there exists a semi-two-valued interpretation for D_Θ that assigns f to* $\mathbf{t}$, *and g to* $\mathbf{f}$.
- *If there exists a semi-two-valued interpretation for D_Θ that assigns f to* $\mathbf{t}$ *and g to* $\mathbf{f}$, *then Θ is valid.*

Proof. Assume that Θ is valid. We aim to show that there exists an interpretation v s.t. $v \in semi\text{-}mod(D_\Theta)$ and $v(f) = \mathbf{t}$. Since Θ is valid, there exist an assignment I_X on X s.t. for each extension to Y there is an extension to Z that evaluates θ to true. We construct a semi-two-valued interpretation v for D_Θ, based on I_X, in which $v(f) = \mathbf{t}$. Let v be as follows. For each $x \in X$, if $x \in I_X$, then $v(x) = \mathbf{t}$ and $v(x^*) = \mathbf{f}$; otherwise, $v(x) = \mathbf{f}$ and $v(x^*) = \mathbf{t}$. For each $x' \in X'$, if $v(x) = \mathbf{t}$, then $v(x') = \mathbf{t}$; otherwise, $v(x') = \mathbf{u}$. For each $x'' \in X''$, if $v(x) = \mathbf{f}$, then $v(x'') = \mathbf{f}$; otherwise, $v(x'') = \mathbf{u}$. Let $v(s_1) = \mathbf{t}$, $v(s_2) = \mathbf{f}$, $v(f) = \mathbf{t}$, $v(g) = \mathbf{f}$, and $v(Z) = v(Y) = v(\bar{Y}) = \mathbf{u}$. Since $v = \Gamma_{D_\Theta}(v)$ and v is $\leq_i$-maximal for D_Θ, $v \in prf(D_\Theta)$. We aim to show that $v \in semi\text{-}mod(D_\Theta)$. Toward a contradiction assume that $v \notin semi\text{-}mod(D_\Theta)$, that is, there exists $w \in prf(D_\Theta)$ s.t. $v^* \subset w^*$. If $v^* \subset w^*$, then either $w(f) = \mathbf{t}$ or $w(f) = \mathbf{f}$. By Corollary 1, $w(f) \neq \mathbf{t}$ (i.e., $w(f) = \mathbf{f}$).

Assume that $w(f) = \mathbf{f}$. Since $w \in adm(D_\Theta)$, it follows that $w(s_1) = \mathbf{f}$. Moreover, by construction of the acceptance condition of f, it holds that any completion of w does not satisfy θ (otherwise f cannot be false in w). Furthermore, by the assumption of $v^* \subset w^*$, $v(X') = w(X') = X^{\mathbf{t}}$ and $v(X'') = w(X'') = X^{\mathbf{f}}$. Therefore, $v(X) = w(X)$. Based on the acceptance condition of each y, preferred interpretation w assigns y either to $\mathbf{t}$ or $\mathbf{f}$ if both $w(s_1) = w(f) = \mathbf{f}$. The false truth value of f in w, give us a freedom to choose an arbitrary truth values for each $y \in Y$ and $y' \in Y'$. Thus, if there is $w \in prf(D_\Theta)$ s.t. $v^* \subseteq w^*$, then w has to be an interpretation similar to the second line of Table 2 in which $w(f) = \mathbf{f}$.

Since all z are assigned $\mathbf{u}$ by w, it follows that any two-valued interpretation u assigning to X and Y variables the same values as w, and an arbitrary value to variables Z, that $u \not\models \theta$. This contradicts Θ being valid. By assumption, for the presumed assignment on the X variables (same as w), for all assignments on Y there is an assignment on Z such that θ is satisfied. However, it holds that for this assignment on X, there is an assignment on Y such that for all assignments on Z, θ is falsified, a contradiction. Thus, the assumption that there exists w s.t., $w \in prf(D_\Theta)$ and $v^* \subseteq w^*$ is a contradiction.

Now we show the second item. Assume that there exists a semi-two-valued model for D_Θ that assigns f to $\mathbf{t}$. We show that Θ is valid. Let $v \in semi\text{-}mod(D_\Theta)$ s.t., $v(f) = \mathbf{t}$. Since $v \in prf(D_\Theta)$, for each $x \in X$, either $v(x) = \mathbf{t}$ or $v(x) = \mathbf{f}$. Toward a contradiction assume that Θ is not valid. That is, for any $I_X \subseteq X$, there exists $I_Y \subseteq Y$, s.t. for all $I_Z \subseteq Z$, it holds that $I_X \cup I_Y \cup I_Z \models \neg\theta$.

We construct interpretation w as follows. Let $w(X) = v(X)$, $w(y) = \mathbf{t}$ if $y \in I_Y$; otherwise, let $w(y) = \mathbf{f}$, and let $w(z) = \mathbf{u}$. Let $w(s) = w(f) = \mathbf{f}$. The remaining arguments in X^*, X' and X'' are assigned in a similar way as above. This interpretation is an admissible interpretation of D_Θ, and $v^* \subset w^*$. Thus, $v \notin semi\text{-}mod(D_\Theta)$. This is a contradiction by the assumption that $v \in semi\text{-}mod(D_\Theta)$. Thus, our assumption was wrong, that is, if $Cred_{semi\text{-}mod}(f, \mathbf{t}, D_\Theta)$: yes and $Cred_{semi\text{-}mod}(g, \mathbf{f}, D_\Theta)$: yes, then Θ is valid. □

We show that f is never assigned to undecided in a semi-two-valued interpretation of D_Θ.

Proposition 3. *Given a QBF $\Theta = \exists X \forall Y \exists Z, \theta(X,Y,Z)$, let $RED(\Theta)$ be an encoding of Θ in ADFs. Let v be an arbitrary semi-two-valued interpretation of D_Θ, $v(f) \neq \mathbf{u}$.*

Proof. Towards a contradiction, assume that there exists $v \in semi\text{-}mod(D_\Theta)$, such that $v(f) = \mathbf{u}$. Note that, if $v(f) = \mathbf{u}$, then $v(s_1) = \mathbf{f}$, and $v(y) = v(\bar{y}) = \mathbf{u}$, for each $y \in Y$ and for each $\bar{y} \in \bar{Y}$. Let w be an interpretation s.t. $w(s_1) = \mathbf{t}$, $w(s_2) = \mathbf{f}$, $w(f) = \mathbf{t}$, $w(g) = \mathbf{f}$, $w(X) = v(X), w(X^*) = v(X^*), w(X') = v(X'), w(X'') = v(X'')$. It is straightforward to check that $w \in adm(D_\Theta)$, and $v^* \subset w^*$. However, this is a contradiction by the assumption that $v \in semi\text{-}mod(D_\Theta)$. Thus, the assumption that there exists $v \in semi\text{-}mod(D_\Theta)$ s.t. $v(f) = \mathbf{u}$ is wrong. Hence, for each $v \in semi\text{-}mod(D_\Theta)$ it holds that $v(f) \neq \mathbf{u}$ □

We show that all semi-two-valued interpretations of D_Θ are in fact also semi-stable interpretations.

Proposition 4. *Given a QBF $\Theta = \exists X \forall Y \exists Z, \theta(X,Y,Z)$, let $RED(\Theta) = D_\Theta$. Let v be a semi-two-valued interpretation for D_Θ. It holds that v is a semi-stable interpretation for D_Θ.*

Proof sketch. Let $v \in \textit{semi-mod}(D_\Theta)$. By Proposition 3, $v(f) \neq \mathbf{u}$. Thus, v corresponds to the truth values depicted in either the first or the second line of Table 2. In either case, it is straightforward to verify that $v^{\mathbf{t}} = w^{\mathbf{t}}$, where w represents the grounded interpretation of the *stb*-reduct $D^v = (A^v, L^v, C^v)$. Hence, v is a semi-stable interpretation of D_Θ. □

Next, we consider hardness of credulous reasoning.

Theorem 4. *Let $\sigma \in \{\textit{semi-mod}, \textit{semi-stb}\}$. It holds that $Cred_\sigma$ is Σ_3^{P}-hard.*

Proof. Given a QBF $\Theta = \exists X \forall Y \exists Z, \theta(X,Y,Z)$, let $RED(\Theta)$ be an encoding of Θ in ADFs, as presented in Reduction 1. By Proposition 2, Θ is valid, iff there exists $v \in \textit{semi-mod}(D_\Theta)$ such that $v(f) = \mathbf{t}$ and $v(g) = \mathbf{f}$. By Proposition 4, each semi-two-valued interpretation for D_Θ is a semi-stable interpretation for D_Θ. Finally, if f in the reduction is assigned to $\mathbf{t}$ then g is assigned to $\mathbf{f}$, and vice versa. Thus, $Cred_\sigma$ is Σ_3^{P}-hard. □

Finally, we address the complexity of $Skept_\sigma(a,x,D)$, again for $\sigma \in \{\textit{semi-mod}, \textit{stb}\}$.

Theorem 5. *Let $\sigma \in \{\textit{semi-mod}, \textit{semi-stb}\}$. It holds that $Skept_\sigma$ is Π_3^{P}-complete.*

Proof. First we show that $Skept_\sigma$ is in Π_3^{P}. Given an ADF $D = (A,R,C)$, an argument a, and the truth value x where $x \in \{\mathbf{t},\mathbf{f}\}$, answering $Skept_\sigma(a,x,D)$ involves considering the complementary problem. In this case, we determine whether there exists a σ model v in which a is not assigned to x. As per Theorem 1 and Theorem 2, the task of checking if v is a σ model in D is a Π_2^{P}-complete problem.

Next we show that $Skept_\sigma$ is Π_3^{P}-hard. Given a quantified Boolean formula $\Theta = \exists X \forall Y \exists Z, \theta(X,Y,Z)$, let $RED(\Theta)$ be an encoding of Θ in ADFs. We know that $Skept_\sigma(g,\mathbf{t},D_\Theta)$: yes, iff $Skept_{\textit{semi-mod}}(f,\mathbf{f},D_\Theta)$: yes. Furthermore, by Proposition 3, for each $v \in \textit{semi-mod}(D_\Theta)$, $v(f) \in \{\mathbf{t},\mathbf{f}\}$. Thus, $Skept_{\textit{semi-mod}}(f,\mathbf{f},D_\Theta)$: yes iff $Cred_{\textit{semi-mod}}(f,\mathbf{t},D_\Theta)$: no. In a similar way; $Skept_{\textit{semi-mod}}(f,\mathbf{f},D_\Theta)$: yes, iff $Skept_{\textit{semi-mod}}(g,\mathbf{t},D_\Theta)$: yes, iff $Cred_{\textit{semi-mod}}(g,\mathbf{f},D_\Theta)$: no. By Proposition 4, the same holds for semi-stable semantics. □

4. Conclusion

We studied the computational properties of the semi-two-valued and semi-sable semantics of ADFs. When compared to AFs, computational complexity for ADFs increases by at least one step in the polynomial hierarchy for nearly all reasoning tasks [7,8].

As future work, we aim to study the complexity of further decision problems. For instance, the complexity of the smallest witness problem, which involves determining whether a given argument is assigned to x in a semi-two-valued interpretation or semi-stable interpretation v such that $|v^*| < k$. Furthermore, studying fragments of ADFs, such as bipolar ADFs [5] appears as an interesting research direction for semi-stable semantics.

Acknowledgements

This publication has been partly supported by the Netherlands Organisation for Scientific Research (NWO) through the Hybrid Intelligence Gravitation Programme with project number 024.004.022 and by the Austrian Science Fund (FWF) P35632.

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