

The Development and Application Research of Grey Wolf Optimization Algorithm

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Abstract. The Grey Wolf Optimization (GWO) algorithm is an emerging swarm intelligence optimization technique known for its simplicity, minimal control parameters, fast convergence, and ease of implementation. This paper investigates the search mechanism and implementation process of the GWO algorithm, analyzing its shortcomings, including poor population diversity, slow convergence in later stages, and susceptibility to local optima. It provides an overview of improvement strategies for the Grey Wolf Optimization algorithm, encompassing enhancements to the population, parameters, search mechanism, and its integration with other optimization algorithms. Furthermore, the paper discusses the applications of the Grey Wolf Optimization algorithm in various domains.

Keywords. Grey Wolf Optimization Algorithm, Algorithm Principles, Algorithm Enhancements, Algorithm Applications

1. Introduction

In recent years, swarm intelligence optimization algorithms have been widely applied to solving complex problems due to their simplicity and ease of implementation. The Grey Wolf Optimization algorithm is a novel swarm intelligence optimization algorithm inspired by grey wolves' behavior in their natural habitat, introduced by scholars from Griffith University in Australia, led by Mirjalili [1] in 2014. The GWO algorithm relies on the collaboration and communication among individual grey wolves within a group to simulate their hunting process, effectively tackling real-world complex problems. It exhibits complex global behavior derived from the simple behaviors of individual grey wolves and can adapt to unstable and intricate environments.

Due to its characteristics of simplicity, few control parameters, fast convergence, and ease of implementation [2], the GWO algorithm has attracted numerous researchers who have successfully applied it to various domains, including job scheduling, path planning, and neural network training. It has become one of the widely adopted emerging swarm intelligence optimization algorithms. This paper elucidates the fundamental principles of the Grey Wolf Optimization algorithm, provides an overview.

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2. The fundamental principles of the Grey Wolf Optimization algorithm.

The traditional Grey Wolf Optimizer algorithm involves three leading wolves, denoted as alpha, beta, and delta wolves, representing the top three best solutions. These leading wolves guide the movement of the remaining wolves to ultimately find the global optimum solution. The hunting behavior of grey wolves consists of three main steps: encircling, hunting, and attacking prey.

2.1. The Encircling Phase

Grey wolves, during the hunting process, utilize the following position update formula to encircle their prey.

$$X_{i,j}^{t+1} = X_{p,j}^t - A \cdot D \quad (1)$$

Where $X_{i,j}^{t+1}$ represents the $j \in \{1, 2, \dots, Dim\}$ th-dimensional position of the $i \in \{1, 2, \dots, SN\}$ th grey wolf at time $t+1$, SN is the number of individuals in the population, $X_{p,j}^t$ stands for the j th-dimensional position of the prey at time t , where p denotes the prey, and D signifies the distance between the grey wolf and the prey.

$$D = |C \cdot X_{p,j}^t - X_{i,j}^t| \quad (2)$$

Where $X_{i,j}^t$ represents the j th-dimensional position of the i th grey wolf at time t , and A and C are coefficients.

$$A = 2 \cdot r_1 \cdot a - a \quad (3)$$

$$C = 2 \cdot r_2 \quad (4)$$

Where r_1 and r_2 are uniformly distributed random numbers in the range (0, 1), and the coefficient a linearly decreases from 2 to 0 and can be expressed as follows.

$$a = 2 - \frac{2t}{T_{\max}} \quad (5)$$

Where T_{\max} represents the maximum number of iterations.

2.2. The Chase Phase

During the hunting process of the wolf pack, it is generally believed that alpha wolf, beta wolf, and gamma wolf have a better understanding of the prey's location. In this manner, the positions of alpha, beta, and delta wolves are treated as the prey's position, and these three positions are used to estimate the prey's actual location. Simultaneously, other

individual grey wolves are guided to update their positions based on the optimal individual's position, gradually closing in on the prey.

Each grey wolf updates its position based on the positions of the alpha, beta, and delta wolves.

$$X_{i,j}^{t+1} = \frac{X_{1,j}^t + X_{2,j}^t + X_{3,j}^t}{3} \tag{6}$$

Where $X_{1,j}^t$, $X_{2,j}^t$, and $X_{3,j}^t$ are computed as follows:

$$X_{1,j}^t = X_{\alpha,j}^t - A_1 \cdot D_\alpha \tag{7}$$

$$X_{2,j}^t = X_{\beta,j}^t - A_2 \cdot D_\beta \tag{8}$$

$$X_{3,j}^t = X_{\delta,j}^t - A_3 \cdot D_\delta \tag{9}$$

In the equation, $X_{\alpha,j}^t$, $X_{\beta,j}^t$, and $X_{\delta,j}^t$ represent the j th-dimensional positions of the alpha, beta, and delta wolves, which are the top three in terms of fitness within the current population at time t . A_1 , A_2 , and A_3 are coefficients, and their calculation formula can be derived from Equation (3). D_α , D_β , and D_δ represent the distances between the grey wolf and the alpha, beta, and delta wolves, calculated as follows:

$$D_\alpha = |C_1 \cdot X_{\alpha,j}^t - X_{i,j}^t| \tag{10}$$

$$D_\beta = |C_2 \cdot X_{\beta,j}^t - X_{i,j}^t| \tag{11}$$

$$D_\delta = |C_3 \cdot X_{\delta,j}^t - X_{i,j}^t| \tag{12}$$

Where C_1 , C_2 , and C_3 are coefficients, derived from Equation (4).

2.3. The Attack Phase

When the prey ceases to move, the grey wolves proceed to attack the prey, a process controlled by parameter A . As indicated by Equation (3), the range of values for A is $[-a, a]$, i.e., between -2 and 2. When $|A| > 1$, the wolf pack is relatively dispersed, engaging in global search to locate the prey's general area, favoring exploration. As a decreases, A also decreases. When $|A| < 1$, the grey wolves approach the prey, entering the local search phase, and initiate an attack, favoring exploitation [3].

3. Improvements in the Grey Wolf Optimization Algorithm

Based on the performance testing of the Grey Wolf Optimization algorithm conducted by Mirjalili et al. [1] and comparisons with other algorithms such as PSO and DE, several shortcomings of the GWO algorithm can be identified:

- 1) **Poor Population Diversity:** The GWO algorithm typically initializes its population randomly, which can sometimes lead to a lack of diversity in the population. Initial solutions may cluster in a specific local region, causing the algorithm to get stuck in a local optimum and fail to discover a better global optimum.
- 2) **Slow Convergence in Later Stages:** The GWO algorithm's search mechanism relies on the distances between individuals in the wolf pack and the alpha, beta, and delta wolves to determine their positions. While this mechanism can effectively help the wolf pack converge quickly to the optimal solution in the early stages, in the later stages, as the wolf pack gets closer to the optimal solution, the distances between pack members shrink, resulting in slower search progress and reduced convergence speed.
- 3) **Susceptibility to Local Optima:** The GWO algorithm depends on the guidance provided by the alpha, beta, and delta wolves for its search. However, the alpha wolf does not necessarily represent the global optimum. During the algorithm's iterations, other members of the wolf pack continually move closer to the alpha, beta, and delta wolves. This tendency causes the search process to focus on the vicinity of local optima, neglecting the broader solution space, making it challenging for the GWO algorithm to converge to the global optimum at times.

3.1. Population Improvement

The quality of the initial population directly impacts the global convergence speed and accuracy of the algorithm, and an initial population with high diversity can enhance the optimization capability of the algorithm. Since the initial population is generated based on random initialization, it cannot guarantee a high level of population diversity.

In 2023, Pan et al. [4] introduced an enhanced Grey Wolf Optimization algorithm for feature selection in high-dimensional data. This algorithm incorporates the ReliefF algorithm and Coupla entropy during the initialization process, effectively improving the quality of the initial population. In the same year, Luo et al. [5] employed chaotic mapping for population initialization to maintain population diversity. Ma et al. [6] proposed an initial population enhancement strategy using a chaotic grouping mechanism. Additionally, they periodically recombined the populations at specific iteration intervals to increase population diversity during the search process.

3.2. Parameter Improvement

In the Grey Wolf Algorithm, the control factor a linearly decreases and regulates the algorithm's trade-off between exploration and exploitation, Therefore, a is of paramount importance to the algorithm. Research indicates that non-linear control factors can prevent premature convergence during the iteration process [7].

In 2021, Fan et al. [8] introduced a non-linear control factor strategy based on the cosine function, significantly influencing the ability of the improved Grey Wolf Optimization algorithm to attain optimal solutions.

In 2022, Singh et al. [9] introduced a non-linear control factor a as:

$$a = (a_{start} - a_{end}) \times E + a_{end} \tag{13}$$

$$E = \exp\left(-\frac{t^2}{(k \times T_{max})^2}\right) \tag{14}$$

Where the value of a_{start} is 2, the value of a_{end} is 0, and the value of k is 0.3. The non-linear convergence factor incorporates 25% global exploration and 75% local exploitation, thereby enhancing local search capabilities and reducing the chances of missing the optimal solution.

In 2023, Shial et al. [10] proposed that the exponential decay equation helps in transforming the exploration to exploitation process from initial iterations to final iterations, with proportions of 70% and 30% respectively, enhancing the exploration capability significantly.

3.3. Improvement in the Search Mechanism

In the GWO algorithm, the control factor a can balance exploration and exploitation abilities but may still lead to local optima. Therefore, improving the algorithm's search mechanism is necessary to enhance its ability to find the optimal solution.

In 2020, Yu et al. [11] proposed a reverse learning strategy that generates reverse solutions to enhance the likelihood of approaching the optimal solution.

In 2023, Rezaei et al. [12] introduced the Velocity-Aided Grey Wolf Optimizer (VAGWO). Traditional GWO lacked a velocity term in its position update process, which weakened its global search capability. Therefore, VAGWO incorporates a velocity term into the Grey Wolf position update formula, enhancing its global search capability.

In 2023, Yin et al. [13] proposed increasing the inertia weight to modify the grey wolves' position vectors. They also introduced the Levy flight strategy to optimize the position update formula, enhancing the algorithm's global search capability.

3.4. Improvements through Hybridization with Other Algorithms

Combining two algorithms into a hybrid approach allows for the full utilization of each algorithm's strengths, leveraging diverse search strategies, and enhancing global search capabilities while reducing the risk of falling into local optima.

In 2020, Yue et al. [14] proposed a fusion of the development capability of GWO and the exploration capability of the Fireworks Algorithm to achieve enhanced global optimization. In 2021, Lan et al. [15] introduced an improved Grey Wolf Optimizer based on the utilization of chaos theory and the strengths of the Ocean Predator Algorithm. This enhancement aims to overcome the issue of low convergence accuracy during the optimization process. Simulation experiments have shown a significant improvement in both search accuracy and search speed. In 2022, Ma et al. [16], inspired by the Aquila Optimizer, introduced a flight factor into the GWO. This addition not only

extends the search range and enhances global search capabilities but also reduces the likelihood of getting trapped in local optima. In 2023, Duan et al. [17] introduced improvements using the Sine Cosine Algorithm to enhance the exploration of alpha, beta, and delta wolves, and utilizing weight allocation to guide the position updates of the omega wolf, the modification of the hunting mechanism through hybridization with Sine Cosine Algorithm can further promote development.

4. Applications of the Grey Wolf Optimization Algorithm

Currently, the GWO algorithm has gained extensive research and application in numerous fields. This article primarily focuses on analyzing its applications in scheduling problems, path planning, and neural network training.

4.1. Scheduling Problems

Komaki et al. [18] applied the GWO to solve job sequencing and scheduling problems in assembly workshops, with the aim of minimizing job completion times. Experimental results indicated that the GWO provided a more effective solution to this problem. Chen et al. [19] introduced the use of a multi-objective multi-population GWO to solve the multi-machine collaborative workshop scheduling problem. Cloud computing has become an efficient system for delivering services to users by utilizing available resources. This generates significant workloads on resources, ultimately leading to a decline in network performance. To overcome workload issues, the task scheduling process is utilized. Indhumathi et al. [20] proposed task execution implemented by GWO and demonstrated the superiority of the proposed method over traditional techniques.

4.2. Path Planning

Ding et al. [21] proposed an Improved Grey Wolf Optimizer (IGWO) to facilitate flight path planning for unmanned aerial vehicles in crop disease and pest monitoring, compared to the GWO algorithm, IGWO has reduced the total cost of path distance in the map model by 8.3%, 16.7%, 28.6%, and 39.6% respectively. The accuracy levels achieved were 15, 20, 25, and 30. Kumar et al. [22] Unmanned aerial vehicle path planning involves finding the optimal route between the source point and the destination point, utilized an improved GWO algorithm to offer a feasible and efficient three-dimensional path planning solution for unmanned aerial vehicles. Luo et al. [23] applied the enhanced GWO for autonomous path planning of mobile robots executing unmanned delivery tasks in complex environments. Yu et al. [24] introduced a HGWODE algorithm to address the drone path planning problem, The paths generated by the proposed HGWODE algorithm excel in terms of both length and smoothness, further substantiating the competitiveness of HGWODE. Zhang et al. [25] proposed an improved GWO algorithm for the path planning problem of patrol robots, The results indicate, the improved GWO demonstrates superior convergence, stability, and accuracy in patrol robot path planning. In addressing the agricultural drone trajectory planning problem, Liu et al. [26] proposed a multi-mechanism collaborative improved GWO algorithm (NAS-GWO). Compared to other metaheuristic algorithms in this study, the average cost function values of NAS-GWO decreased by 27.93%, 38.15%, 32.32%,

34.11%, 10.63%, and 13.48%. This demonstrates the effectiveness and significance of NAS-GWO in agricultural drone trajectory planning.

4.3. Neural Network Training

Ahmed et al. [27] integrated the GWO algorithm with a supervised Artificial Neural Network classifier to enhance the classification accuracy of brain magnetic resonance images by selecting the optimal parameters for the artificial neural network. Xie et al. [28] introduced an enhanced GWO algorithm for optimizing Convolutional Long Short-Term Memory Networks (CNN-LSTM) in time series analysis. The network topology and hyperparameters for time series prediction and classification tasks were optimized. The optimized CNN-LSTM network exhibited enhanced learning capabilities. Grace et al. [29] introduced a novel model for wind speed prediction by combining wavelet transformation, backpropagation neural networks, and the GWO algorithm. They utilized the GWO to optimize the parameters of the backpropagation neural network, improving convergence and enhancing the model's performance. Arumuga et al. [30] proposed the use of the GWO algorithm to optimize convolutional neural network models, effectively identifying areas of Diabetic Foot Ulcers. They compared the results with existing algorithms and found that the proposed algorithm improved accuracy. Liu et al. [31] introduced an improved GWO algorithm (SGWO) to optimize the Elman network structure, proposing SGWO-Elman. SGWO significantly optimizes the network structure, and SGWO-Elman demonstrates accurate predictive performance. Irshad et al. [32] presented a novel Internet of Things (IoT)-based medical monitoring platform and an improved Grey Wolf Optimization-based Deep Convolutional Neural Network (DCNN) model for lung cancer detection with superior performance.

5. Summary

Due to its simplicity and efficiency, the GWO algorithm has found widespread applications in various fields. Additionally, various improvement strategies have been proposed to address the limitations of the GWO algorithm, aiming to enhance its performance. When facing complex real-world problems, the use of the GWO or its enhanced versions can lead to improved solutions.

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