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A VMD Harmonic Detection Method Based on Improved SVD Denoising

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Abstract. Harmonic detection is a vital problem in power system, but the accuracy of harmonic detection is affected by noise. In order to suppress noise, this paper proposes the improved singular value decomposition(SVD) to denoise. Then, the optimal number of decomposition modes of variational mode decomposition(VMD) is determined with the aim of minimizing the residual of signal energy after Schmidt orthogonalization. The amplitude and frequency of each harmonic are detected by the Hilbert transform. The simulation shows that the algorithm can accurately identify the information characteristics of each harmonic and inter-harmonic, and has the advantages of high accuracy and small error.

Keywords. Harmonic detection, SVD denoising, VMD, K selection, schmidt orthogonalization, hilbert transform

1. Introduction

The existence of harmonics seriously harms the power quality in the power system and affects the safe operation of the power system [1-3].

At present, the commonly used harmonic detection methods exists some problems: Fast Fourier transform (FFT)[4] is prone to spectrum leakage and fence effect ;wavelet transform[5] failed to select wavelet basis and avoid frequency band aliasing. Hilbert-Huang transform [6] is prone to mode aliasing when using EMD to decompose signals [7].

In addition to the above methods, variational mode decomposition (VMD) is also an effective harmonic detection. In order to denoise, the improved SVD is used, and the optimal decomposition mode K of VMD is determined with the minimum energy residual of Schmidt orthogonalization as the evaluation index. Finally, the amplitude and frequency of the signal are detected by Hilbert transform.

2. The Basic Theory of VMD and SVD

2.1. The basic theory of VMD

The VMD method is used by the following steps for signal decomposition [8-9]:

(1) Hilbert transform is used to analysis the correlation signal of different mode.

(2) Initializing the center frequency of different mode.

(3) Transforming the constrained modal bandwidth problem into a constrained variational problem which can be transformed into the following question:

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$$\min_{\{u_k\},\{\omega_k\}}\left\{\sum_{k=1}^{K}\left\|\partial_t\left[\left(\delta(t)+\frac{\mathrm{j}}{\pi t}\right)*u_k(t)\right]e^{-\mathrm{j}\omega_k t}\right\|_2^2\right\} \tag{1}$$

$$\begin{aligned}L(\{u_k\},\{\omega_k\},\lambda)=\alpha\sum_{k=1}^{K}\left\|\partial_t\left[\left(\delta(t)+\frac{\mathrm{j}}{\pi t}\right)*u_k(t)\right]e^{-\mathrm{j}\omega_k t}\right\|_2^2+\left\|f(t)-\sum_{k=1}^{K}u_k(t)\right\|_2^2\\+\left\langle\lambda(t),f(t)-\sum_{k=1}^{K}u_k(t)\right\rangle\end{aligned} \tag{2}$$

The convex optimization problem is decomposed by alternating direction multiplier method and u_k^{n+1} is updated in the iterative process as follow:

$$u_k^{n+1}=\arg\min_{u_k\in X}\left\{\alpha\left\|\partial_t\left[\left(\delta(t)+\frac{\mathrm{j}}{\pi t}\right)*u_k(t)\right]e^{-\mathrm{j}\omega_k t}\right\|_2^2+\left\|x(t)-\sum_i u_i(t)+\frac{\lambda(t)}{2}\right\|_2^2\right\} \tag{3}$$

Repeat iteration, until the iterative constraint condition is satisfied:

$$\sum_k\left\|\hat{u}_k^{n+1}-\hat{u}_k^{n}\right\|_2^2/\left\|\hat{u}_k^{n}\right\|_2^2<\varepsilon \tag{4}$$

2.2. The basic theory and improvement method of SVD

For any given m×n matrix A which rank is r, by singular value decomposition theory [10], it is shown that A can be represented as:

$$\boldsymbol{A}=\sum_{i=1}^{r}\sigma_i\boldsymbol{u}_i\boldsymbol{v}_i^{\boldsymbol{H}} \tag{5}$$

Where σ_i is the singular values of A. u_i with i=1,2,…,n being the first n of m left singular vectors, and v_i is the right singular vectors[11].

For a signal y(i) (i=1,2,…,N), we can construct matrix A as follows[12]:

$$\boldsymbol{A}=\begin{bmatrix}y(1) & y(2) & \mathrm{L} & y(N-L+1)\\ y(2) & y(3) & \cdots & y(N-L+2)\\ \vdots & \vdots & \ddots & \vdots\\ y(L) & y(L+1) & \cdots & y(N)\end{bmatrix} \tag{6}$$

The size of singular values reflects the situation of energy concentration. The non-zero singular values are arranged in descending order, usually large singular values represent useful signals, and small singular values represent noise. By selecting large singular values and setting small singular values to 0, the noise can be removed.

3. The Proposed Detection Method

3.1. Rank order selection based on singular value area

In order to select appropriate effective rank order to denoise, a rank order selection method based on singular value area is proposed in this paper:

Since the singular values of useful signals appear in pairs and differ greatly from each other, the singular values of noisy signals differ little and their distribution is concentrated [13]. By changing the value of the number of rows of the matrix in (6) between L1 and L2, the corresponding singular values can be calculated. The area of the singular values can be expressed as follows:

$$S_{\mathrm{n}} = \sum_{i=\mathrm{L}_1}^{\mathrm{L}_2} \left(\frac{\sigma_{i(2n-1)} + \sigma_{i(2n)}}{2} - \frac{\sigma_{\mathrm{i}(2n+1)} + \sigma_{\mathrm{i}(2n+2)}}{2} \right) \tag{7}$$

Compared with the size of the singular value area, the singular value area changes greatly at the beginning, and when the adjacent singular value area is almost unchanged, it is the best effective rank order.

3.2. The selection of K value of VMD

Since each modal component obtained from VMD decomposition cannot be completely orthogonal [14-15], Schmidt orthogonalization is adopted to make each component completely orthogonal. After Schmidt orthogonalization, each component can be represented as follows:

$$\hat{u}_k(t) = \hat{u}_k(t) - \frac{\langle \hat{u}_k(t), \hat{u}_1(t) \rangle}{\langle \hat{u}_1(t), \hat{u}_1(t) \rangle} \hat{u}_1(t) - \cdots - \frac{\langle \hat{u}_k(t), \hat{u}_{(k-1)}(t) \rangle}{\langle \hat{u}_{(k-1)}(t), \hat{u}_{(k-1)}(t) \rangle} \hat{u}_{(k-1)}(t) \tag{8}$$

The residual of VMD decomposition after Schmidt orthogonalization is:

$$\delta(t) = f(t) - \sum_{k=1}^{K} \hat{u}(t) \tag{9}$$

The decomposed signals are decomposed several times, and the residual energy corresponding to each VMD decomposition after Schmidt orthodontic processing is calculated. When the residual energy is the lowest, the corresponding K value is determined to be the optimal number of mode decomposition of VMD decomposition.

3.3. VMD method of harmonic detection based on improved SVD

In order to suppress ground noise in power system and detect harmonics more accurately, The basic steps of the proposed method are as follows:

(1) Singular value decomposition. L=N/4~3N/4 is taken as the number of rows of reconstructed matrix (8), and the singular value decomposition is carried out respectively, and the singular value area S_i is calculated.

(2) Determine the effective rank order. By comparing the area of each singular value calculated in step (1), when the area of adjacent singular value Si does not change much, it is determined that the effective rank order is 2*(i-1).

(3) Remove noise and reconstruct signal. According to formula(7), the signal is reconstructed with the first 2*(i-1) large singular values.

(4) Variational mode decomposition. The optimal number of decomposition modes is determined by Schmidt orthogonalization residual method, the signal after removing noise is decomposed, and K IMF components are obtained.

(5) Hilbert transform was carried out for each IMF to calculate the instantaneous frequency and amplitude of each IMF.

4. Simulation and Analysis

Case1: in order to verify the harmonic detection effect of the proposed method, the follow signal is used to carry out simulation verification :

$$x_1(t) = \cos(100\pi t + \pi/4) + 0.3\cos(206\pi t + \pi/2) + 0.15\cos(300\pi t) + 0.1\cos(500\pi t + \pi/4) + \varepsilon(t) \tag{10}$$

Where ε(t) is gaussian white noise(SNR=20dB). By calculation,it is known that the reconstruction matrix r=(5-1)×2=8.Figure1 shows that the signal waveform becomes smooth after denoising.

In order to measure the denoising effect of the improved singular value method in this paper, the proposed method is compared with sparse decomposition denoising and wavelet denoising. The results are shown in Table 1. It can be seen from the table that the proposed method has better denoising effect than both wavelet denoising and sparse decomposition, and the signal obtained is more stable.

Table 1. Comparison of denoising effect of different methods

	SNR(dB)	RMSE
proposed method	40.4263	0.3901
wavelet threshold	26.0242	2.0479
sparse decomposition	32.3172	0.9923

The result of VMD decomposition of signal after denoising is shown in Figure2. It can be seen that after improved singular value denoising, VMD can effectively identify each IMF component, the waveform is smooth and clear. And then Hilbert transformation is performed on each IMF component. The results of harmonic detection are shown in Table 2.

As can be seen from Table 2, the VMD harmonic detection method proposed in this paper based on improved singular value denoising can still identify each harmonics and inter-harmonics relatively accurately with high precision and small error under the condition of strong noise.

Table 2. Harmonic detection result

Amplitude			Frequency /Hz		
Original	VMD	Error /%	Original	VMD	Error/%
1	1.0005	0.05	50	50.0181	0.0362
0.3	0.3011	0.3667	103	103.0638	0.0619
0.15	0.1492	0.5333	150	150.0427	0.0285
0.1	0.1001	0.1	250	250.0798	0.0316

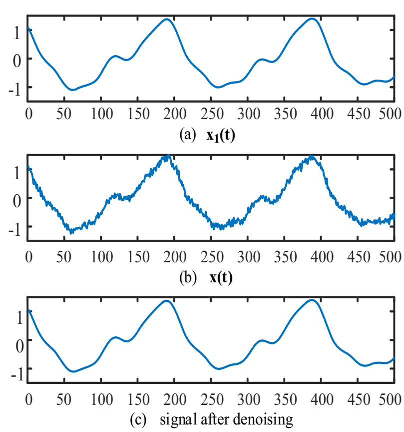

Figure 1. x1(t),x(t)and denoised signal

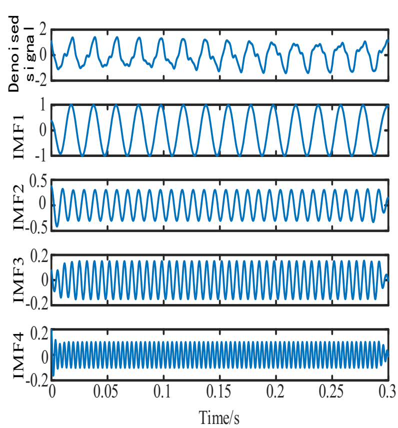

Figure 2. Denoised signal and IMFs

Case2: In the actual power system, the amplitude and frequency of harmonics are often time-varying. The simulated signals are shown as follows:

$$x(t) = \varepsilon(t) + \begin{cases} \sin(100\pi t), & 0 < t < 1 \\ 0.2\sin(500\pi t), & 0.2 \le t < 0.4 \\ 0.6\sin(220\pi t), & 0.4 \le t < 0.55 \\ 0.5\sin(350\pi t), & 0.55 \le t < 0.7 \end{cases} \tag{11}$$

Where ε(t) is gaussian white noise with a SNR of 20dB, sampling rate fs=4096Hz, sampling number N=4096, and sampling time is 1s.

As can be seen from Figure3, the first four IMFs decomposed by VMD can correspond to each component in the original time-varying signal.

Figure4 shows the instantaneous amplitude and instantaneous frequency obtained by Hilbert transformation of the first four IMFs components. The instantaneous amplitude and instantaneous frequency of IMF1 are almostly always stable, and the fundamental wave can be accurately detected. The instantaneous amplitude of IMF2~4 shows the characteristics of gate signal. Only in a specific period of time, the amplitude of the signal is not 0, which represents a time-varying signal. The amplitude of the disturbance signal can be obtained by averaging, and the starting and ending moments of the time-varying harmonic signal can be accurately located according to the characteristics of the gate signal. From Figure4 (a), it is shown that the instantaneous frequency (the red part) in the time period corresponding to the variable signal is relatively stable. The harmonic detection results are shown in Table 3. It can be seen from the data in the table that under the background of strong noise, the proposed method can still accurately detect the amplitude and frequency of fundamental wave and various time-varying interharmonics with small error and high precision.

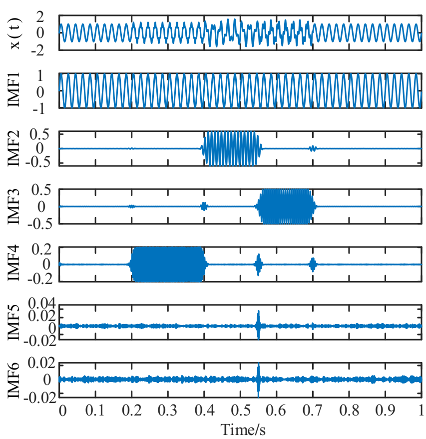

Figure 3. x(t) and IMFs

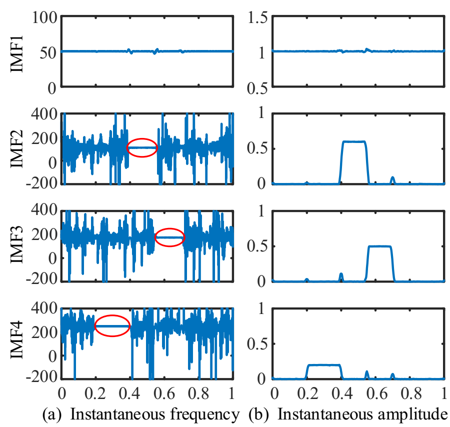

Figure 4. Instantaneous amplitude and frequency of IMFs

Table 3. Time-varying harmonic detection result

Amplitude			Frequency /Hz		
Original	VMD	Error /%	Original	VMD	Error/%
1	1.0011	0.11	50	50.0132	0.0264
0.6	0.6013	0.2167	110	110.0204	0.0185
0.5	0.4996	0.08	175	175.0905	0.0517
0.2	0.1995	0.25	250	250.0102	0.0041

5. Conclusion

In order to solve the difficulty of selecting effective rank order in singular value denoising, the concept of singular value area is proposed. Using singular value area as a measure index, the optimal effective rank order can be determined accurately and effectively, and the denoising effect can be significantly improved.

Aiming at the difficulty of selecting K value in VMD decomposition, the minimum residual energy Er after Schmidt orthodontic is used as a measure index to determine the optimal K value.

The simulation results show that the proposed variational mode decomposition harmonic detection method based on improved singular value denoising has strong anti-noise performance, and can accurately identify various harmonics, inter-harmonics and time-varying signals.

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