

Green's Function Associated with a Vertical Jump of a Person Using a Load Cell Platform

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Abstract. The spring-mass system is a simple physics problem that has been widely discussed due to its importance and wide applications in different problems, particularly the waist-legs-feet system of a person could be represented by a spring-mass model, and the leap of a person can also be represented by the compression-release phenomena of the spring-mass system previously mentioned, this phenomenon is of great importance in different areas such as human health, and robotics. Thus, a model of damped spring-mass with a deformable load cell platform was used for getting a characteristic Green's Function of a vertical jump of a person.

Keywords. Dynamic calibration load cell platform, Green Function spring-mass system, Curve adjustment.

1. Introduction

When a person walks, it modifies the sole, giving a particular shape to the surface of the shoe. The signal shows that each person applies a different force to the surface in the search for the proper characterization of each person we pretend to find this starting with a function that surrounds the weight of the person, with the cushioning differential equation, this is only the beginning of this search.

The damped mass spring prototype is a good approach to approximately describe the vertical jump and the two feet walk. Hence, if we place a load cell platform, the forces applied to it describe the kinetics of the movements for different action and reaction forces of a damped mass spring. These phenomena were captured by our load cell sensor.

First at all, a simple physical pattern prototype of mass and spring with predetermined elastic constant is used as a standard prototype reference before a person performs the jumps on the platform with the load cell. In addition, the voltage-time load cell signal is proportional to the applied deformation force $F(t)$ [1]. The force is recorded by the Biopac Student Lab Basics system and the person's jump along with a video camera that records 120 frames per second, all within the experimental setup.

Thus, several variables come into consideration when our System captures the action-reaction mechanisms in-situ.

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More appropriate models for the dynamic movement of two walk systems are described by a matrix-second order differential equation. This is the case for the center of mass position, inertia moment J , such as the well-known model “Asymmetric Spring-Loaded Inverted Pendulum” (ASLIP) [2], where three generalized coordinates (leg length, leg angle respect to the line center de mass and the angle of orientation of such a center the mass).

The upward movement model considers the variable leg length since the center the mass will always be on the vertical axis where:

For the movement in the *action-reaction* over the cell sensor was proposed by:

$$F_{all\ time} = m \frac{dv}{dt} = -Rv + F(t) \quad (1)$$

where $v(t)$ and m are the velocity and mass acting on the resistive platform (coefficient R), under an external *action* force $F(t)$ that exist only during a very small period of time.

Thus, the well-known solution [3] for $v(t)$ is given by:

$$v(t) = \int_{-\infty}^t G_{speed}(t, \tau) F(\tau) d\tau \quad (2)$$

Where Green's Function $G_{speed}(t, \tau)$ for variable speed,

And for Eq. (1)

$$\begin{aligned} F_{all\ Time} &= \int_{-\infty}^t \left[\frac{-R}{m} \exp^{\frac{-R}{m}(t-\tau)} + \frac{\partial}{\partial \tau} \right] F(\tau) d\tau \\ &= \int_{-\infty}^t G_{Reaction}(t, \tau) F(\tau) d\tau \end{aligned} \quad (3)$$

$$G_{Reaction}(t, \tau) = \left\{ \begin{array}{ll} 0 & \text{for } t < \tau \\ \left[\frac{-R}{m} \exp^{\frac{-R}{m}(t-\tau)} + \frac{\partial}{\partial \tau} \right] & \text{for } t > \tau \end{array} \right\} \quad (4)$$

Here, Green's Function: $G_{Reaction}$ represents the response to an *action force* acting at very short time.

2. Materials and Method

First, with a simple mathematical structure we begin with the equivalent object of the vertical jump upward with the spring masses at the ends within the experimental setup (Figure 1 and 2). The legs are assumed to be massless.

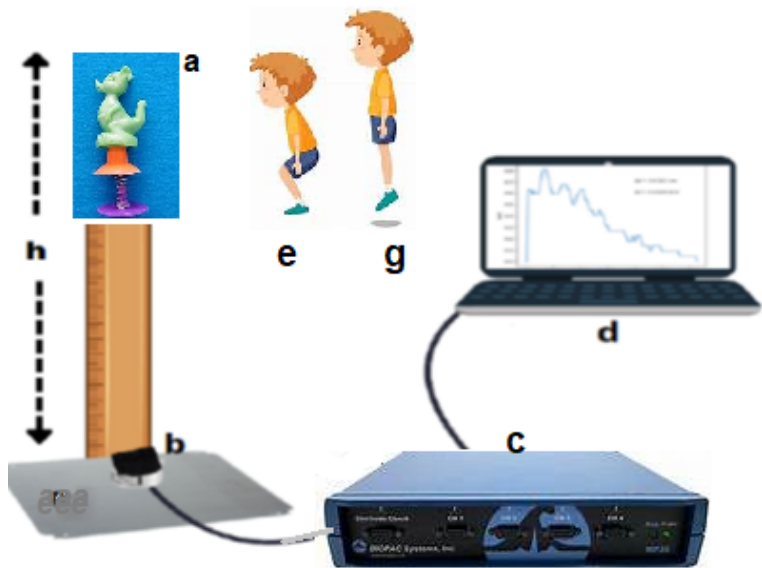


Figure 1. a) Physical sample prototype; b) Load cell platform; c) Biopac Student Lab -Basic system; d) PC for data capture; e) Pre-Action in the vertical jump; g) Reaction in the vertical jump.



Figure 2. Complete experimental system.

Notes: a) Physical sample prototype; b) Load cell platform; c) Biopac Student Lab -Basic system; d) PC for data capture.

We will always divide the load function $V(t)$ registered by the system into two parts:

1. In a deformation load $0 \leq t \leq \Delta\tau$, which is the interaction between the instant of Object-Sensor interaction and the maximum deformation reached, $F(t)$ takes action here precisely.
2. In a relaxation load $\Delta\tau \leq t \leq t_1$. Which is the maximum deformation and the recovery to the original geometry of the load cell sensor. This part will give us the characterization of the jump of the object of mass m in question.

For the case that concerns us in the deformation interval, a standard linear model will be used between the voltage $V(t)$ given by the sensor and the load expressed by $F(t) = K \cdot V(t)$ [1], where there is also an instantaneous elastic response followed by an exponential relaxation function

Here, some experimental results with the *sample patron prototype* (Figure 3 and 4).

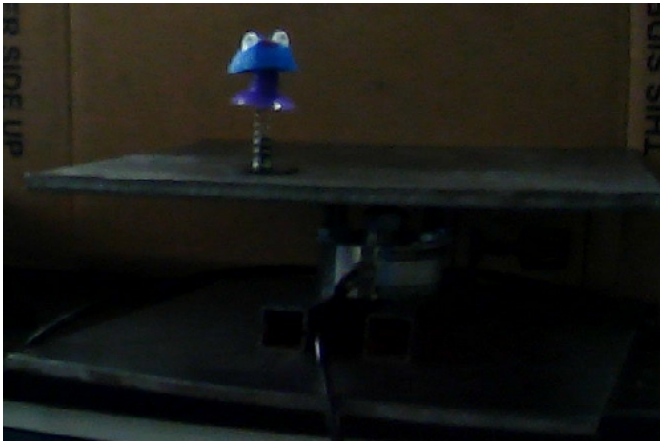


Figure 3. The sample patron prototype spring-mass object above load cell platform.

3. Results

It was obtained the following experimental outcomes:

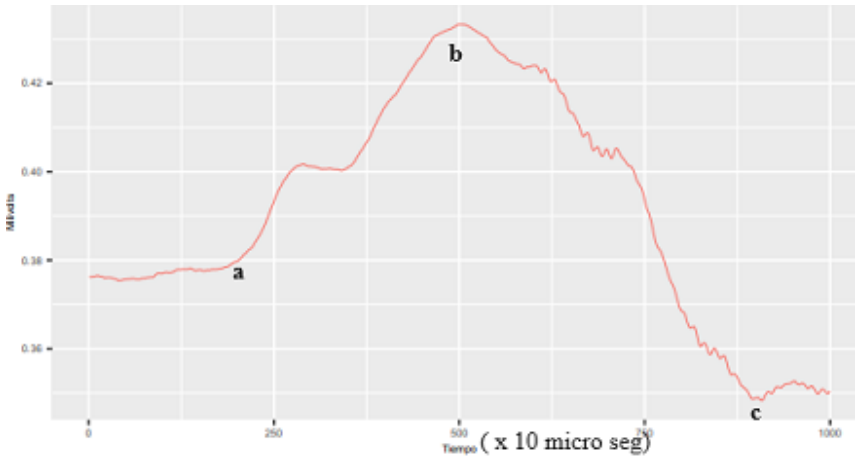


Figure 4. Voltage vs time response for the prototype mass-spring sample.

Where the difference in time between point b and point a is the action time $\Delta\tau=3.333 \times 10^{-3}$ seg of $F(t)$, and the voltage difference between point a and point c is precisely the weight $w= 0.1661$ Nw and $K= F(t)/V(t)= 5.2454$ Nw/mV for the prototype mass-spring sample.

Now we continue with the jump of a 86.1 Kg person placed on the load cell platform and we proceed to obtain its corresponding voltage-time response (Figure 5).

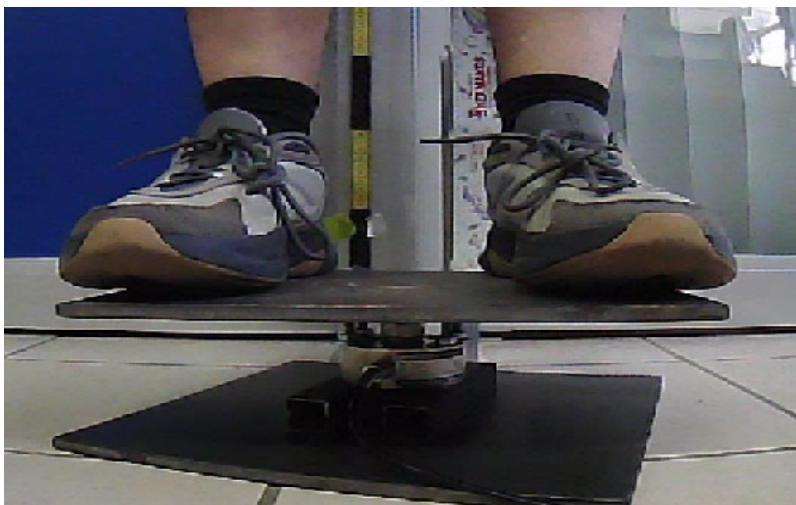


Figure 5. Pre-Action in the vertical jump for a person over the load cell platform.

It is important to note that load cell is flat and the action of the gravitational force is orthogonal everywhere; hence a “Zero-Moment Point, (ZMP)” is guaranteed and the net force moment over the load cell is equal to zero is ensured [4].

For a repetition of five vertical jumps, we have the following graphs (Figure 6).

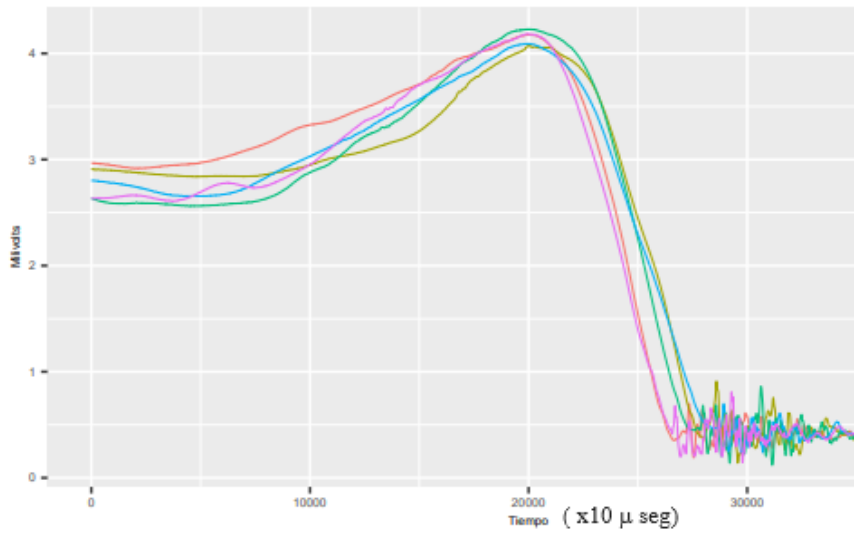


Figure 6. Voltage-time response in five vertical jumps of a person.

Considering only the average graph (Figure 7), we then have:

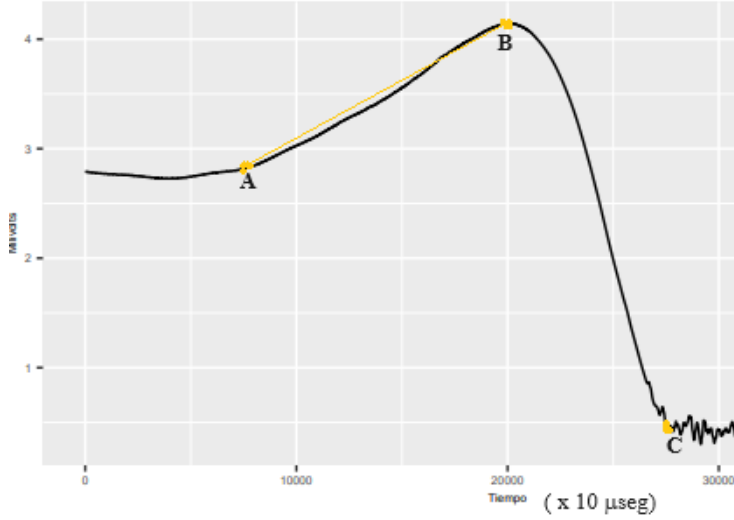


Figure 7. Voltage vs time average response for person's vertical jump.

In the same way, following the same voltage-time response of the prototype mass-spring model, we can find that for the vertical jump the action force $F(t)$ acts between points A and B obtaining $\Delta\tau = 0.125$ seconds. Where the voltage difference between points A and C correspond to the weight of the person 844.64Nw and $K = F(t)/V(t) = 283.7\text{Nw/mV}$.

Considering as a linear function the *action force* $F(t)$ from point A to point B

$$F(t) = (11.096 \frac{\text{mV}}{\text{sec}})(283.7\text{Nw/mV}) * t + \text{Constant} \quad (5)$$

Finally, substituting Eq. 5 into Eq. 3 and adjusting the experimental values of points B and C in Figure 6, we obtain our desired expression of the reaction force of the vertical jump for a person with a mass of 86.1 kg.

4. Conclusions

As a final step for our result for the vertical jump of an 86.1 Kg person, we substitute Eq. (5) in Eq. (3), and adjust our Green's function for points B and C of the graph in Figure 7 taking as a conclusion the following very particular expression:

$$G_{\text{Reaction}}(t, \tau) = \begin{cases} 0 & \text{for } t < \tau \\ \left[\frac{-0.885974\text{Kg/seg}}{86.1\text{Kg}} \exp \frac{-0.885974\text{Kg/seg}}{86.1\text{Kg}}(t-\tau) + \frac{\partial}{\partial \tau} \right] & \text{for } t > \tau \end{cases} \quad (6)$$

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