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Vagueness in Predicates and Objects

Brandon BENNETT [a] and Lucia GOMEZ ALVAREZ [b]
[a] *University of Leeds*
[b] *TU Dresden*
ORCiD ID: Brandon Bennett https://orcid.org/0000-0001-5020-6478, Lucia Gomez Alvarez https://orcid.org/0000-0002-2525-8839

Abstract. Standard first-order logic interprets reference, predication and quantification in terms of fixed denotations with respect to a domain of precise objects. We explore ways to generalise this semantics to account for variability of meaning due to factors such as vagueness, context and diversity of definitions or opinions. We present *Variable Reference Logic* (VRL), an elaboration of *Standpoint Logic*, which is a multi-modal logic based on a variety of *Supervaluation Semantics*. VRL can accommodate several modes of variability in relation to both predicates and objects. Its principal novelty is that its semantics incorporates a domain of *indefinite individuals*, whose precise properties (such as spatial extension) are not fully determinate. Each indefinite individual is associated with a set of precise entities corresponding to possible precise versions of the individual.

Keywords. Vagueness, Semantics, Vague Objects, Standpoint Logic

1. Introduction

Reference, predication and quantification are fundamental to classical first-order logic. Its semantics assumes a fixed domain of objects, with name constants referring to unique elements of the domain, predicates associated with subsets of the domain and quantifiers ranging over all the elements of the domain. Thus, if $\mathcal{D}$ is the domain of objects, then a constant name, say c, will denote an object $\delta(\mathsf{c})$, such that $\delta(\mathsf{c}) \in \mathcal{D}$, and an unary predicate P will be taken to denote a set of objects $\delta(P)$, with $\delta(P) \subseteq \mathcal{D}$. In this way, one can straightforwardly interpret $P(\mathsf{c})$ as a proposition that is true if and only if $\delta(\mathsf{c}) \in \delta(P)$. However, in natural languages there is considerable variability in the interpretation of symbols: a name may not always refer to a unique, precisely demarcated entity; a predicate need not always correspond to a specific set of entities.

Building on [1], this paper explores how the classical semantics can be modified to account for denotation, predication and quantification in the presence of semantic indeterminacy. We present a formal framework, based on *standpoint semantics* [2,3,4], which is a multi-modal elaboration of supervaluation semantics, where different modal operators correspond to different standpoints with respect to possible interpretations of vague terminology. The principal novelty of the current paper is the introduction of a domain of vague individuals as referents of names and values of variables. We motivate this by considering propositions whose meaning involves indeterminacy with regard to both the individuals referred to and also the precise properties of the referent objects.

Hence, for a vague predicate such as ‘desert’, it may be indeterminate what entities count as deserts and even when we identify something as being definitely a desert it may still be indeterminate what is the exact spatial extension of this desert.

2. Theories of Vagueness

The literature on vagueness has generally assumed that the phenomena of vagueness could arise from three potential sources [5]: (a) indeterminacy of representation (linguistic a.k.a *de dicto* vagueness), (b) indeterminacy of things in the world (known as *ontic* of *de re* vagueness) and (c) limitations of knowledge (*epistemic* vagueness). The epistemic view has some strong advocates (e.g. [6]) and it is apparent that semantics of multiple possible interpretations take a similar form to logics of knowledge and belief [7]. Indeed, the *standpoint semantics*, which will be used in our following analysis can be regarded as being of this form. But the question of whether vagueness is an epistemic phenomena, or merely shares a similar logic, does not seem to have any direct bearing on our semantic analysis, so in the current paper we shall not further consider the epistemic view.

2.1. Vagueness as exclusively De Dicto

A widely held view is that all vagueness is essentially *de dicto*. That is, it is entirely a property of language, not of the world: it consists in the variability or lack of precision in the way that words and phrases correspond to the precise entities and properties of the real world that they aim do describe. According to this view, forms of natural language that seem to attribute vagueness directly to things in the world are misleading and can be paraphrased in *de dicto* terms [8,9]. A fairly typical version of such an attitude is that of Varzi who (focusing on the domain of geography, within which vagueness is pervasive) takes a strong position against ontic vagueness. Varzi’s view of the relationship between vague terms and their referents is summarised in the following quotation:

> “[To] say that the referent of a geographic term is not sharply demarcated is to say that the term vaguely designates an object, not that it designates a vague object.” [9]

The view that vagueness is a linguistic phenomenon leaves open many possibilities regarding which aspects of language are involved, and how one might model vagueness within a theoretical semantic framework. These will be explored in detail below.

2.2. The possibility of De Re *or* Ontic *Vagueness*

Natural language sentences commonly describe objects as ‘vague’: ‘The smoke formed a vague patch on the horizon’; ‘The vaccine injection left a vague mark on my arm’; ‘He saw the vague outline of a building through the fog’. Here ‘vague’ means something like ‘indefinite in shape, form or extension’. If we take such sentences at face value, they seem to indicate that vagueness can be associated with an object in virtue of its spatio-temporal extension, its material constituency or even abstract characteristics. One may then argue that such properties are intrinsic to objects, and that, if an object has a vague intrinsic property, this indicates vagueness ‘of the thing’, often called *de re* vagueness. And, in so far as *things* are identified with entities existing in the world, such vagueness is often described as *ontic* (or *ontological*) meaning that it pertains to what exists in reality.

Note that the view that vagueness may be *de re* or *ontic* does not require one to deny that vagueness is often *de dicto*. Even if the world consisted entirely of precisely demarcated objects with exact physical properties, there could still be vague terminology (e.g. adjectives such as ‘large’) and hence vague ways to describe the world. Proponents of *de re* vagueness typically only claim that *some* kinds of vagueness are ontic [10,11].

The idea that vagueness of objects is primarily associated with vagueness of *spatial extension* has been endorsed and examined by Tye [10], who gives the following criterion for identifying vague objects: “A concrete object *o* is vague if and only if: *o* has borderline spatio-temporal parts; and there is no determinate fact of the matter about whether there are objects that are neither parts, borderline parts, nor non-parts of *o*.” The second, rather complex condition concerns the intuition that we cannot definitely identify borderline parts of a vague object. The current paper will not consider this second-order aspect of vagueness. However, we will be presenting a semantics in which there can be objects whose spatial extension, and hence material constituents, are indeterminate.

The case against *ontic* vagueness is typically based on two contentions: (a) the idea of an object in the world being in itself indeterminate is mysterious and implausible; (b) statements apparently implying *ontic* vagueness are readily paraphrased into forms where the vagueness is evidently *de dicto*. Although, at the level of atomic particles, we may accept that the position of an electron might be indeterminate (as is strongly indicated by quantum mechanics), this kind of indeterminacy seems very far from what we would need to account for vagueness in the demarcation of macroscopic physical objects. Consider a pile of twigs upon a twig-strewn forest floor. The indeterminacy of whether a particular twig is part of the pile does not arise from any indeterminacy in the physical locations of twigs. Rather, it seems that, for exactly the same physical situation, there can be different judgements of which twigs should be counted as part of the pile.

So perhaps it is reasonable to consider that a ‘thing’ such as a pile of twigs could have an indeterminate extension, even though there is no *ontic* indeterminacy in the reality within which the twigs are manifest. Hence, if a pile of twigs is considered a ‘thing’, the indeterminacy of its extension would be *de re*, though not *ontic*. Here, we see that there is an ambiguity in the meaning of ‘thing’ (and hence in the term *de re*). One can interpret it in the sense of an entity existing in a concrete reality or in the sense of an object of discourse, whose physical manifestation need not be completely determinate.

2.3. Established Accounts of Vagueness: Fuzzy Logic and Supervauationism

Among computer scientists *fuzzy logic* [12] has been the dominant approach to modelling vagueness. This theory modifies the classical denotations of expressions. Vague concepts denote *fuzzy sets*, such that the degree of membership of an object in a fuzzy set is graded (typically represented by a real number in the range [0..1]). This degree of membership also determines fuzzy truth values: the truth value of $\mathsf{C}(c)$ is simply the degree of membership of the object denoted by c in the fuzzy set denoted by C. The truth values of complex formulae are then determined by interpreting logical operators in terms of fuzzy truth functions. Although the original presentation of fuzzy logic does not treat objects as vague, many researchers have also modelled vague objects as fuzzy sets of points [13].

Many philosophers favour some variety of *supervaluationist* account, which models linguistic vagueness in terms of variability in the relation between vocabulary terms and their semantic reference. This is modelled in terms of a set of possible precise interpretations (often called *precisifications*). An early proposal that vagueness can be analysed in terms of multiple precise senses was made by Mehlberg [14], and a formal semantics based on a multiplicity of classical interpretations was used by van Fraassen [15] to explain 'the logic of presupposition'. This kind of formal model was subsequently applied to the analysis of vagueness by Fine [16] and a similar approach was proposed by Kamp [17].

An attraction of supervaluationism is its account *penumbral connections* [16], which many believe to be an essential characteristic of vagueness. This is the phenomenon whereby logical laws (such as the principle of non-contradiction) and semantic constraints (such as mutual exclusiveness of properties such as '... is short' and '... is tall') are maintained even for statements involving vague concepts. The solution, in a nutshell, being that, even though words may have multiple different interpretations, each admissible precisification of a language makes precise all vocabulary in a way that ensures mutual coherence of the interpretation of distinct but semantically related terms.

3. Semantic Aspects of Predicates and Objects in Relation to Vagueness

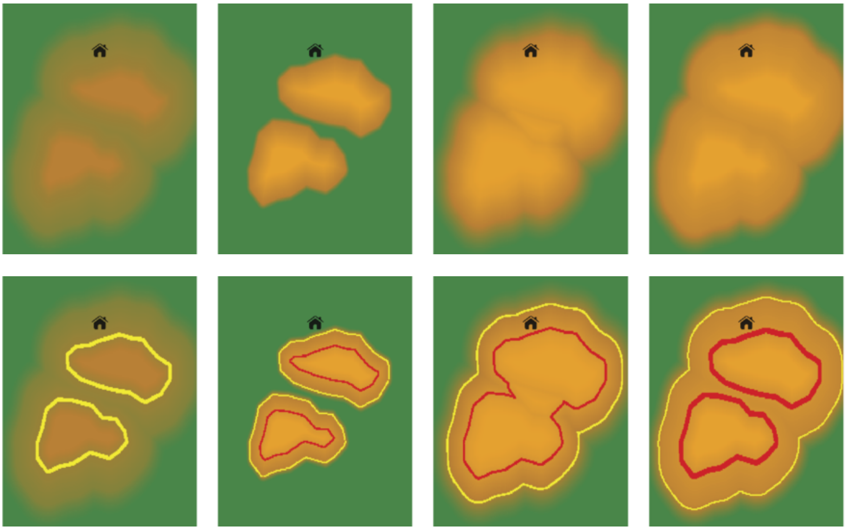
The semantic content of a predicate can be analysed into many aspects. Like in [18], we pick out the following as being of particular relevance to the study of vagueness:

Classification: Given a set of objects, a predicate classifies them into members or non-members of the set.

Individuation: Given a relevant information about a state of the world (its spatial and material properties) the meaning of a *sortal* predicate provides the criteria by which individual instances of the predicate are recognised. (In terms of axioms these criteria can often be divided into *existential conditions* and *identity criteria*.)

Demarcation/Constituency: The meaning of *sortal* predicates also provides criteria for spatially demarcating and determining the material constituents of its instances.

In classical logic 'classification' is the most prominent, perhaps even the only, aspect of predication. This is because in the usual semantics for first-order logic, the 'universe' or 'domain of quantification' is typically presented as if ontologically prior to predication, in that we 'interpret' predicates in relation to a given domain. However, in order to name an object or count instances of a category of object we need to be able to individuate the object or objects to which we refer, and this implies that individuation is necessary for (and one could say ontologically prior to) classification. Furthermore, as illustrated by the 'desert' examples below, individuating objects as instances of a certain kind does not necessarily require that they be identified with precisely demarcated portions of the real world. Hence, the objects of discourse referred to by names or by quantification may not be completely definite in all their attributes. Accounting for the independence between criteria for individuation, classification and demarcation in the presence of vagueness is a motivation for the semantics that we shall present below.



(a) (b) (c) (d)

Figure 1. Illustration of individuation, demarcation and classification in relation to deserts.

3.1. Classifying, Individuating and Demarcating Deserts

To illustrate how the different aspects of predication operate, we consider the concept ‘desert’. Several factors may be relevant to deciding what is a desert, in particular: lack of precipitation (aridity), temperature and to a lesser extent vegetation, land form and land use. For simplicity, we consider a situation where a measure of aridity is taken to be the only relevant factor (we may assume that other factors are either uniform or co-vary with aridity within the region under consideration). Consider the situations illustrated in Fig. 1 depicting several states of a region with ‘my house’ indicated by a black icon:

(a) The region was cool and fertile although two of the most arid areas were arguably deserts. *My house was near to one arid area that was arguably a desert.*

(b) The climate warmed and the arid areas became true deserts, but with a fertile valley between them. *There was a desert that my house was unequivocally near to but not part of.*

(c) Eventually the desert areas expanded and merged into one big desert. *Then, my house was at its edge, arguably though not unequivocally within the desert.*

(d) Recently a little moisture has returned along the valley. *Some argue that the whole arid area is still a desert. Others refer to two deserts in that area, separated by the valley. Either way, my house is still arguably in a desert.*

The narrative sequence is not relevant to present concerns, except that it shows that situations of the same general type may exhibit different aspects of vagueness depending on specific details. What we should particularly note is that: in (a) the classification of the arid area as desert is equivocal; in (b) we definitely have two deserts but each has an indeterminate boundary; in (c) we definitely have exactly one desert, but it is debatable whether my house is within the desert area; and, in (d) the number of deserts present is indeterminate. The semantics that we shall develop is motivated by considering situations such as those depicted Fig. 1, where there is a separation between the aspects of classification, individuation and demarcation with respect to vagueness.

3.2. *Possibilities for Semantic Modelling of Vague Predicates and Objects*

Vagueness of both predicates and names can be modelled either in terms of the mapping from vocabulary to some form of 'semantic denotation' or in the correspondence between semantic denotations and the real world. Or, indeed, each of these relationships could model a different aspect of vagueness. Fig. 2 illustrates some possibilities. Fig. 2(a) depicts a hilly region with rocky crags, where the terrain is irregular and there is no clear way of dividing it into separate 'crag' objects. The name 'Arg Crag' has been given to one of the rocky outcrops. However, there may be different opinions regarding exactly which outcrop is Arg Crag. Indeed, some people may use the name to refer to the whole of this rocky area, whereas others consider that it refers to a more specific rock structure.

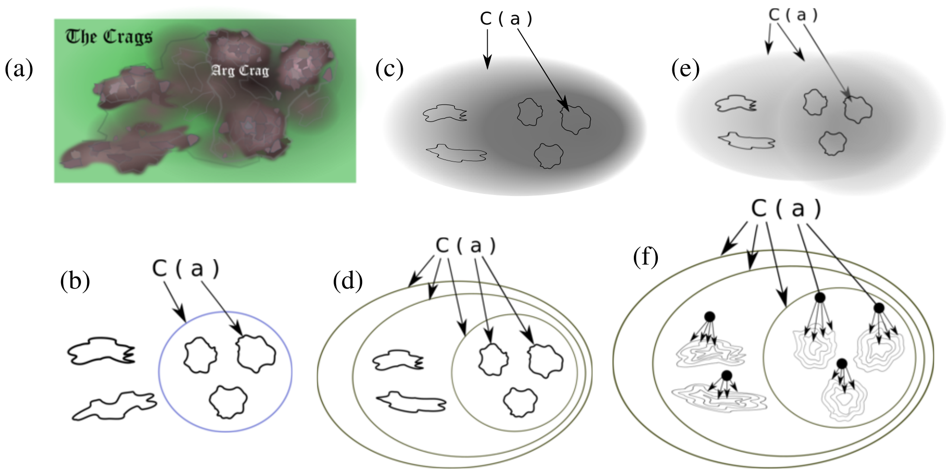


Figure 2. (a) Arg Crag and surrounding area. (b) Classical semantics. (c) Fuzzy model of a vague predicate. (d) Multiple denotation model, such as supervaluation semantics. (e) Concepts can have multiple extensions, each of which is a fuzzy set. (f) multiple reference for predicates and also multiple extensions for objects.

In classical semantics each conceptual term denotes a fixed set of entities and each name refers to a single precise entity. Thus, even before specifying denotations, we will need to divide the craggy region into specific individual objects, to make a set of possible referents. Figs. 2(c)–(f) depict various ways in which vaguenes of predicates and objects can be modelled. Sub-figures (c) and (d) show fuzzy-logic and supervaluation semantics models. In (c) C denotes a fuzzy set, whereas in (d) both the predicate and object have a variable reference (with each *precisification* determining a unique reference). Sub-figure (e) depicts a possible combination of the fuzzy and supervaluation approaches, where predicate symbols can have multiple references, each of which is a fuzzy set.

A limitation of models (c)–(e) is that fuzziness/variability is only associated with the predicate and/or the reference relation but not the object referred to. However, both approaches can be modified to incorporate vague referents, that might correspond to objects with indeterminate physical boundaries. In fuzzy logic, objects with indefinite extensions can be modelled as fuzzy sets of points [13]. And within a multiple-reference semantics, one could also associate a set of different extensions for different precise versions of an object (see e.g. the 'egg-yolk' representation of [19]).

Fig. 2(f) depicts an extended form of multi-reference semantics (*The Full Multi*). Here we see not only that the predicates and names can have variable reference, but also there can be multiple possible precise versions of each reference object. This variability in the objects could be called *de re* vagueness, but we regard it as semantic vagueness, with the objects being vague semantic objects (visualised as the small black discs) that potentially correspond to many precise physical extensions.

It may appear that the two stages of multiple reference are redundant. A name refers to one or more indefinite objects, each of which corresponds to one or more precise extensions. This may seem to give the same effect as if the name simply has multiple precise extensions, without any intervening indefinite object. However, this would force us to conflate the different types of vagueness illustrated in the desert examples above: we could not have something that was definitely a desert or a mountain but whose extension was indefinite. Indeterminacy regarding which object a name refers to and also indeterminacy in of referent objects themselves are both modelled explicitly in the semantics for the logic $\mathbb{V}_1$, that we present below (Sec 4.2).

Figure 2(f) still considerably simplifies the potential semantic variability that might arise. It presupposes that the global set of vague objects, together with their associations to precise entities, remains fixed, even though the subset associated with predicate C may vary. In other words, C only varies in how it *classifies* objects, not in how it *individuates* them. The logic $\mathbb{V}_1$ is more general. It allows different senses of sortal predicates to be associated with different ways of individuating objects (for instance under some interpretations of 'Crag', all three of the roundish craggy objects within the innermost circle of the diagram might be considered as parts of a single large crag).

4. A Semantic Theory of Variable Reference

We now consider what kind of semantics can account for the general form of variable denotation illustrated in Fig. 2(f). *Standpoint Semantics* provides a reasonably general framework within which semantic variability can be modelled in terms of the symbols of a formal language having multiple possible denotations. We shall start by introducing First-Order Standpoint Logic and then elaborate this to the formalism *Variable Reference Logic*, within which we can model predication and quantification of vague objects.

4.1. Standpoint Logic

Standpoint Semantics is based on a formal structure that models semantic variability in terms of the following two closely connected components:

- A *precisification* is a precise and consistent interpretation of a vague language, and it coherently assigns precise denotations to all its vague elements. A model for a vague sentence contains a set of admissible precisifications of that sentence.
- A *standpoint* is modelled as a set of *precisifications* which are compatible with a point of view or context of language understanding. A standpoint can capture both explicit specifications of terminology and implicit constraints on meanings that arise in conversation (e.g. "That person is tall" constrains the meaning of tallness).

4.1.1. Syntax of First-Order Standpoint Logic

First-Order Standpoint Logic ($\mathbb{S}_{\text{FO}}$) is based on a *vocabulary* $\langle \mathcal{P}, \mathcal{N}, \mathcal{S} \rangle$, consisting of *predicate symbols* $\mathcal{P}$ (each associated with an arity $n \in \mathbb{N}$), *constant symbols* $\mathcal{N}$ and *standpoint symbols* $\mathcal{S}$, usually denoted with s, s', such that $* \in \mathcal{S}$, where $*$ is used to designate the *universal standpoint*. There is also a set $\mathcal{X} = \{x, y, ...\}$ of *variables*, and the set $\mathcal{T} = \mathcal{N} \cup \mathcal{X}$ of *terms* contains all constants and variables. The set $\mathbb{S}_{\text{FO}}$ of first-order standpoint *formulae* is given by

$$\phi, \psi := \mathsf{P}(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_s \phi ,$$

where $\mathsf{P} \in \mathcal{P}$ is a k-ary predicate symbol, $t_1, \ldots, t_k \in \mathcal{T}$ are terms, $x \in \mathcal{X}$, and $s \in \mathcal{S}$. The definable connectives and operators $\mathbf{t}$, $\mathbf{f}$, $\phi \vee \psi$, $\phi \rightarrow \psi$, $\exists x \phi$, and $\Diamond_s \phi$ are as usual.

4.1.2. Semantics of First-Order Standpoint Logic

Given a vocabulary $\langle \mathcal{P}, \mathcal{N}, \mathcal{S} \rangle$, a *first-order standpoint structure* $\mathcal{M}$ is a tuple $\langle \Delta, \Pi, \sigma, \delta \rangle$, where Δ is the (non-empty) *domain* of $\mathcal{M}$, Π is a non-empty set of *precisifications*, σ is a function mapping each standpoint symbol from $\mathcal{S}$ to a set of precisifications (i.e., a subset of Π), and δ is a function mapping each precisification from Π to an ordinary first-order structure $\mathcal{I}$ over the domain Δ, whose interpretation function $\cdot^{\mathcal{I}}$ maps:

- each predicate symbol $\mathsf{P} \in \mathcal{P}$ of arity k to an k-ary relation $\mathsf{P}^{\cdot\mathcal{I}} \subseteq \Delta^k$,
- each constant symbol $a \in \mathcal{N}$ to a domain element $a^{\cdot\mathcal{I}} \in \Delta$.

The semantics of [20] required that $a^{\cdot\delta(\pi_1)} = a^{\cdot\delta(\pi_2)}$ for any $\pi_1, \pi_2 \in \Pi$ and $a \in \mathcal{N}$. Here we allow *non-rigid constants* that may denote different objects in different precisifications. But (in contrast to the more expressive $\mathbb{V}_1$ that will be developed below) all precisifications of a first-order standpoint structure will implicitly share the same interpretation domain Δ; that is, the *constant domain assumption* is adopted. The most distinctive elements of the model are the s_i, which model the notion of standpoint as a set of precisifications that are *admissible* for that standpoint. A proposition is *unequivocally true* according to standpoint s_i iff it is true at every precisification $\pi \in s_i$.

Let $\mathcal{M} = \langle \Delta, \Pi, \sigma, \delta \rangle$ be a first-order standpoint structure for the vocabulary $\langle \mathcal{P}, \mathcal{N}, \mathcal{S} \rangle$ and $\mathcal{X}$ be a set of variables. A *variable assignment* is a function $v : \mathcal{X} \rightarrow \Delta$ mapping variables to domain elements. Given a variable assignment v, we denote by $v_{\{x \mapsto \varepsilon\}}$ the function mapping x to $\varepsilon \in \Delta$ and any other variable y to $v(y)$.
An interpretation function $\cdot^{\mathcal{I}}$ and a variable assignment specify how to interpret terms as domain elements:[1]

$$t^{\cdot\mathcal{I},v} = \begin{cases} v(x) & \text{if } t = x \in \mathcal{X}, \\ a^{\cdot\mathcal{I}} & \text{if } t = a \in \mathcal{N}. \end{cases}$$

Then, let $\pi \in \Pi$ and $v : \mathcal{X} \rightarrow \Delta$ be a variable assignment. The satisfaction relation $\models$ is:

[1]Here, and in the following, $\sigma^{\cdot\gamma}$ is a concise notation for $\gamma(\sigma)$. It gives the semantic denotation of symbol σ according to a function γ.

$$
\begin{array}{rcll}
\mathcal{M},\pi,v & \models & \mathsf{P}(t_1,\ldots,t_k) & \text{iff} \quad (t_1^{\cdot\delta(\pi),v},\ldots,t_k^{\cdot\delta(\pi),v}) \in \mathsf{P}^{\cdot\delta(\pi)} \\
\mathcal{M},\pi,v & \models & \neg\phi & \text{iff} \quad \mathcal{M},\pi,v \not\models \phi \\
\mathcal{M},\pi,v & \models & \phi \wedge \psi & \text{iff} \quad \mathcal{M},\pi,v \models \phi \text{ and } \mathcal{M},\pi,v \models \psi \\
\mathcal{M},\pi,v & \models & \forall x\phi & \text{iff} \quad \mathcal{M},\pi,v_{\{x\mapsto\varepsilon\}} \models \phi \text{ for all } \varepsilon \in \Delta \\
\mathcal{M},\pi,v & \models & \Box_s\phi & \text{iff} \quad \mathcal{M},\pi',v \models \phi \text{ for all } \pi' \in \sigma(s) \\
\mathcal{M},\pi & \models & \phi & \text{iff} \quad \mathcal{M},\pi,v \models \phi \text{ for all } v : \mathcal{X} \to \Delta \\
\mathcal{M} & \models & \phi & \text{iff} \quad \mathcal{M},\pi \models \phi \text{ for all } \pi \in \Pi
\end{array}
$$

When $\mathcal{M} \models \phi$ we say that $\mathcal{M}$ is a *model* for ϕ.

The logic $\mathbb{S}_{\text{FO}}$ enables one to formalise the content of some statements discussed in Section 3.1, namely (a), *"My house was near to one arid area that was arguably a desert"* and (b), "*There was a desert that my house was unequivocally near to but not part of.*" We assume that Desert and AridArea are vague predicates, but (my) house has definite coordinates and PartOf is a definite predicate.

$$\exists x\, \Box_*(\mathsf{AridArea}(x) \wedge \mathsf{Near}(\mathsf{house},x) \wedge \Diamond_*\mathsf{Desert}(x)) \tag{a}$$

$$\exists x\, (\mathsf{Desert}(x) \wedge \Box_*(\mathsf{Near}(\mathsf{house},x) \wedge \neg\mathsf{PartOf}(\mathsf{house},x)) \tag{b}$$

It should be apparent that addressing examples involving vague objects with $\mathbb{S}_{\text{FO}}$ is not straightforward but can in many cases be achieved by means of paraphrasing into a form where the vagueness is captured by the indeterminacy of a predicate applied to the object. However, consider sentence (c): *Then, my house was [at its edge,] arguably though not unequivocally within the desert.* It may seem that we can capture the core of this claim with a formula such as:

$$\exists x(\ \mathsf{Desert}(x) \wedge \Diamond_*\mathsf{PartOf}(\mathsf{house},x) \wedge \neg\Box_*\mathsf{PartOf}(\mathsf{house},x)) \tag{c}$$

But, according to the semantics of $\mathbb{S}_{\text{FO}}$, variables are *rigid*, they do not vary according to the precisification. Hence, a model that satisfies $\Diamond_*\mathsf{PartOf}(\mathsf{house},x)$ cannot satisfy $\neg\Box_*\mathsf{PartOf}(\mathsf{house},x)$ unless either 'house' or 'PartOf' are not rigid (i.e. vague). But this does not seem to fit with the intended meaning of the sentence. There is no implication that the location of the house is indeterminate; and, since PartOf is a definite geometrical relationship we would expect this to also have a determinate value for any precise entities. I.e., we would expect: $\forall x(\Diamond_*(\mathsf{house} = x) \to \Box_*(\mathsf{house} = x))$ and $\forall x,y(\Diamond_*\mathsf{PartOf}(x,y) \to \Box_*\mathsf{PartOf}(x,y))$. With these axioms (c) is unsatisfiable in $\mathbb{S}_{\text{FO}}$.

Of course, the possible values of x in (c) depend on the interpretation of 'Desert', which is vague and can have be satisfied by different entities for different precisifications. But this does not solve the problem because any value given to x must be a determinate entity. Hence, it must be determinate whether the referent of 'house' is a part of that entity. This is a paradigm example, illustrating that expressing certain types of vagueness requires indeterminate objects as well as indeterminate predicate (and constant) symbols.

4.2. Variable Reference Logic

We now generalise standpoint semantics to define Variable Reference Logic, $\mathbb{V}_1$, that can represent predication and quantification over indefinite objects. The most significant complication of $\mathbb{V}_1$ in relation to $\mathbb{S}_{\text{FO}}$ is that rather than the domain being comprised only of atomic entities, we also introduce 'indefinite individuals' that are modelled as functions from precisifications to precise entities. Another complication is that predicates are divided into: 'sortals' (corresponding to count nouns such as 'desert' or 'cat') that determine subdomains of the domain of indefinite individuals, 'indefinite predicates' that classify or relate indefinite individuals, and 'precise entity predicates' that represent properties or relations of exact spatio-temporal and/or material extensions in reality.

With respect to sortals, individual properties and quantification, indefinite individuals are treated as basic objects (e.g. a particular mountain or cloud). But in the application of a 'precise entity predicate' (e.g. '... contains 7.3×10^{26} atoms') the individual is evaluated with respect to a precisification to obtain a precise entity (the individual's extension) that determines the truth of the precise predicate. Although our semantics allows indefinite individuals to be any function from precisifications to precise entities, instances of sortals will typically by highly constrained by axioms expressing conditions arising from the meaning of the sortal. For example, if a desert is a maximal area of the Earth's surface satisfying certain conditions (e.g. low precipitation) then, even if exact thresholds on the parameters of these conditions are not given, the set of indefinite individuals that could correspond to a desert is hugely restricted.

4.2.1. Syntax

The language of $\mathbb{V}_1$ is built from a vocabulary $\mathcal{V} = \langle \mathcal{K}, \mathcal{A}, \mathcal{Q}, \mathcal{N}, \mathcal{S} \rangle$, comprising:

- $\mathcal{K} = \{\ldots, \mathsf{K}_i, \ldots\}$, sortal predicates (e.g. Desert, Mountain, Cat),
- $\mathcal{A} = \{\ldots, \mathsf{A}_i, \ldots\}$, indefinite predicates (e.g. Dry, Fluffy, Climbed),
- $\mathcal{Q} = \{\ldots, \mathsf{Q}_i, \ldots\}$, precise entity predicates (e.g. exact spatial properties),
- $\mathcal{N} = \{\ldots, \mathsf{n}_i, \ldots\}$, proper name symbols (e.g. everest, tibbles).
- $\mathcal{S} = \{\ldots, s_i, \ldots, *\}$, standpoint symbols.

Let $\mathcal{P} = \mathcal{K} \cup \mathcal{A} \cup \mathcal{Q}$ be the set of predicate symbols, containing all sortals, indefinite predicates and precise entity predicates. Again, there is a set of variables $\mathcal{X}$, and the set $\mathcal{T} = \mathcal{N} \cup \mathcal{X}$ contains all proper name symbols and variables. Then, the set $\mathbb{V}_1$ of Variable Reference Logic *formulae* is given by

$$\phi, \psi \ := \ \mathsf{P}(\tau) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_s \phi \ ,$$

where $\mathsf{P} \in \mathcal{P}$, $\tau \in \mathcal{T}$, $x \in \mathcal{X}$, $\mathsf{K} \in \mathcal{K}$ and $s \in \mathcal{S}$.

4.2.2. Semantics

Given a vocabulary $\mathcal{V} = \langle \mathcal{K}, \mathcal{A}, \mathcal{Q}, \mathcal{N}, \mathcal{S} \rangle$, a *Variable Reference Logic structure* is a tuple $\langle E, \Pi, \sigma, \delta \rangle$ where:

- E is the set of precise entities.
- Π is the set of precisifications.
- σ is a function mapping standpoint symbols to precisifications, and
- $\delta = \langle \delta_{\mathcal{K}}, \delta_{\mathcal{A}}, \delta_{\mathcal{Q}}, \delta_{\mathcal{N}} \rangle$ is a denotation function divided into components specifying the denotations for each type of non-logical symbol (see below).

We define the set of *indefinite individuals* $I_* = Maps(\Pi, E)$ as the set of all mappings from precisifications to precise entities. For each indefinite individual $i \in I_*$ and $\pi \in \Pi$, $i(\pi)$ will be an element of E, which constitutes the precise version of individual i according to precisification π. At each precisification $\pi \in \Pi$, each name symbol $\mathsf{n} \in \mathcal{N}$

denotes some element of I_* and each sortal $\mathsf{K} \in \mathcal{K}$ and individual attribute predicate $\mathsf{A} \in \mathcal{A}$ denotes a subset of I_*. The set of indefinite individuals at a precisification π depends on the interpretation of the sortal predicates at π. The denotation functions map:

- $\delta_{\mathcal{K}}$: each precisification $\pi \in \Pi$ into a function from each sortal symbol $\mathsf{K} \in \mathcal{K}$ to a subset $\mathsf{K}^{\cdot\delta_{\mathcal{K}}(\pi)} \subseteq I_*$ — i.e. a set of indefinite individuals.

  On the basis of $\delta_{\mathcal{K}}$ we define $I_\pi = \bigcup\{\mathsf{K}^{\cdot\delta_{\mathcal{K}}(\pi)} \mid \mathsf{K} \in \mathcal{K}\}$, the domain of (indefinite) individuals according to precisification π. I_π is the set of all individuals of any sort, where the sortals are interpreted according to precisification π.
- $\delta_{\mathcal{A}}$: each precisification $\pi \in \Pi$ into a function from each indefinite predicate symbol $\mathsf{A} \in \mathcal{A}$ of arity k to a k-ary relation $\mathsf{A}^{\cdot\delta_{\mathcal{A}}} \subseteq I_\pi^k$,
- $\delta_{\mathcal{Q}}$: each entity predicate symbol $\mathsf{Q} \in \mathcal{Q}$ of arity k to k-ary relations $\mathsf{Q}^{\cdot\delta_{\mathcal{Q}}} \subseteq E^k$,
- $\delta_{\mathcal{N}}$: each precisification $\pi \in \Pi$ into a function from each indefinite name symbol $\mathsf{N} \in \mathcal{N}$ to an indefinite individual $\mathsf{N}^{\cdot\delta_{\mathcal{N}}(\pi)} \in I_\pi$.

Let us now deal with the variable assignments in *Variable Reference Logic*. Let $\mathcal{M} = \langle E, \Pi, \sigma, \delta\rangle$ be a model for the vocabulary $\langle \mathcal{K}, \mathcal{A}, \mathcal{Q}, \mathcal{N}, \mathcal{S}\rangle$ and $\mathcal{X}$ be a set of variables. A *variable assignment* is a function $v : \mathcal{X} \to I_*$ mapping variables to individuals. Given a variable assignment v, we denote by $v_{\{x \mapsto i\}}$ the function mapping x to $i \in I_*$ and any other variable y to $v(y)$. The interpretation function $\delta_{\mathcal{N}}$ and a variable assignment specify how to interpret terms by domain elements in each precisification $\pi \in \Pi$:

$$\tau^{\cdot\pi,v} = \begin{cases} v(x) & \text{if } \tau = x \in \mathcal{X}, \\ \mathsf{n}^{\cdot\delta_{\mathcal{N}}(\pi)} & \text{if } \tau = \mathsf{n} \in \mathcal{N}. \end{cases}$$

Then, let $\pi \in \Pi$ and $v : \mathcal{X} \to \Delta$ be a variable assignment. The satisfaction relation $\models$ is:

$$\begin{array}{ll}
\mathcal{M},\pi,v \models \mathsf{K}(\tau) & \text{iff } \tau^{\cdot\pi,v} \in \mathsf{K}^{\cdot\delta_{\mathcal{K}}(\pi)} \\
\mathcal{M},\pi,v \models \mathsf{A}(\tau_1,\ldots,\tau_k) & \text{iff } \langle \tau_1^{\cdot\delta(\Pi),v},\ldots,\tau_k^{\cdot\delta(\Pi),v}\rangle \in \mathsf{A}^{\cdot\delta_{\mathcal{A}}(\pi)} \\
\mathcal{M},\pi,v \models \mathsf{Q}(\tau_1,\ldots,\tau_k) & \text{iff } \langle \tau_1^{\cdot\delta(\Pi),v}(\pi),\ldots,\tau_k^{\cdot\delta(\Pi),v}(\pi)\rangle \in \mathsf{Q}^{\cdot\delta_{\mathcal{Q}}} \\
\mathcal{M},\pi,v \models \neg\phi & \text{iff } \mathcal{M},\pi,v \not\models \phi \\
\mathcal{M},\pi,v \models \phi \wedge \psi & \text{iff } \mathcal{M},\pi,v \models \phi \text{ and } \mathcal{M},\pi,v \models \psi \\
\mathcal{M},\pi,v \models \forall x\, \phi(x) & \text{iff } \mathcal{M},\pi,v_{\{x\mapsto i\}} \models \phi(x) \text{ for all } i \in I_\pi \\
\mathcal{M},\pi,v \models \Box_s\phi & \text{iff } \mathcal{M},\pi',v \models \phi \text{ for all } \pi' \in \sigma(s)
\end{array}$$

To understand the interpretation of atomic predications, one must be aware that the evaluation of a symbol may require one or two levels of de-referencing in relation to the precisification index π. First, notice that the K and A predications are interpreted in the same way. When applied to a name symbol n, both the constant and the predicate get evaluated with respect to a precisification, so that the names denote particular individuals and the predicate denotes a set of such individuals. When the argument is a variable rather than a name symbol, the variable directly denotes an individual without any need for evaluation relative to a precisification. In the case of (exact) Q predications, individuals need to be further evaluated relative to the precisification in order to obtain precise entities, which can be tested for membership of the precise set denoted by the property

Q. So, although Q predicates are not themselves subject to variation in relation to π, they require an extra level of disambiguation in the interpretation of their argument symbol.

4.2.3. Classification, Individuation and Demarcation with $\mathbb{V}_1$

The elaboration in the denotation functions for the different kinds of symbol and the semantics given above for the different cases of predication allow quantification to operate at an intermediate level in the interpretation of reference. The *individuation* of potential referents occurs prior to quantification by establishing individuals in relation to a given interpretation of sortal predicates. But these individuals can still be indeterminate in that they may correspond to different exact entities. The formulas (a) and (b) in Section 4.1.2 are also satisfiable $\mathbb{V}_1$ formulas, and correctly formalise the scenarios described. Furthermore, the formula (c), that was unsatisfiable in $\mathbb{S}_{\mathrm{FO}}$, is satisfiable in $\mathbb{V}_1$, because area is modelled as an indefinite individual that is unequivocally classified as a Desert and at some but not all precisifications is associated to a precise extension containing house. Finally, statement (d) can be formalised as follows:

$$\exists x\,(\Box_{*}\mathsf{AridArea}(x) \;\wedge\; \Diamond_{some}\mathsf{Desert}(x) \;\wedge$$
$$\Diamond_{others}(\exists y,z\;(y \neq z) \;\wedge\; \mathsf{PartOf}(y,x) \;\wedge\; \mathsf{PartOf}(z,x) \;\wedge\; \mathsf{Desert}(y) \;\wedge\; \mathsf{Desert}(z)))$$
$$\wedge\,\Box_{*}\exists x\;(\mathsf{Desert}(x) \;\wedge\; \Diamond_{*}\mathsf{PartOf}(\mathsf{house},x))$$

Here, the final conjunct illustrates how the indefinite individuals in the semantics of $\mathbb{V}_1$ permit forms of expression that do not have a natural representation in $\mathbb{S}_{\mathrm{FO}}$. The construct '$\Box_{*}\exists x\,(\mathsf{Desert}(x)\ldots)$' ensures that under any interpretation of Desert (in particular including the '*some*' and '*others*' standpoints) there is an entity that satisfies the condition $\Diamond_{*}\mathsf{PartOf}(\mathsf{house},x)$. This only makes sense if x refers to an indeterminate object.

5. From $\mathbb{V}_1$ back to First Order Standpoint Logic $\mathbb{S}_{\mathrm{FO}}$

We next turn to the question of the expressive power of $\mathbb{V}_1$, specifically in comparison to $\mathbb{S}_{\mathrm{FO}}$. We will provide a translation from $\mathbb{V}_1$ into $\mathbb{S}_{\mathrm{FO}}$, which not only settles the above question but is also interesting in its own right, since it spells out the restrictions that $\mathbb{V}_1$ imposes on the semantics. First, we consider the set Φ of axioms in $\mathbb{S}_{\mathrm{FO}}$ such that:

$$\begin{aligned}
\Phi = \{\;\; & \forall x(\mathsf{ind}(x) \leftrightarrow \neg\mathsf{ext}(x)), \qquad \forall x(\mathsf{ind}(x) \leftrightarrow \Box_{*}\mathsf{ind}(x)) \wedge (\mathsf{ext}(x) \leftrightarrow \Box_{*}\mathsf{ext}(x)), && (1)\\
& \textstyle\bigwedge_{\mathsf{Q}\in\mathcal{Q}} \forall \vec{x}(\mathsf{Q}(\vec{x}) \leftrightarrow \Box_{*}\mathsf{Q}(\vec{x})), \quad \forall x,y(\mathsf{prec}(x,y) \rightarrow (\mathsf{ind}(x) \wedge \mathsf{ext}(y))), && (2)\\
& \forall x \exists y(\mathsf{ind}(x) \rightarrow \mathsf{prec}(x,y)), \qquad \forall x,y(\mathsf{prec}(x,y) \;\wedge\; \mathsf{prec}(x,z) \rightarrow y = z), && (3)\\
& \textstyle\bigwedge_{\mathsf{A}\in\mathcal{A}} \forall x_1,..,x_k(\mathsf{A}(x_1,..,x_k) \rightarrow \mathsf{ink}(x_1) \wedge \ldots \wedge \mathsf{ink}(x_k)), && (4)\\
& \textstyle\forall x\;(\mathsf{ink}(x) \rightarrow \mathsf{ind}(x)) \wedge (\mathsf{ink}(x) \leftrightarrow \bigvee_{\mathsf{K}\in\mathcal{K}} \mathsf{K}(x)) \;\; \} && (5)
\end{aligned}$$

The axioms in line (1) divide the domain into two kinds of elements, namely the individuals (denoted by ind) and their extensions (denoted by ext). This is done by stating that every domain element is either an individual or an extension but not both and by making ind and ext rigid predicates. Axioms in line (2) establish that every precise entity property predicate is rigid, following the intended semantics, and that the predicate $\mathsf{prec}(x,y)$ always relates an individual $\mathsf{ind}(x)$ to its precise extension $\mathsf{ext}(y)$ at a given precisification. Axioms in line (3) ensure that every individual $\mathsf{ind}(x)$ has (at each precisification) a unique precise extension $\mathsf{prec}(x,y)$ and, in line (4), that every domain element with an indefinite predicate A must be an instantiated sortal individual in that precisification (denoted by ink). Finally, axiom (5) ensures that an instantiated individual $\mathsf{ink}(x)$ has indeed been instantiated by some sortal predicate $\mathsf{k} \in \mathsf{K}$.

Then, the function ***trans*** : $\mathbb{V}_1 \to \mathbb{S}_{\mathrm{FO}}$, mapping $\mathbb{V}_1$ formulae to formulae in First-Order Standpoint Logic, is recursively defined as follows:

$$\begin{aligned}
&\boldsymbol{trans}(\mathsf{K}(\tau)) = \mathsf{K}(\tau) && \boldsymbol{trans}(\neg\phi) = \neg\boldsymbol{trans}(\phi)\\
&\boldsymbol{trans}(\mathsf{A}(\vec{\tau})) = \mathsf{A}(\vec{\tau}) && \boldsymbol{trans}(\phi_1 \wedge \phi_2) = \boldsymbol{trans}(\phi_1) \wedge \boldsymbol{trans}(\phi_2)\\
&\boldsymbol{trans}(\forall x\, \phi(x)) = \forall x\ (\mathsf{ink}(x) \to \boldsymbol{trans}(\phi)) && \boldsymbol{trans}(\Box_s\phi) = \Box_s\boldsymbol{trans}(\phi)
\end{aligned}$$

$$\boldsymbol{trans}(\mathsf{Q}(\tau_1,...,\tau_k)) = \exists e_1,...,e_k(\mathsf{Q}(e_1,...,e_k) \wedge \mathsf{prec}(\tau_1,e_1))\ \wedge\ ...\ \wedge\ \mathsf{prec}(\tau_k,e_k))$$

There are two non-trivial steps of ***trans***, the translations of quantification and of precise entity predicates. The quantification step, $\boldsymbol{trans}(\forall x\, \phi(x))$ restricts the application of $\boldsymbol{trans}(\phi)$ to the domain elements that correspond to instantiated count nouns (at the given precisification), which is denoted by $\mathsf{ink}(x)$. The translation of precise entity predicates associates to each individual τ_i its precise extension e_i, to which the predicate Q applies.

Finally, for a $\mathbb{V}_1$ formula ψ, we specify:

$$\boldsymbol{Trans}(\psi) := \boldsymbol{trans}(\psi) \wedge \textstyle\bigwedge_{\phi \in \Phi} \Box_* \phi$$

A $\mathbb{V}_1$ formula ψ and its translation $\mathrm{Trans}(\psi)$ are equisatisatisfiable, that is, ψ is $\mathbb{V}_1$-satisfiable if and only if $\mathrm{Trans}(\psi)$ is $\mathbb{S}_{\mathrm{FO}}$-satisfiable. Correctness of the translation can be shown by establishing a correspondence between the two kinds of model in each direction. The proof, by induction, can be found in the arXived version of this paper[2].

As well as showing that the notion of an 'indefinite individual' can be reconstructed indirectly within the more restricted semantics of $\mathbb{S}_{\mathrm{FO}}$, this result has practical applications. The translation, which is linear in the size of the formula, preserves the monodic fragment of Standpoint Logic, where modalities occur only in front of formulas with at most one free variable. Monodic modal logics are known to be decidable for numerous decidable fragments of first-order logic [21], and tight complexity bounds have been shown for monodic fragments of standpoint description logic [22][3], for which practical reasoning algorithms have also been provided [23]. This highlights the usefulness of the translation since reasoning support for $\mathbb{V}_1$ can be provided through its implementation.

6. Some Philosophical Views Considered in relation to the $\mathbb{V}_1$ Semantics

From the translation, we may observe that we move from having a domain with only precise entities to also having a potentially large collection of *indefinite entities* that are abstract in nature and only relate to their extension via the predicate prec. This issue regarding whether multiple precise versions of vague things can exist simultaneously (especially fuzzy boundaried objects like clouds) is known to philosophers as 'The Problem of the Many' [24], and has been the subject of considerable debate [25].

[2] https://arxiv.org/abs/2302.13189v2

[3] These results assume rigid constants, thus their application would involve adopting this assumption in $\mathbb{V}_1$.

One view is that the issue can be resolved by modifying the identity relation. According to Geach [26], the different precise versions of a cat are identical relative to the high-level count noun ‘cat’, whereas Lewis [27] suggests that physical objects that are almost the same with respect to spatial extension and material composition are ordinarily regarded as identical. Our semantics has some affinity with Geach’s approach in that we consider that it is count nouns that determine individuation and hence the sets of possible precise versions of individuals. However, in our theory there is no need to modify identity, since quantification is over individuals not their possible precise extensions. We believe that Lewis’s identity condition cannot account for many examples of objects with vague boundaries, where there can be large difference between what we may take to be their spatial extension. For instance, small variations in our interpretation of ‘desert’ may in certain circumstances result in very different spatial extensions being ascribed.

Lewis, along with Varzi [9], also suggests a supervaluationist account where the different candidate extensions of a vague object correspond to their extension in different precisifications. So, at a given precisification there is always a single physical extension of each object. But what is distinctive about our semantics is that we model variability in the extension of an individual as an additional kind of vagueness, not subsumed by the indeterminacy of a name (or sortal) in referring to an individual (or class of individuals).

Lowe [28] posits an ontological difference between objects of a high-level category such as ‘cat’ and physical portions of feline tissue present in the world, which he calls ‘cat-constituters’. This is very much in line with our semantics, especially as presented in the flat (translated) form that contains both definite and indefinite basic elements. Critics of Lowe (e.g. Lewis) have argued that the existence of both cats and cat-constituters is ontologically profligate (or incoherent). We consider instances of the high-level categories to be semantic objects modelling the way language operates rather than carrying ontological commitment. This is clearest in the semantics of $\mathbb{V}_1$, where modelling indefinite objects as functions from precisifications to extensions is a way of packaging indeterminacy within a semantic object. In the flat version of our semantics, indefinite objects are present (along with precise extensions) as basic elements of the domain; however we still consider them semantic rather than *ontic* elements.

7. Conclusions and Further Work

We have analysed the phenomenon of vagueness in both predicates and objects, and considered examples suggesting that the aspects of individuation, classification and demarcation associated with predication need to be considered separately. We have proposed the framework of *Variable Reference Logic*, which takes the general form of a supervaluation semantics but allows variability both in the set of individuals falling under a predicate and also in the spatial extension and physical constitution of ‘objects’ denoted by names or quantified variables. This separation not only supports flexible expressive capabilities of our formal language, but enables the representation to capture definite aspects of information despite other aspects being affected by vagueness.

There remains much interesting work to be done investigating what can be expressed and inferred by means of $\mathbb{V}_1$. We envisage reasoning support for $\mathbb{V}_1$ being of practical interest for specific kinds of application, such as querying of geographical information systems, and plan to develop a case study in this area. The linear translation to flat Standpoint Logic and ongoing progress on implementations for that logic are a strong indicator that query answering functionality may be feasible.

References

[1] Bennett B. Reference, Predication and Quantification in the Presence of Vagueness and Polysemy. In: Proc. Joint Ontology Workshop (JOWO-22). vol. 3249 of CEUR; 2022. .

[2] Bennett B. Standpoint Semantics: a framework for formalising the variable meaning of vague terms. In: Cintula P, Fermüller C, Godo L, Hájek P, editors. Understanding Vagueness — Logical, Philosophical and Linguistic Perspectives. Studies in Logic. College Publications; 2011. p. 261-78.

[3] Gómez Álvarez L, Bennett B. Dealing with conceptual indeterminacy: A framework based on supervaluation semantics. In: Proc. MedRACER and WOMoCoE 2018. vol. 2237. CEUR; 2018. p. 38-50.

[4] Gómez Álvarez L, Rudolph S. Standpoint logic: Multi-perspective knowledge representation. In: Neuhaus F, Brodaric B, editors. Procs. of the 12th Int. Conf. on Formal Ontology in Information Systems. vol. 344. IOS Press; 2022. p. 3-17.

[5] Barnes E. Ontic Vagueness: A Guide for the Perplexed. Noûs. 2010;44(4):601-27.

[6] Williamson T. Vagueness and Ignorance. Procs of the Aristotelian Society. 1992;Supp. Vol. 66:145-62.

[7] Lawry J, Tang Y. Uncertainty modelling for vague concepts: A prototype theory approach. Artificial Intelligence. 2009;173(18):1539-58.

[8] Lewis DK. On the Plurality of Worlds. Oxford: Blackwell; 1986.

[9] Varzi AC. Vagueness in Geography. Philosophy and Geography. 2001;4:49-65.

[10] Tye M. Vague objects. Mind. 1990;99:535-57.

[11] Barnes E. Ontic Vagueness: A Guide for the Perplexed. Noûs. 2010;44(4):601-27.

[12] Zadeh LA. Fuzzy Sets. Information and Control. 1965;8:338-53.

[13] Schockaert S, De Cock M, Kerre EE. Spatial reasoning in a fuzzy region connection calculus. Artificial Intelligence. 2009;173(2):258-98.

[14] Mehlberg H. The Reach of Science. University of Toronto Press; 1958.

[15] van Fraassen BC. Presupposition, supervaluations and free logic. In: Lambert K, editor. The Logical Way of Doing Things. New Haven: Yale University Press; 1969. p. 67-91.

[16] Fine K. Vagueness, truth and logic. Synthèse. 1975;30:263-300.

[17] Kamp H. Two theories about adjectives. In: Keenan EL, editor. Formal Semantics of Natural Language. Cambridge, England: Cambridge University Press; 1975. .

[18] Gómez Álvarez L, Bennett B. Classification, Individuation and Demarcation of Forests: Formalising the Multi-Faceted Semantics of Geographic Terms. In: Clementini E, Donnelly M, Yuan M, Kray C, Fogliaroni P, Ballatore A, editors. Procs. of the 13th Int. Conf. on Spatial Information Theory. vol. 86 of LIPIcs. Schloss Dagstuhl; 2017. p. 8:1-8:15.

[19] Cohn AG, Gotts NM. Representing Spatial Vagueness: a mereological approach. In: L C Aiello JD, Shapiro S, editors. Procs. of the 5th Int. Conf. on Principles of Knowledge Representation and Reasoning. Morgan Kaufmann; 1996. p. 230-41.

[20] Gómez Álvarez L, Rudolph S, Strass H. How to Agree to Disagree: Managing Ontological Perspectives using Standpoint Logic. In: Sattler U, Hogan A, Keet M, Presutti V, Almeida J, Takeda H, et al., editors. Procs. of the 21st Int. Semantic Web Conf. vol. 13489 of LNCS. Springer; 2022. p. 125-41.

[21] Wolter F, Zakharyaschev M. Decidable fragments of first-order modal logics. The Journal of Symbolic Logic. 2001;66(3):1415-38.

[22] Gómez Álvarez L, Rudolph S, Strass H. Tractable Diversity: Scalable Multiperspective Ontology Management via Standpoint EL. In: Proc. of the 32 Int. Joint Conf. on Artificial Intelligence; 2023. p. [TBP].

[23] Gómez Álvarez L, Rudolph S, Strass H. Pushing the Boundaries of Tractable Multiperspective Reasoning: A Deduction Calculus for Standpoint EL+; 2023.

[24] Unger P. The Problem of the Many. Midwest Studies in Philosophy. 1980;5:411-67.

[25] Weatherson B. The Problem of the Many. In: The Stanford Encyclopedia of Philosophy; 2016. .

[26] Geach PT. Reference and Generality. 3rd ed. Ithaca: Cornell Universiy Press; 1990.

[27] Lewis DK. Many, but Almost One. In: Bacon J, Campbell K, Reinhardt L, editors. Ontology, Causality, and Mind: essays in honour of D.M. Armstrong. Cambridge University Press; 1993. p. 23-38.

[28] Lowe EJ. The Paradox of the 1,001 Cats. Analysis. 1982;42(1):27-30.