Fuzzy Systems and Data Mining IX
A.J. Tallón-Ballesteros and R. Beltrán-Barba (Eds.)

doi:10.3233/FAIA231080

# Data-Driven E-Commerce End-to-End Inventory Optimization Algorithm

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**Abstract.** In the field of inventory management, due to the rapid development of artificial intelligence technology, especially data mining and machine learning, new research paradigm has been added to inventory decision-making. Compared with demand forecasting based on sales volume in previous studies, existing research aims to more fully utilize various ancillary information related to products to assist decision-making. In addition, compared with the traditional two-step decision-making (predict first and then optimize), the end-to-end (E2E) proposed in recent years can effectively avoid errors caused by the intermediate process. Based on the idea of E2E, this paper builds an end-to-end integration model E2E-Weighted on how to make optimal ordering decisions for e-commerce companies under the conditions of inventory backlog and service level target constraints. This paper also iteratively developed the model solution method KNN-Weighted based on the KNN algorithm. Results proves that as the number of samples increases, the KNN-Weighted algorithm converges to the theoretical optimal value and is better than other traditional algorithms. Furthermore, the E2E-Weighted model is more suitable for situations with high inventory target service levels.

**Keywords.** Data-driven, inventory optimization, KNN-weighted algorithm, end-to-end, machine learning

## 1. Introduction

In the context of big data, with the full penetration of information technology, machine learning and artificial intelligence technologies are being applied in various scenarios. Inventory management is a very important part of modern enterprise operations. How enterprises currently use data to cope with increasing competition and control various uncertainties, such as uncertainty in demand, has become a top priority for the sustainable development of enterprises. Most of the previous analyses assumed that the demand distribution was known [1]. However, with the dramatic changes in the business environment, the uncertainty of demand has greatly increased, therefore inventory control based on random demand emerged. In order to avoid losses caused by stockout, safety stock is added to reduce potential forecast errors [2]. In addition, time series-based demand forecast models are used in inventory management. However, demands are easily affected by various exogenous variables (such as weather conditions, promotional activities) and endogenous variables (such as seasonality). Thus indicates that forecast models based only on time series may cause in large errors, leading to out-of-stocks or inventory accumulation, thereby incurring large costs [3].

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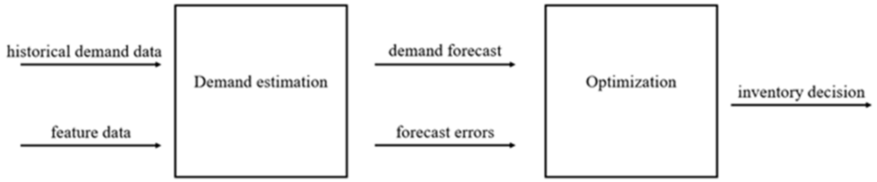
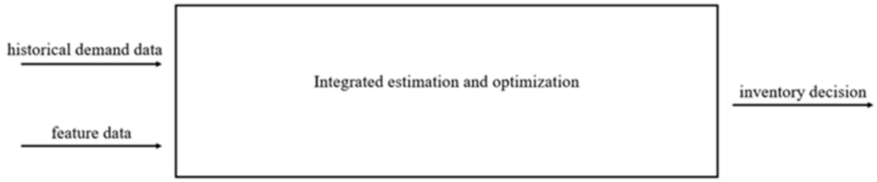
Previously research mainly provides a two-step procedure for inventory optimization: 1) Assume the demand distribution and obtain the corresponding parameters; 2) solve the optimal order quantity based on the set distribution and parameters, as shown in Figure 1. However, the application results of the distribution hypothesis in practice may not be satisfied. In recent years, data-driven inventory management has become a key research direction. Liyanage and Shanthikumar [4] uses the concept of "Operational Statistics" to make integrated decisions on the estimation of demand parameters and inventory optimization in the newsvendor problem. Bertsimas and Kallus [5,6] applied five common non-parametric machine learning methods to holistic framework of data-driven integrated decision-making, i.e., K-nearest neighbor regression (KNN), kernel optimization (KO), local regression (LOESS), classification regression tree (CART), and random forest (RF). Qin et al. [7] aimed at the data-driven multi-period inventory replenishment problem. They considered the user demand and vendor lead time (VLT) on the premise of factors, thus designing a single-stage end-to-end (E2E) decision-making framework based on the deep learning model [8]. By integrating the demand forecasting and optimization models into one model, optimal inventory decisions are obtained based on historical demand and characteristic data, thus avoiding systematic errors caused by the link between forecasting and optimization in the two-step process [9]. In order to further optimize the classical machine learning algorithms that are difficult to apply to solve the problem of lack of correctly formatted input-output historical data, reinforcement learning methods have been applied to research related to inventory management. Boute et al. [10] detailed the roadmap and future direction of reinforcement learning in inventory management in their study.




(a) Separate estimation and optimization




(b) Integrated estimation and optimization

**Figure 1**.The two modes of data-driven inventory decision

Referring to the study of Bertsimas and Kallus [6], this paper is to propose a new integrated decision-making model. This model integrates data-driven demand forecasting and inventory optimization, and utilizes machine learning methods to design a model solving algorithm. Inventory decisions can be output based solely on characteristic data and historical demand data, thus providing a reference for e-commerce companies' operational decisions. This paper considers the inventory service level constraints faced by e-commerce enterprise warehouses instead of the classic newsvendor model. Meanwhile, this paper also expands the application of the framework method proposed by Bertsimas and Kallus [6]. The KNN algorithm used is improved by adding service constraints and constructing conditional expectation distributions by assigning weights to historical samples based on their feature data.

## 2. Methodology

### *2.1. Problem description*

The distribution of consumer demand is constantly changing, decision makers are not aware of the distribution of real consumer demand, and demand is easily affected by external factors, such as promotional activities. Decision makers need to determine the optimal order quantity in the next few days based on the characteristics of the product and historical related data. The relevant assumptions are as follows:

- The data is assumed to be stored and updated after the end of each sales cycle, and the relevant data of each product in the current warehouse will be recorded comprehensively.
- For a certain product $j$, the sales data and the related feature vector $x_i^j$ of the $N$ day are available, assuming that the sales cycle is 1 day.
- For a product $j \in J = \{1,2,3,\cdots,j\}$, the characteristic data set of $T$ days in the future is $X_N^j = \{X_{N+1}^j, X_{N+2}^j, \cdots, X_{N+T}^j\}$, but the future demand is unknown.
- After the end of each sales period, the remaining inventory cannot be entered into the next period of sales, resulting in remaining inventory costs.
- The ordering strategy is set to $q_z^j$, assuming that within the next $T$ days, the probability of product shortage will not exceed $\alpha^j$, then the inventory service level is $1 - \alpha^j$.

### *2.2. List of Symbols*

**Table 1.** List of symbols

| Symbol | Description |
|---|---|
| $J$ | Index of product, $j \in J = \{1,2,3,\cdots,U\}$ |
| $I$ | Index of historical days, $i \in I = \{1,2,3,\cdots,N\}$ |
| $Z$ | Index of days to be decided in the future, $z \in Z = \{N+1, N+2, \cdots, N+T\}$ |
| $q_z^j$ | The order quantity of product $j$ on the $z$ day in the future |
| $\gamma_i^j$ | Binary variable, equal to 1 if product $j$ is out of stock on the $i$ day in history, otherwise 0 |
| $y_i^j$ | The remaining inventory of product $j$ on the $i$ day in history,$y_i^j \geq 0$ |
| $D_i^j$ | The demand for product $j$ on the $i$ day in history |
| $x_i^j$ | $p$-dimensional feature vector of product $j$ on the $i$ day in history |
| $x_z^j$ | $p$-dimensional feature vector of product $j$ on the z day in the future |
| $M$ | Penalty coefficient |
| $\alpha^j$ | Out of stock rate, $\alpha^j \in (0,1)$ |
| $1-\alpha^j$ | Target inventory service level |

### *2.3. Proposed model*

In the theoretical model, when the service level (SL) is $1 - \alpha^j$, the random demand is $D$, and the order quantity is set to $q$, if the expected inventory is minimized, then:

$$\min_{q\geq 0}\{E(q-D)^{+}: P[q \geq D] \geq 1-\alpha\} \tag{1}$$

The above formula does not consider feature vectors. When the data set $S_N^j = \{(d_1^j, x_1^j), (d_2^j, x_2^j), \cdots, (d_N^j, x_N^j)\}$is available, then the minimum expected remaining inventory is:

$$\min_{q\geq 0}\{E(q-D)^{+}|x: P[q \geq D|x] \geq 1-\alpha\} \tag{2}$$

In reality, when the demand distribution is unknown, characteristic data can be used to link the unsolved product order quantity with historical demand of "similar" situations in the form of a weight function. Historical demand with "similar" scenarios and their weights are then used to approximate the conditional expected remaining inventory. As for the inventory service level constraint, it is required to meet the out-of-stock rate target at a selected confidence level while minimizing the conditional expected remaining inventory level. Then the model can be described as:

$$\min \textstyle\sum_{i=1}^{N}\left[w(x_z^j, x_i^j) y_i^j\right] \tag{3}$$

$$s.t. \qquad y_i^j \geq q_z^j - D_i^j \tag{4}$$

$$y_i^j \geq 0 \tag{5}$$

$$q_z^j + \gamma_i^j M \geq D_i^j \tag{6}$$

$$\textstyle\sum_{i=1}^{N}\left[w(x_z^j, x_i^j) \gamma_i^j\right] \leq \alpha \tag{7}$$

$$\textstyle\sum_{i=1}^{N} w(x_z^j, x_i^j) = 1 \tag{8}$$

$$y_i^j \in \{0,1\} \tag{9}$$

$$q_z^j \geq 0 \tag{10}$$

Among them, $w(x_z^j, x_i^j)$ represents the similarity between future product characteristics and historical characteristics represented by weights. Formulas (4) and (5) together constitute the decision variable remaining inventory $y_i^j = (q_z^j - D_i^j)^{+}$, $\gamma_i^j$ is a Binary variable. When it is 0, formula (6) becomes $q_z^j \geq D_i^j$, indicating that the demand is met, otherwise, a shortage occurs. Formula (8) indicates that the service level constraint is met within the historical $N$ days.

This model makes full use of the information of feature data and apply machine learning methods to establish the relationship between features and sales (including linear and nonlinear relationships), which can be quickly solved by optimization solvers such as Gurobi.

To give a relatively specific example, an e-commerce company wants to determine the A brand of black jumpers to determine the optimal order quantity for the next day. Known jumper price, discount, brand, sales channels, categories and other attribute features, the need to find in the existing database and these features are very similar to the historical features corresponding to the historical sales, and give weight based on the set level of service, to find the jumper's optimal inventory decision. Then it can be seen that the key to solving the model is how to define the similarity between the samples to be decided and the historical samples, i.e. how to determine the weight function in the constructed model.

### *2.4. Algorithm development*

K-Nearest Neighbor (KNN) learning is a commonly used supervised learning method in machine learning. This paper proposes the KNN-Weighted method based on the original KNN method to solve the above integrated inventory problem. The differences and advantages between it and KNN can be stated into two ways. Firstly, KNN uses the square loss formula as the loss function, while KNN-Weighted uses the objective function in the above model as the loss function. Secondly, KNN-Weighted directly outputs the order quantity instead of the demand quantity. The detailed steps are as follows:

**Step1:** Enter the $K$value range and select a value;

**Step2:** Select the characteristic data $x$ of a sample in the verification set;

**Step3:** Calculate the Euclidean distance between $x_i$and $K$, sort the values of $K$ by distance. Then select the $K$ with the shortest distance for sales training, thus assigning the sales volume of the training set feature with a weight of $1/K$. According to these $K$ sales, calculate the optimal order quantity that meets the service level by weight and record it;

**Step4:** Determine whether the verification set has been calculated, if so, proceed to step6, otherwise jump to step2;

**Step5:** Determine whether the $K$ value has been traversed, if so, proceed to step6, otherwise jump to step1;

**Step6:** Summarize and select the $K$ value corresponding to the minimum inventory remaining quantity as the optimal $K$ value;

**Step7:** Select the feature data to be tested in the test set and repeat the same operation as step3;

**Step8:** Determine whether the test set has been calculated, if so, proceed to step9, otherwise jump to step7;

**Step9:** Output the optimal $K$ value, optimal ordering decision, actual service level and remaining inventory level.

## 3. Experiment and Results

### *3.1. Data sets*

This paper applies the sales data of an e-commerce company in June 2019, including 260 SKUs and a total of 4,234 pieces of data. Table 2 shows each product contains 9 attributes, among which Facility, Channel Name, SKU Name, and Discount are used as characteristic attributes. Apply one-hot encoding feature attributes corresponding to the word text for processing before solving.

**Table 2.** List of characteristic attributes

| Attribute | Description |
|---|---|
| SKU_ID | Product unique identifier |
| Facility | Sales address |
| Channel Name | Sales channels |
| SKU Name | SKU category name |
| Quality | Sales volume |
| Total Price | Total sales price |
| Discount | Discount |
| Date | Sales date |
| Vendibility | Binary variable, equal to 1 when the day's sales generate inventory, otherwise 0 |

The data is divided into training set, validation set and test set, the details are shown in table 3:

**Table 3.** Data partition

| Data set | Data amount | Number of identification codes |
|---|---|---|
| Training set | 3387(80%) | 245 |
| Validation set | 212(5%) | 253 |
| Test set | 635(15%) | 260 |

The verification set is used to tune the weight parameters in the E2E-Weighted model, and the test set is used to calculate the final results after parameter tuning and to evaluate the performance of the integrated decision-making model.

### *3.2. Comparison of results*

Since the service level in practice is generally higher than 50% [11], this experiment assumes that the target service levels are located at 50%, 70%, 90%, and 95% respectively. The solution methods used include: ARIMA, LR (linear regression), KNN, DTR (decision tree regression), and the integrated model KNN-Weighted algorithm proposed in this article.

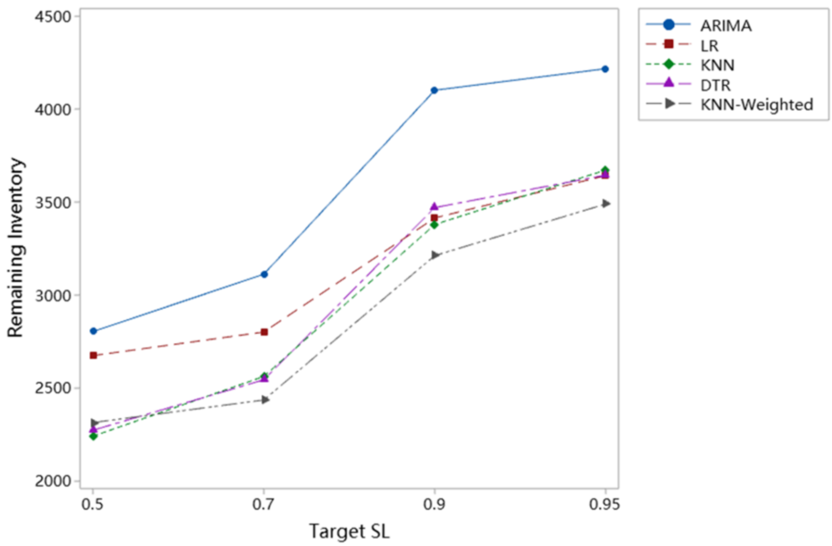



**Figure 2** Results of different methods (Remaining inventory)

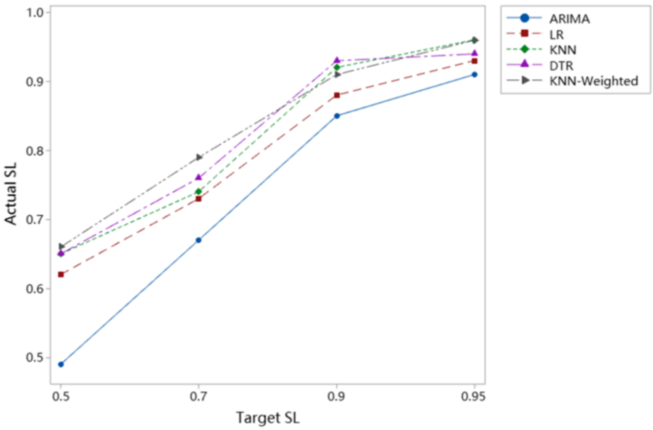



**Figure 3** Results of different methods (Remaining inventory)

The results shown in Figure 3 and Figure 4 reveal that KNN-Weighted performs well when the target service level is higher than 70%, and is suitable for situations with high inventory service level requirements. Whether it is the actual inventory service level or the remaining inventory level, the performance of the integrated decision-making method is better than that of other methods.

### *3.3. Asymptotic optimality*

The sample sizes are set to 10, 100, 1000, 5000, and 10000 respectively, and the target inventory service level is 0.5 [9]. This paper compares the running performance of various methods for different sample sizes.

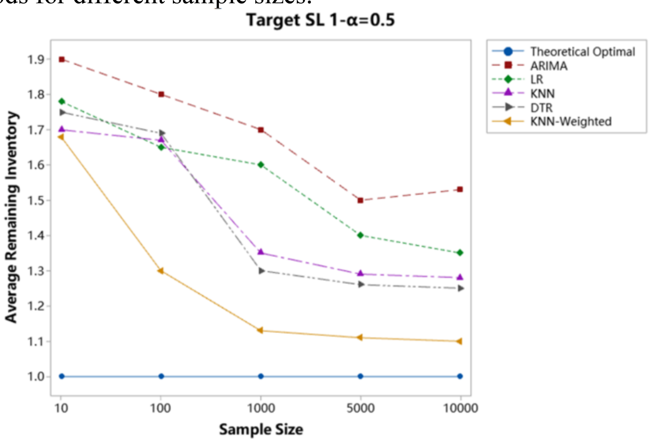



**Figure 4** Asymptotic optimality of various methods

Overall, when the sample size is between 100 and 1,000, the performance of each method can be significantly improved. When the sample size exceeds 10,000, as the sample continues to increase, it is hard to improve the performance of the model. It can also be concluded from Figure 4 that the KNN-Weighted method proposed in this article has better performance than the other four methods, followed by the decision tree regression (DTR) method.

## 4. Conclusion

This article is based on the idea of end-to-end (E2E) and considers how e-commerce companies make optimal ordering decisions under inventory backlog and service level target constraints. Through data mining and machine learning technology, the research uses the minimum amount of remaining inventory as the optimization goal to build an end-to-end integrated model E2E-Weighted with service level constraints. This article iterates the KNN-Weighted solution algorithm suitable for this model based on the KNN algorithm. In using the obtained historical sales data and characteristic data of products on sale in e-commerce, inventory decisions rather than demand forecasts are obtained in one step. In addition, this study proves that as the number of samples increases, the KNN-Weighted algorithm converges to the theoretical optimal value, and compared with other traditional methods, it reflects the advantages of feature data in searching optimal inventory decisions. Actual data experimental results demonstrate that the E2E-Weighted model is more suitable for high target service levels. When the target service level is higher than 70%, the corresponding decision algorithm can reduce more inventory remaining amounts. This article has the following follow-up research directions. Firstly, more complex inventory scenarios can be considered. Secondly, this article mainly concluded inventory level constraints, there are other constraints that can be added in subsequent research. Thirdly, more suitable solution algorithms or comparisons with other more complex algorithms can be considered to provide better decision-making solutions for the enterprise.

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