Fuzzy Systems and Data Mining IX
A.J. Tallón-Ballesteros and R. Beltrán-Barba (Eds.)

doi:10.3233/FAIA231079

Heterogeneous Multi-Attribute Group Decision-Making Integrating Multi-Granulation Weighting Model and Improved VIKOR in Uncertain Linguistic Environment

Jifang PANG [a,1], Peng SONG [b], Baoli WANG [c], and Chao ZHANG [a]

[a] *Key Laboratory of Computational Intelligence and Chinese Information Processing of Ministry of Education, School of Computer and Information Technology, Shanxi University, Taiyuan 030006, Shanxi, China.*

[b] *School of Economics and Management, Shanxi University, Taiyuan 030006, Shanxi, China*

[c] *School of Mathematics and Information Technology, Yuncheng University, Yuncheng 044000, Shanxi, China*

Abstract. Heterogeneous multi-attribute group decision-making (HMAGDM) is a complex decision-making problem that widely exists in the real world. However, there is relatively little research on the HMAGDM problems when the attribute set and alternative set are both heterogeneous, and the existing studies still have some limitations, such as the weight calculation is too simple, lacking objectivity and comprehensiveness; the ranking methods does not consider the utility of both group and individuals simultaneously, lacking flxibility and practicality. In order to obtain more effective decision results, a HMAGDM method integrating multi-granulation weighting model and improved VIKOR in uncertain linguistic environment is proposed in this paper. Our contributions can be identified as follows: (1) On the basis of the uncertainty and closeness of uncertain linguistic terms (ULTs), a measure indicator for the effectiveness of experts' opinions is proposed, and a finest-granulation weight optimization model for experts is established by maximizing the effectiveness; (2) Based on comprehensive consideration of effectiveness and deviation, a bi-objective optimization model is proposed to obtain the multi-granulation weights of attributes; (3) An improved VIKOR method combining the boundedness of ULTs and the multi-granulation weights of attributes is proposed to obtain more stable and effective ranking results. Finally, the case study and comparative analysis illustrate the feasibility and characteristics of the proposed method.

Keywords. heterogeneous multi-attribute group decision-making, uncertain linguistic term, probabilistic uncertain linguistic term set, multi-granulation weight, improved VIKOR method

[1]Corresponding Author: Jifang Pang, School of Computer and Information Technology, Shanxi University, Taiyuan 030006, Shanxi, China; E-mail: purplepjf@sxu.edu.cn.

1. Introduction

Multi-attribute group decision-making (MAGDM) can be used to deal with many practical decision-making problems such as development of large project [1], urban waste recycling partner selection[2], photovoltaic power station evaluation [3] and so on. In the real decision-making environment, experts prefer to select some attributes they are familiar with and the alternatives belonging to their professional fields for evaluation. In addition, experts may use different types of evaluation information to express their opinions. This type of MAGDM problem with different types and/or structures of evaluation information is collectively referred to as heterogeneous MAGDM (HMAGDM). The existing research on HMAGDM can be classified into two frameworks: heterogeneous type of evaluation information [4,5,6] and heterogeneous sets of attributes and/or alternatives. The second framework can be subdivided into two cases: one is that only the attribute set is heterogeneous[7,8,9,10,11,12]; The other is that the attribute set and alternative set are both heterogeneous[13,14]. At present, research on HMAGDM with heterogeneous sets of attributes and alternatives is relatively insufficient. This is mainly because the decision space structure of such problems has characteristics of diversity and personalization, further increasing the difficulty of information aggregation and decision analysis. How to effectively aggregate such complex heterogeneous information and obtain reasonable and reliable decision results is one of the important challenges faced by current research.

In order to better solve the complex HMAGDM problems in uncertain linguistic environment, a HMAGDM method integrating multi-granulation weighting model and improved VIKOR will be proposed in this paper. The research motivation of this paper is summarized as follows:

(1) ULTs can well reflect the uncertainty in the decision process [15] and the HMAGDM problems in uncertain linguistic environment are widespread in the real world. However, there are relatively few related studies.

(2) In existing HMAGDM research, weights are mostly subjectively given by experts and lack objectivity [7,8,9,10]. Moreover, the objective weighting methods currently used are too simple[11,12,13,14], and the obtained weights lack comprehensiveness, which is not conducive to obtaining reasonable decision results.

(3) Most HMAGDM methods use weighted aggregation [7,10,11,12,13] or TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) methods [8,9] to rank the alternatives. However, both of them do not consider the utility of group and individuals simultaneously and cannot well reflect the decision-maker's subjective preferences. Compared with them, VIKOR (VIsekriterijumska Optimizacija I Kompromisno Resenje) method is more flexible and practical, which can obtain a compromise solution finally accepted by experts [16,17]. However, the classical VIKOR method is prone to encounter reverse ranking situations when adding, deleting, or replacing an alternative [18], so it is necessary to develop an improved VIKOR method.

Our contributions can be mainly summarized in the following three aspects:

(1) On the basis of uncertainty and closeness of ULTs, an effectiveness measure is proposed, which provides theoretical support for further analyzing the effectiveness of decision results.

(2) Two multi-granulation weighting models with the goal of maximizing effectiveness measures are established for experts and attributes respectively, which can obtain more flexible and comprehensive weight and are more suitable for the aggregation of heterogeneous decision matrices.

(3) An improved VIKOR method combined with the boundedness of ULTs and the multi-granulation weights of attributes is proposed to effectively improve the stability and reliability of the ranking results.

The rest of the paper is organized as follows. Section 2 reviews some basic concepts. Section 3 provides an introduction to the HMAGDM method proposed in this paper. Section 4 demonstrates the feasibility of the method through a supplier selection case. Section 5 conducts experimental analysis from three aspects: sensitivity, stability, and effectiveness. Section 6 draws our conclusions and points out future research directions.

2. Preliminaries

Definition 1 *[19] Let $S=\{s_\alpha|\alpha=0,1,\cdots,l\}$ be a finite and totally ordered discrete linguistic term set, where l is an even value, s_α represents a linguistic term.*

Xu [20] extended S into the continuous set $\overline{S}=\{s_\alpha|\alpha\in[0,q]\}$, where $q(q\geq l)$ is a sufficiently large natural number. $I(s_\alpha)$ denote the term index of s_α in $\overline{S}$, i.e., $I(s_\alpha)=\alpha$.

Definition 2 *[20] Let $\tilde{s}=[s_L,s_R]$, where $s_L,s_R\in\overline{S}$, s_L and s_R are the lower and upper limits of $\tilde{s}$, respectively. We call $\tilde{s}$ the uncertain linguistic term (ULT).*

For two ULTs $\tilde{s}_1=[s_{L_1},s_{R_1}]$, $\tilde{s}_2=[s_{L_2},s_{R_2}]$, the operational laws are [20]: (1) $\tilde{s}_1\oplus\tilde{s}_2=[s_{L_1},s_{R_1}]\oplus[s_{L_2},s_{R_2}]=[s_{L_1+L_2},s_{R_1+R_2}]$; (2) $\rho\tilde{s}_1=\rho[s_{L_1},s_{R_1}]=[s_{\rho L_1},s_{\rho R_1}]$, $\rho\geq 0$.

Definition 3 *Let $\tilde{s}_1=[s_{L_1},s_{R_1}]$ and $\tilde{s}_2=[s_{L_2},s_{R_2}]$ be two ULTs, the Euclidean distance between $\tilde{s}_1$ and $\tilde{s}_2$ is given by $d(\tilde{s}_1,\tilde{s}_2)=\frac{1}{l}\sqrt{\frac{1}{2}[(I(s_{L_1})-I(s_{L_2}))^2+(I(s_{R_1})-I(s_{R_2}))^2]}$.*

Xu [21] also proposed the uncertain linguistic weighted averaging (ULWA) operator.

Definition 4 *[21] The ULWA operator is defined as $ULWA_\lambda(\tilde{s}_1,\tilde{s}_2,\cdots,\tilde{s}_n)=\lambda_1\tilde{s}_1\oplus\lambda_2\tilde{s}_2\oplus\cdots\oplus\lambda_n\tilde{s}_n$, where $\lambda=(\lambda_1,\lambda_2,\cdots,\lambda_n)$ is the weighting vector of ULTs $\tilde{s}_j(j=1,2,\cdots,n)$, and $\lambda_j\in[0,1]$, $\Sigma_{j=1}^n\lambda_j=1$.*

Definition 5 *[22] A PULTS is defined as $S(p)=\{\langle\tilde{s}^k,p^k\rangle|p^k\geq 0,k=1,2,\cdots,|S(p)|,\sum_{k=1}^{|S(p)|}p^k\leq 1\}$, where $\tilde{s}^k=[s_L^k,s_R^k]$ is an ULT, s_L^k and $s_R^k(s_L^k,s_R^k\in S,s_L^k\leq s_R^k)$ are the lower and upper limits of $\tilde{s}^k$, respectively. $|S(p)|$ is the cardinality of $S(p)$, and p^k is the probability of $\tilde{s}^k$.*

PULTS is an effective tool to depict uncertain linguistic opinions [23], which provides a new means of modeling ULTs by computing their occurrence probabilities.

3. HMAGDM method integrating multi-granulation weighting model and improved VIKOR in uncertain linguistic environment

3.1. Problem description

For a HMAGDM problem in uncertain linguistic environment, let $X=\{x_1,x_2,\cdots,x_m\}$ be an alternative set, $A=\{a_1,a_2,\cdots,a_n\}$ be an attribute set, $E=\{e_1,e_2,\cdots,e_f\}$ be an expert set, $S=\{s_\alpha|\alpha=0,1,\cdots,l\}$ be a linguistic term set. $X^h=\{x_i^h|x_i^h\in X\}$ represents a subset of the alternatives selected by e_h, $A^h=\{a_j^h|a_j^h\in A\}$ represents a subset of the attributes selected by e_h. The decision matrix of e_h can be expressed as $R^h=[\tilde{s}_{ij}^h]_{|X^h|\cdot|A^h|}$, where $|X^h|$ is the number of alternatives in X^h, $|A^h|$ is the number of attributes in A^h, $\tilde{s}_{ij}^h=[s_{ijL}^h,s_{ijR}^h]$ represents the evaluation value of x_i under a_j given by e_h.

Let $E_{ij}=\{e_h|\tilde{s}_{ij}^h\neq\emptyset,e_h\in E\}$ represent the set of experts who evaluate x_i under a_j. The number of experts in E_{ij} is denoted as $|E_{ij}|$. The set of evaluation value given by the experts in E_{ij} is denoted as $V_{ij}=\{\tilde{s}_{ij}^h|e_h\in E_{ij}\}=\{[s_{ijL}^h,s_{ijR}^h]|e_h\in E_{ij}\}$

In order to ensure the objectivity and comprehensiveness of decision results, a HMAGDM method needs to meet the following conditions:(1) $X^1\cup X^2\cup\cdots\cup X^f=X$; (2) $A^1\cup A^2\cup\cdots\cup A^f=A$; (3) $|E_{ij}|\geq 3(i=1,2,\cdots,m,j=1,2,\cdots,n)$.

3.2. Multi-granulation weighting model for experts

In this subsection, the ULT evaluation values provided by different experts are first aggregated into PULTS. Then, the finest-granulation weights of experts are calculated from the perspectives of uncertainty and closeness. Finally, a bi-objective optimization model is established to fuse the two kinds of weights together.

First, by counting the number of occurrences of each $\tilde{s}_{ij}^h$ in $V_{ij}=\{\tilde{s}_{ij}^h|e_h\in E_{ij}\}$, the set V_{ij} can be transformed into a PULTS $S_{ij}(p)$, that is,

$$S_{ij}(p)=\{\langle\tilde{s}_{ij}^k,p_{ij}^k\rangle|p_{ij}^k=\frac{\#\tilde{s}_{ij}^k}{|V_{ij}|},\tilde{s}_{ij}^k\in V_{ij},k=1,2,\cdots,|S_{ij}(p)|,\sum_{k=1}^{|S_{ij}(p)|}p^k=1\} \tag{1}$$

where $\tilde{s}_{ij}^k=[s_{ijL}^k,s_{ijR}^k]$, $\#\tilde{s}_{ij}^k$ represents the number of occurrences of each $\tilde{s}_{ij}^k$ in V_{ij}, $|V_{ij}|$ represents the cardinality of V_{ij}, $|S_{ij}(p)|$ is the cardinality of $S_{ij}(p)$.

Then, calculate the multi-granulation weights of experts.

(1) The expert weight based on uncertainty degree

Step 1. Calculate he uncertainty degree $UND(\tilde{s}_{ij}^h)$ of expert e_h as follows

$$UND(\tilde{s}_{ij}^h)=\frac{I(s_{ijR}^h)-I(s_{ijL}^h)}{l} \tag{2}$$

Step 2. Calculate the uncertainty degree $UND(S_{ij}(p))$ of subgroup E_{ij} as follows

$$UND(S_{ij}(p))=\sum_{k=1}^{|S_{ij}(p)|}p_{ij}^k UND(\tilde{s}_{ij}^k) \tag{3}$$

Step 3. Calculate the weight ω_{ijh}^u of expert e_h in E_{ij} based on uncertainty degree

$$\omega_{ijh}^u=\frac{1-UND(\tilde{s}_{ij}^h)}{|E_{ij}|\times(1-UND(S_{ij}(p)))} \tag{4}$$

(2) The expert weight based on closeness degree

Step 1. Calculate the closeness degree $CLD(\tilde{s}_{ij}^h,S_{ij}(p))$ between expert e_h and the subgroup E_{ij} as follows

$$CLD(\tilde{s}_{ij}^{h}, S_{ij}(p)) = \sum_{k=1}^{|S_{ij}(p)|} p_{ij}^{k}(1 - d(\tilde{s}_{ij}^{h}, \tilde{s}_{ij}^{k})) \tag{5}$$

Step 2. Calculate the weight ω_{ijh}^{c} of expert e_h in E_{ij} based on closeness degree

$$\omega_{ijh}^{c} = \frac{CLD(\tilde{s}_{ij}^{h}, S_{ij}(p))}{\sum_{e_h \in E_{ij}} CLD(\tilde{s}_{ij}^{h}, S_{ij}(p))} \tag{6}$$

(3) Expert weight optimization model

Let ω_{ijh} be the multi-granulation weight of e_h under a_j w.r.t x_i. Then the weighted overall uncertainty of E_{ij} under a_j w.r.t x_i is $UND_{ij} = \sum_{e_h \in E_{ij}} \omega_{ijh} UND(\tilde{s}_{ij}^{h})$. Using ULWA operator and ω_{ijh}, the weighted overall evaluation value of E_{ij} can be obtained and denoted by $\tilde{s}_{ij}$. Then the consensus degree of E_{ij} can be expressed as $COD_{ij} = \sum_{e_h \in E_{ij}} \omega_{ijh}(1 - d(\tilde{s}_{ij}^{h}, \tilde{s}_{ij}))$. The effectiveness degree of subgroup E_{ij}'s opinions can be expressed as $EFD_{ij} = COD_{ij} - UND_{ij}$. By solving the following model (M-1) with the goal of maximizing EFD_{ij}, the weight ω_{ijh} of expert can be obtained.

$$\max EFD_{ij} = COD_{ij} - UND_{ij} = \sum_{e_h \in E_{ij}} \omega_{ijh}(1 - d(\tilde{s}_{ij}^{h}, \tilde{s}_{ij})) - \sum_{e_h \in E_{ij}} \omega_{ijh} UND(\tilde{s}_{ij}^{h})$$

$$\text{s.t.} \begin{cases} \omega_{ijh} \geq \min\{\omega_{ijh}^{u}, \omega_{ijh}^{c}\}, e_h \in E_{ij} \\ \sum_{e_h \in E_{ij}} \omega_{ijh} = 1 \end{cases} \tag{M-1}$$

3.3. Multi-granulation weighting model for attributes

Through the above calculation, the effectiveness under each attribute can be expressed as $EFD_j = \frac{1}{m} \sum_{i=1}^{m} EFD_{ij}$. Let v_j be the final weight of attribute, the total effectiveness of group opinions can be expressed as $EFD = \sum_{j=1}^{n} v_j EFD_j$. Next, we will give an objective comprehensive weighting method for attributes.

(1) The attribute weight based on effectiveness degree

The weight v_{ij}^{r} of a_j based on effectiveness under x_i can be calculated as follows:

$$v_{ij}^{r} = \frac{EFD_{ij}}{\sum_{j=1}^{n} EFD_{ij}} \tag{7}$$

(2) The attribute weight based on maximizing deviation

By using the weights of experts and ULWA operator, the individual decision matrices of experts can be aggregated into the group decision matrix $\tilde{R} = [\tilde{s}_{ij}]_{m \times n}$, where $\tilde{s}_{ij} = [s_{ijL}, s_{ijR}]$ represents the overall evaluation value of the subgroup under a_j w.r.t x_i.

The weight v_j^{d} of a_j based on maximizing deviation can be calculated as follows[24]:

$$v_j^{d} = \frac{\sum_{i=1}^{m} \sum_{t=1}^{m} d(\tilde{s}_{ij}, \tilde{s}_{tj})}{\sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{t=1}^{m} d(\tilde{s}_{ij}, \tilde{s}_{tj})} \tag{8}$$

(3)Attribute weight optimization model

Let v_j denote the synthetic weight of attribute and $DEV_j = \sum_{i=1}^{m}\sum_{t=1}^{m} d(\tilde{s}_{ij}, \tilde{s}_{tj})$ represent the deviation under a_j. Then, the total deviation under all attributes can be expressed as $DEV = \sum_{j=1}^{n} v_j DEV_j$. Combined with the total effectiveness $EFD = \sum_{j=1}^{n} v_j EFD_j$, a bi-objective optimization model (M-2) with the goal of maximizing both EFD and DEV can be establised to fuse the above two kinds of attribute weights.

$$\max EFD * DEV = (\sum_{j=1}^{n} v_j EFD_j) * (\sum_{j=1}^{n} v_j DEV_j)$$

$$\text{s.t.} \begin{cases} v_{ij} = \dfrac{v_{ij}^r v_j^d}{\sum_{j=1}^{n} v_{ij}^r v_j^d}, i = 1,2,\cdots,m, j = 1,2,\cdots,n \\ \min\{\min\limits_i v_{ij}^r, v_j^d, \min\limits_i v_{ij}\} \le v_j \le \max\{\max\limits_i v_{ij}^r, v_j^d, \max\limits_i v_{ij}\}, j = 1,2,\cdots,n \\ \sum\limits_{j=1}^{n} v_j = 1 \end{cases} \tag{M-2}$$

3.4. The improved VIKOR method

In order to effectively improve the stability of the ranking results, an improved VIKOR method is proposed in this subsection, which replaces the maximum and minimum values in the current alternative set with the upper and lower limits of the linguistic term set as PIS and NIS, and uses the multi-granulation weights of attributes to calculate the group utility and individual regret respectively. The specific ranking method is as follows:

Step 1. Let $\tilde{s}^+ = [s_l, s_l]$ be the PIS and $\tilde{s}^- = [s_0, s_0]$ be the NIS under all attributes.

Step 2. Calculate the group utility value $S_i = \sum_{j=1}^{n} v_j d(\tilde{s}^+, \tilde{s}_{ij})$ and individual regret value $R_i = \max_j\{v_j d(\tilde{s}^+, \tilde{s}_{ij})\}$ of alternative x_i $(i = 1,2,\cdots,m)$.

Step 3. Calculate the overall evaluation value Q_i of alternative $x_i (i = 1,2,\cdots,m)$

$$Q_i = \mu \frac{S_i - S^-}{S^+ - S^-} + (1-\mu)\frac{R_i - R^-}{R^+ - R^-} \tag{9}$$

where $S^+ = \max\limits_i S_i$, $S^- = \min\limits_i S_i$, $R^+ = \max\limits_i R_i$, $R^- = \min\limits_i R_i$. $\mu \in [0,1]$ is the compromise coefficient.

Step 4. Arrange the alternatives in ascending order according to the values of S_i, R_i and Q_i, respectively. Suppose the ranking result obtained from Q_i is $x^{(1)}, x^{(2)}, \cdots, x^{(m)}$, the determination process of the optimal alternative is as follows:

If $x^{(1)}$ satisfies condition 1: $Q(x^{(2)}) - Q(x^{(1)}) \ge \dfrac{1}{m-1}$, and condition 2: $x^{(1)}$ is still the optimal alternative according to the ascending order of S_i or R_i. Then $x^{(1)}$ is a stable optimal alternative in the decision process.

If the above two conditions cannot be met at the same time, the compromise solution can be generated according to the following two situations: if condition 2 is not met, $x^{(1)}$ and $x^{(2)}$ are both compromise solutions; If condition 1 is not met, the compromise solution set is $X = \{x^{(1)}, x^{(2)}, \cdots, x^{(J)}\}$, where J is the maximum positive integer calculated by $Q(x^{(J)}) - Q(x^{(1)}) < \dfrac{1}{m-1}$.

4. An illustrative example

The following takes an e-commerce enterprise as an example to illustrate the feasibility of the method. $X = \{x_1, x_2, \cdots, x_6\}$ are six candidate suppliers, and the corresponding evaluation indices are $A = \{a_1, a_2, \cdots, a_5\}$, where a_1–service quality, a_2–logistics cost, a_3–enterprise capability, a_4–informatization degree, a_5–enterprise development prospect. Five experts are $E = \{e_1, e_2, \cdots, e_5\}$. The linguistic term set adopted by the experts is $S = \{s_0 =$ extremely poor, $s_1 =$ very poor, $s_2 =$ poor, $s_3 =$ slightly poor, $s_4 =$ fair, $s_5 =$ slightly good, $s_6 =$ good, $s_7 =$ very good, $s_8 =$ extrimly good$\}$. The decision matrices $R^h (h = 1, 2, \cdots, 5)$ provided by the experts are shown in Tables 1-3.

Table 1. The decision matrix R^1 provided by e_1

X^1	A^1				
	a_1	a_2	a_3	a_4	a_5
x_1	$[s_7, s_7]$	$[s_6, s_7]$	$[s_6, s_7]$	$[s_4, s_4]$	$[s_6, s_7]$
x_2	$[s_5, s_8]$	$[s_7, s_7]$	$[s_3, s_3]$	$[s_4, s_5]$	$[s_2, s_5]$
x_5	$[s_6, s_8]$	$[s_4, s_5]$	$[s_4, s_6]$	$[s_5, s_5]$	$[s_6, s_7]$
x_6	$[s_0, s_3]$	$[s_5, s_7]$	$[s_5, s_6]$	$[s_4, s_6]$	$[s_6, s_8]$

Table 2. The decision matrices provided by e_2 and e_3

	X^2	A^2					X^3	A^3			
		a_2	a_3	a_4	a_5			a_1	a_2	a_4	a_5
e_2	x_2	$[s_5, s_6]$	$[s_4, s_6]$	$[s_6, s_6]$	$[s_2, s_5]$	e_3	x_1	$[s_5, s_8]$	$[s_7, s_8]$	$[s_3, s_4]$	$[s_5, s_5]$
	x_3	$[s_5, s_6]$	$[s_3, s_5]$	$[s_4, s_5]$	$[s_3, s_5]$		x_2	$[s_5, s_8]$	$[s_7, s_8]$	$[s_6, s_6]$	$[s_2, s_5]$
	x_4	$[s_5, s_6]$	$[s_5, s_7]$	$[s_4, s_4]$	$[s_4, s_5]$		x_3	$[s_1, s_3]$	$[s_6, s_6]$	$[s_5, s_5]$	$[s_4, s_6]$
	x_5	$[s_4, s_5]$	$[s_5, s_5]$	$[s_4, s_4]$	$[s_6, s_6]$		x_4	$[s_1, s_4]$	$[s_5, s_8]$	$[s_4, s_5]$	$[s_5, s_5]$
	x_6	$[s_4, s_5]$	$[s_4, s_6]$	$[s_5, s_5]$	$[s_5, s_7]$		x_5	$[s_4, s_7]$	$[s_5, s_5]$	$[s_4, s_6]$	$[s_3, s_5]$

Table 3. The decision matrices provided by e_4 and e_5

	X^4	A^4				X^5	A^5			
		a_1	a_3	a_4			a_1	a_2	a_3	a_5
e_4	x_1	$[s_3, s_6]$	$[s_6, s_8]$	$[s_4, s_5]$	e_5	x_1	$[s_7, s_7]$	$[s_7, s_8]$	$[s_8, s_8]$	$[s_5, s_8]$
	x_2	$[s_3, s_5]$	$[s_3, s_5]$	$[s_4, s_5]$		x_3	$[s_4, s_7]$	$[s_5, s_7]$	$[s_4, s_5]$	$[s_4, s_5]$
	x_3	$[s_1, s_3]$	$[s_4, s_5]$	$[s_3, s_4]$		x_4	$[s_1, s_4]$	$[s_5, s_6]$	$[s_5, s_7]$	$[s_4, s_7]$
	x_4	$[s_1, s_4]$	$[s_5, s_5]$	$[s_4, s_5]$		x_5	$[s_3, s_3]$	$[s_3, s_6]$	$[s_2, s_5]$	$[s_2, s_5]$
	x_6	$[s_1, s_4]$	$[s_1, s_3]$	$[s_4, s_5]$		x_6	$[s_5, s_7]$	$[s_4, s_5]$	$[s_5, s_5]$	$[s_3, s_6]$

The specific supplier selection process is as follows:

Step 1. By using Eq.(1), the group PULTS matrix are obtained.

Step 2. By using Eqs.(2)-(4), the weights ω^u_{ijh} of $e_h \in E_{ij}$ are obtained; By using Eqs.(5) and (6), the weights ω^c_{ijh} of $e_h \in E_{ij}$ are obtained. By solving model (M-1), the weights ω_{ijh} of $e_h \in E_{ij}$ are obtained.

Step 3. By using Eqs.(7), the weights v_{ij}^r of attributes under each alternative based on effectiveness are obtained.

Step 4. By using the weights of experts ω_{ijh} and ULWA operator, the group ULT decision matrix $\tilde{R}$ are obtained and shown in Table 4.

Table 4. Group ULT decision matrix $\tilde{R}$

X	A				
	a_1	a_2	a_3	a_4	a_5
x_1	$[s_{5.848}, s_7]$	$[s_{6.676}, s_{7.676}]$	$[s_6, s_{7.3}]$	$[s_{3.682}, s_{4.318}]$	$[s_{5.342}, s_{6.434}]$
x_2	$[s_{4.388}, s_{7.082}]$	$[s_{6.364}, s_7]$	$[s_{3.3}, s_{4.5}]$	$[s_{5.068}, s_{5.534}]$	$[s_2, s_5]$
x_3	$[s_{1.88}, s_{4.175}]$	$[s_{5.385}, s_{6.286}]$	$[s_{4.898}, s_{5.888}]$	$[s_{4.046}, s_{4.682}]$	$[s_{3.684}, s_{5.316}]$
x_4	$[s_1, s_4]$	$[s_5, s_{6.526}]$	$[s_5, s_{6.2}]$	$[s_4, s_{4.636}]$	$[s_{4.411}, s_{5.5}]$
x_5	$[s_{4.211}, s_{5.632}]$	$[s_{4.116}, s_{5.185}]$	$[s_{3.808}, s_{5.403}]$	$[s_{4.396}, s_{4.942}]$	$[s_{4.539}, s_{5.824}]$
x_6	$[s_{1.872}, s_{4.576}]$	$[s_{4.3}, s_{5.6}]$	$[s_{3.918}, s_{5.051}]$	$[s_{4.381}, s_{5.286}]$	$[s_{4.743}, s_{7.037}]$

Step 5. By using Eq.(8), the weights v_j^d of each attribute based on maximizing deviation in $\tilde{R}$ are obtained, $v_1^d = 0.326$, $v_2^d = 0.196$, $v_3^d = 0.193$, $v_4^d = 0.091$, $v_5^d = 0.195$.

Step 6. By solving model (M-2), the synthetic weights v_j of attributes are obtained, $v_1 = 0.324$, $v_2 = 0.269$, $v_3 = 0.155$, $v_4 = 0.091$, $v_5 = 0.161$.

Step 7. Let $\mu = 0.5$, $\tilde{s}^+ = [s_8, s_8]$ and $\tilde{s}^- = [s_0, s_0]$. By using the improved VIKOR method, the values of S_i, R_i and Q_i are obtained (see Table 5). The ranking result obtained from Q_i is $x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$ and the optimal solution is x_1.

Table 5. The values and rankings of the alternatives

X	Values			Rankings		
	S_i	R_i	Q_i	S_i	R_i	Q_i
x_1	0.219	0.068	0	1	1	1
x_2	0.359	0.107	0.406	2	2	2
x_3	0.446	0.207	0.892	4	5	4
x_4	0.463	0.231	1	6	6	6
x_5	0.406	0.128	0.568	3	3	3
x_6	0.460	0.201	0.902	5	4	5

5. Comparative analysis

5.1. Sensitivity analysis

The overall evaluation values $Q_i (1 \leq i \leq 6)$ and ranking results corresponding to different compromise coefficient μ are shown in Table 6.

It can be seen from Table 6 that different μ will result in different ranking results. However, no matter what value μ takes, the optimal alternative is always x_1 and the order of the first three alternatives has not changed. If weighted aggregation or TOPSIS is used instead of the improved VIKOR method, only one ranking result can be obtained, i.e., $x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$. By contrast, the improved VIKOR method is more flexible and practical.

Table 6. The decision results under different compromise coefficient μ

μ	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Ranking results	Optimal alternative
0	0	0.238	0.851	1	0.368	0.817	$x_1 \succ x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$	x_1
0.1	0	0.272	0.860	1	0.408	0.834	$x_1 \succ x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$	x_1
0.2	0	0.305	0.868	1	0.448	0.851	$x_1 \succ x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$	x_1
0.3	0	0.339	0.876	1	0.488	0.868	$x_1 \succ x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$	x_1
0.4	0	0.373	0.884	1	0.528	0.885	$x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$	x_1
0.5	0	0.406	0.892	1	0.568	0.902	$x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$	x_1
0.6	0	0.440	0.900	1	0.607	0.919	$x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$	x_1
0.7	0	0.473	0.909	1	0.647	0.936	$x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$	x_1
0.8	0	0.507	0.917	1	0.687	0.954	$x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$	x_1
0.9	0	0.541	0.925	1	0.727	0.971	$x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$	x_1
1.0	0	0.574	0.933	1	0.767	0.988	$x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$	x_1

5.2. *Stability analysis*

In this subsection, the stability of the proposed method will be verified through ablation experiments. Table 7 lists the differences between the improved VIKOR method (I-VIKOR) and three comparison methods M-VIKOR, D-VIKOR and MD-VIKOR.

Table 7. The improved VIKOR method and three comparison methods

Methods	PIS	NIS	Attribute weights
I-VIKOR (ours)	$[s_l, s_l]$	$[s_0, s_0]$	multi-granulation weight
M-VIKOR	$[\max_i s_{ijL}, \max_i s_{ijR}]$	$[\min_i s_{ijL}, \min_i s_{ijR}]$	multi-granulation weight
D-VIKOR	$[s_l, s_l]$	$[s_0, s_0]$	single-granulation weight based on maximizing deviation
MD-VIKOR	$[\max_i s_{ijL}, \max_i s_{ijR}]$	$[\min_i s_{ijL}, \min_i s_{ijR}]$	single-granulation weight based on maximizing deviation

Tables 8 show the ranking results of the four methods mentioned above. It can be seen that the stability of the proposed I-VIKOR is the best, while the stability of MD-VIKOR is the worst.

Table 8. Ranking results of four methods before and after deleting x_1

Methods	Before deleting x_1	After deleting x_1
I-VIKOR	$x_1 \succ x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$ $(0 \le \mu \le 0.3)$ $x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$ $(0.4 \le \mu \le 1)$	$x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$ $(0 \le \mu \le 0.4)$ $x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$ $(0.5 \le \mu \le 1)$
M-VIKOR	$x_1 \succ x_2 \succ x_6 \succ x_5 \succ x_3 \succ x_4$ $(\mu \le 0.1)$ $x_1 \succ x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$ $(0.2 \le \mu \le 0.5)$ $x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$ $(0.6 \le \mu \le 0.9)$ $x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_4 \succ x_6$ $(\mu = 1)$	$x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$ $(0 \le \mu \le 0.6)$ $x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$ $(0.7 \le \mu \le 1)$
D-VIKOR	$x_1 \succ x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$ $(0 \le \mu \le 0.6)$ $x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$ $(0.7 \le \mu \le 1)$	$x_5 \succ x_2 \succ x_6 \succ x_3 \succ x_4$ $(0 \le \mu \le 0.2)$ $x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$ $(0.3 \le \mu \le 1)$
MD-VIKOR	$x_1 \succ x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$ $(0 \le \mu \le 0.9)$ $x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$ $(\mu = 1)$	$x_5 \succ x_2 \succ x_6 \succ x_3 \succ x_4$ $(0 \le \mu \le 0.4)$ $x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$ $(0.5 \le \mu \le 1)$

5.3. Effectiveness analysis

In this subsection, the advantages of the multi-granulation weighting model for expert will be explained through comparative analysis of effectiveness. The proposed HMAGDM method is called M-HMAGDM. The other three comparison methods are U-HMAGDM using the uncertainty based expert weight ω^{u}_{ijh}, C-HMAGDM using the closeness based expert weight ω^{c}_{ijh}, and E-HMAGDM with equal expert weight. The difference between them is only reflected in the calculation method of expert weights. Table 9 lists the effectiveness and ranking results corresponding to the above four methods.

Table 9. Comparison of effectiveness and ranking results of four HMAGDM methods

Mehtods	EFD_1	EFD_2	EFD_3	EFD_4	EFD_5	EFD	Ranking results
M-HMAGDM (ours)	0.576	0.78	0.686	0.822	0.662	0.684	$x_1 \succ x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$ $(0 \le \mu \le 0.3)$ $x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$ $(0.4 \le \mu \le 1)$
U-HMAGDM	0.572	0.78	0.686	0.822	0.66	0.683	$x_1 \succ x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$ $(0 \le \mu \le 0.7)$ $x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$ $(0.8 \le \mu \le 1)$
C-HMAGDM	0.566	0.764	0.681	0.812	0.637	0.671	$x_1 \succ x_2 \succ x_5 \succ x_6 \succ x_3 \succ x_4$ $(0 \le \mu \le 0.3)$ $x_1 \succ x_2 \succ x_5 \succ x_3 \succ x_6 \succ x_4$ $(0.4 \le \mu \le 1)$
E-HMAGDM	0.559	0.705	0.679	0.811	0.633	0.644	$x_1 \succ x_5 \succ x_2 \succ x_6 \succ x_3 \succ x_4$ $(0 \le \mu \le 0.6)$ $x_1 \succ x_5 \succ x_2 \succ x_3 \succ x_6 \succ x_4$ $(0.7 \le \mu \le 1)$

From Table 9, it can be seen that the proposed method M-HMAGDM has the best performance, while E-HMAGDM has the lowest performance. The other two methods are between M-HMAGDM and E-HMAGDM.

6. Conclusions

The proposed HMAGDM method can effectively overcome the limitations of existing methods in weight setting, information aggregation and alternative ranking, and obtain more reasonable and high-quality decision results. The advantages of the proposed method are demonstrated through comparative analysis of sensitivity, stability, and effectiveness. However, this study is limited to uncertain linguistic environment and cannot be directly used to handle the HMAGDM problems with different forms and granulations of fuzzy linguistic preference information in the open dynamic environment. In the future, we will closely connect with practical decision-making problems and combine factors such as trust relationships, self-confidence, attribute priority, and alternative grade to study consensus analysis models and decision feedback mechanisms for complex HMAGDM problems in different linguistic environments. On this basis, explore more flexible and reliable group decision-making methods to provide decision-makers with more scientific and practical decision support.

Acknowledgements

This work was supported by the National Natural Science Foundation of China [No. 62006148, 72171137, 62272284], the Open Project Foundation of Intelligent Information Processing Key Laboratory of Shanxi Province [No. CICIP2022002] and the 1331 Engineering Project of Shanxi Province, China.

References

[1] Pang K, Martínez L, Li N, Liu J, Zou L, Lu MY. A concept lattice-based expert opinion aggregation method for multi-attribute group decision-making with linguistic information. Expert Systems With Applications. 2024 Mar; 237: 121458.

[2] Haseli G, Torkayesh AE, Hajiaghaei-Keshteli M, Venghaus S. Sustainable resilient recycling partner selection for urban waste management: Consolidating perspectives of decision-makers and experts. Applied Soft Computing. 2023 Apr; 137: 110120.

[3] Zhu JX, Ma XL, Kou G, Herrera-Viedma E, Zhan JM. A three-way consensus model with regret theory under the framework of probabilistic linguistic term sets. Information Fusion. 2023 Jul; 95: 250-274.

[4] Li WW, Yi PT, Li LY. Superiority-comparison-based transformation, consensus, and ranking methods for heterogeneous multi-attribute group decision-making. Expert Systems With Applications. 2023 Mar; 213: 119018.

[5] Zhao CW, Xu XH, Liu RH, He JS. A novel multi-attribute group decision-making method for heterogeneous preferences and attribute interaction contexts. Expert Systems With Applications. 2023 Jul; 221: 119752.

[6] Zheng J, Wang YM, Zhang K, Gao JQ, Yang LH. A heterogeneous multi-attribute case retrieval method for emergency decision making based on bidirctional projection and TODIM. Expert Systems With Application. 2022 Oct; 203: 117382.

[7] Morente-Molinera JA, Wu X, Morfeq A, Al-Hmouz R, Herrera-Viedma E. A novel multi-criteria group decision-making method for heterogeneous and dynamic contexts using multi-granular fuzzy linguistic modelling and consensus measures. Information Fusion. 2020 Jan; 53: 240-250.

[8] Lourenzutti R, Krohling RA. A generalized TOPSIS method for group decision making with heterogeneous information in a dynamic environment. Information Sciences. 2016 Feb; 330: 1-18.

[9] Gao Y, Li DS. A consensus model for heterogeneous multi-attribute group decision making with several attribute sets. Expert Systems With Application. 2019 Jul; 125: 69-80.

[10] Liu X, Xu YJ, Gong ZW, Herrera F. Democratic consensys reaching process for multi-person multi-criteria large scale decision making considering participants' individual attributes and concerns. Information Fusion. 2022 Jan; 77: 220-232.

[11] Ye J, Sun BZ, Chu XL, Zhan JM, Cai JX. Valued outranking relation-based heterogeneous multi-decision multigranulation probabilistic rough set and its use in medical decision-making. Expert Systems With Applications. 2023 Oct; 228: 120296.

[12] Li WW, Yi PT, Zhang DN, Wang L, Dong QK. Stochastic-integration-based decision support methods for heterogeneous multi-attribute group decision making with several attribute sets. Expert Systems With Applications. 2023 Dec; 234: 121100.

[13] Yu SM, Du ZJ, Wang JQ, Luo HY, Lin XD. Trust and behavior analysis-based fusion method for heterogeneous multiple attribute group decision-making. Computers & Industrial Engineering. 2021 Feb; 152: 106992.

[14] Dong YC, Zhang HJ, Herrera-Viedma E. Consensus reaching model in the complex and dynamic MAGDM problem. Knowledge-Based Systems. 2016 Aug; 106: 206-219.

[15] Tan X, Zhu JJ, Palomares I, Liu X. On consensus reaching process based on social network analysis in uncertain linguistic group decision making: Exploring limited trust propagation and preference modification attitudes. Information Fusion. 2022 Feb; 78: 180-198.

[16] Opricovic S, Tzeng GH. Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS, European Journal of Operational Research. 2004 Jul; 156: 445-455.

[17] Liu GZ, Wang XG. A trapezoidal fuzzy number-based VIKOR method with completely unknown weight information. Symmetry. 2023 Feb; 15: 559.

[18] Yang WG, Wu YJ. A new improvement method to avoid rank reversal in VIKOR. IEEE Access. 2020 Apr; 8: 21261-21270.

[19] Herrera F, Herrera-Viedma E, Verdegay JL. A sequential selection process in group decision making with a linguistic assessment approach. Information Sciences. 1995 Jul; 85(4): 223-239.

[20] Xu ZS. A method based on linguistic aggregation operators for group decision making with linguistic preference relations. Information Sciences. 2004 Oct; 166(1-4): 19-30.

[21] Xu ZS. Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment. Information Sciences. 2004 Dec; 168(1-4): 171-184.

[22] Lin MW, Xu ZS, Zhai YL, Yao ZQ. Multi-attribute group decision-making under probabilistic uncertain linguistic environment. Journal of the Operational Research Society. 2018 Dec; 69(2): 157-170.

[23] Bashir Z, Ali J, Rashid T. Consensus-based robust decision making methods under a novel study of probabilistic uncertain linguistic information and their application in Forex investment. Artificial Intelligence Review. 2021 Mar; 54: 2091-2132.

[24] Wu ZB, Fang TF. A consensus and maximizing deviation based approach for multi-criteria group decision making under linguistic setting. 2014 IEEE Irenational Conference on Fuzzy Systems. IEEE Press; c2014. p.469-475.