Fuzzy Systems and Data Mining IX
A.J. Tallón-Ballesteros and R. Beltrán-Barba (Eds.)

doi:10.3233/FAIA231069

A New Interval-Valued Fuzzy Entropy Based on Interval-Valued Q-Rung Orthopair Fuzzy Sets

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Abstract. As an extension of fuzzy sets, the interval-valued q-rung orthopair fuzzy sets (IVq-ROFSs) is a powerful tool for dealing with uncertainty problems. Furthermore, fuzzy entropy is a crucial indicator to measure the fuzzy degree of fuzzy sets. However, the current fuzzy entropy of IVq-ROFSs have some disadvantages. First, for some interval-valued q-rung orthopair fuzzy numbers (IVq-ROFNs), the existing fuzzy entropy cannot accurately measure the fuzzy degree. Second, it is not a reasonable method to utilize exact values as fuzzy entropy in the form of interval values. In this paper, the fuzzy entropy of IVq-ROFSs is characterized by interval values. The axiomatic definitions of IVq-ROFSs fuzzy entropy is given. Strict mathematical proof and a numerical example verify that the proposed axiomatic definition of fuzzy entropy is complete and avoids the loss of interval-valued fuzzy information.

Keywords. Interval-valued q-rung orthopair fuzzy set, Fuzzy entropy, Interval value

1 Introduction

Since Zadeh[1] presented fuzzy sets(FSs), FS has been a hot spot study field for scholars worldwide[2-4]. FS and its extension models had involved multi-attribute decision-making[5], clustering algorithms[6], neural network[7] and so on. Turksen[8] proposed the concept of interval-valued fuzzy sets(IVFS). IVFS takes the interval value as the membership degree (MD) form, which can better reflect the fuzzy concept in fuzzy theory. Furthermore, Takeuti[9] proposed the non-membership degree(NMD) and hesitation degree based on MD. Then the concept of intuitionistic fuzzy set (IFS) was proposed. On the basis of IFS, q-rung orthopair fuzzy set(q-ROFS)[10] and interval-valued q-rung orthopair fuzzy sets(IVq-ROFSs) [11]were presented.

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At the same time, in order to be able to judge the relationship between the degree of fuzziness between different fuzzy numbers, Kosko[12] proposed fuzzy entropy. The proposal of fuzzy entropy can effectively compare the fuzzy degrees of different fuzzy sets or fuzzy elements. Scholars have proposed their axiomatic definitions for different fuzzy sets. Peng[13] proposed q-rung orthopair fuzzy entropy,. For various fuzzy sets whose degrees are exact values, the axiomatic definitions do not change much, but for various fuzzy sets whose data forms are interval values, the axiomatic definitions diverge greatly. Zhang[14] proposed an axiomatic definition based on IVIFSs, using the comparative relations of interval values, and analyzed the judgment of fuzzy entropy under four interval relations step by step. Peng[15] proposed an axiomatic definition based on IVPFSs, which only defined the comparison of fuzzy entropy under the containment relationship.

The current axiomatic definition of fuzzy entropy for various fuzzy sets in the form of interval-valued data is based on the calculation of the endpoint values of MD and NMD to obtain an accurate value of fuzzy entropy to judge the degree of fuzziness. However, in the process of calculating the precise fuzzy entropy from the interval value through the endpoint value, information loss may occur, so we think that the interval value can be directly used as the data form of q-rung orthopair fuzzy entropy. Directly using interval values as the data form of fuzzy entropy, on the one hand, can effectively reduce the information loss in the process of converting interval value data into accurate value data. And on the other hand, it can directly use the size comparison method of interval value data to determine the fuzzy entropy. In this paper, we first propose the axiomatic definitions of interval-valued fuzzy entropy (IVFE) based on the IVq-ROFSs, and explain it through the function graph of fuzzy entropy. And then the IVFE formulas are defined by the axiom, then an example is given to illustrate the effectiveness of the formula.

The structure of the article is as follows: In the Second part, some related concepts based on the fuzzy entropy on IVq-ROFSs are explained. In the Third part the axiomatic definitions of the IVFE on IVq-ROFSs and the related formulas are presented. Its effectiveness is demonstrated by an example compared with existing research. In the Forth part, we summarize the contributions made in this paper.

2 Preliminaries

In this section we introduce the interval-valued size relationship and some related concepts about the fuzzy entropy on IVq-ROFSs.

Definition 1[16] Let$[\varphi_1, \phi_1]$, $[\varphi_2, \phi_2] \in [\mathrm{I}]$, we define

$[\varphi_1, \phi_1] \leq [\varphi_2, \phi_2]$,iff $\varphi_1 \leq \varphi_2, \phi_1 \leq \phi_2$;

$[\varphi_1, \phi_1] \preccurlyeq [\varphi_2, \phi_2]$,iff $\varphi_1 \leq \varphi_2, \phi_1 \geq \phi_2$;

$[\varphi_1, \phi_1] = [\varphi_2, \phi_2]$,iff $\varphi_1 = \varphi_2, \phi_1 = \phi_2$;

$[\varphi_1, \phi_1] \precsim [\varphi_2, \phi_2]$,iff $\begin{cases}[\varphi_1, \phi_1] \leq [\varphi_2, \phi_2] \\ [\varphi_1, \phi_1] \succcurlyeq [\varphi_2, \phi_2]\end{cases}$.

Definition 2[11] Let X be a non-empty set, then the IVq-ROFS A on X can be expressed as:

$$Q = \{< \chi, (\widetilde{\varphi_Q}(x), \widetilde{\phi_Q}(x)) > | \chi \in \mathrm{X}\},$$

where $\widetilde{\varphi_Q}(\chi) = [\widetilde{\varphi_Q}^-(\chi), \widetilde{\varphi_Q}^+(\chi)] \subset [0,1]$, $\widetilde{\phi_Q}(\chi) = [\widetilde{\phi_Q}^-(\chi), \widetilde{\phi_Q}^+(\chi)] \subset [0,1]$and the MD and NMD satisfy$0 \leq \left(\widetilde{\varphi_Q}^+(\chi)\right)^q + \left(\widetilde{\phi_Q}^+(\chi)\right)^q \leq 1, q \geq 1$

Definition 3[11] Let $Q_1, Q_2 \in q - ROIVF(\chi)$, some operations can be defined as follow:

$Q_1 \leq Q_2$,iff $[\widetilde{\varphi_{Q_1}}^-(\chi), \widetilde{\varphi_{Q_1}}^+(\chi)] \leq [\widetilde{\varphi_{Q_2}}^-(\chi), \widetilde{\varphi_{Q_2}}^+(\chi)]$, $[\widetilde{\phi_{Q_1}}^-(\chi), \widetilde{\phi_{Q_1}}^+(\chi)] \geq [\widetilde{\phi_{Q_2}}^-(\chi), \widetilde{\phi_{Q_2}}^+(\chi)], \forall \chi \in X$;

$Q_1 \preccurlyeq Q_2$,iff $[\widetilde{\varphi_{Q_1}}^-(\chi), \widetilde{\varphi_{Q_1}}^+(\chi)] \preccurlyeq [\widetilde{\varphi_{Q_2}}^-(\chi), \widetilde{\varphi_{Q_2}}^+(\chi)]$, $[\widetilde{\phi_{Q_1}}^-(\chi), \widetilde{\phi_{Q_1}}^+(\chi)] \succcurlyeq [\widetilde{\phi_{Q_2}}^-(\chi), \widetilde{\phi_{Q_2}}^+(\chi)], \forall \chi \in X$;

From definition 2, we can see that IVq-ROFSs degenerates to IVIFS, IVPFS and IVFFS when the values of q are 1, 2 and 3, respectively. So, the axiomatic definition of fuzzy entropy based on IVIFS and IVPFS is given here as an example of the existing research for IVFE.

Definition 4[14] A real function E:$IVIF(X) \to [0,1]$ is named as fuzzy entropy on IVIFS, and E has to satisfy following properties:

(I1)$E(A^*) = 0$ if A^* is a crisp set.

(I2)$E(A^*) = 1$ iff $\mu_{A^*}(\chi_i) = \nu_{A^*}(\chi_i), \forall \chi_i \in X$;

(I3)$E(A^*) = E(B^*)$ if $A^*$is less fuzzy than B^*which is defined as:

$\varphi_{A^*}(\chi_i) \leq \varphi_{B^*}(\chi_i)$, $\phi_{A^*}(\chi_i) \geq \phi_{B^*}(\chi_i)$, for $\varphi_{B^*}(\chi_i) \leq \phi_{B^*}(\chi_i)$

$\varphi_{A^*}(\chi_i) \geq \varphi_{B^*}(\chi_i)$, $\phi_{A^*}(\chi_i) \leq \phi_{B^*}(\chi_i)$, for $\varphi_{B^*}(\chi_i) \geq \phi_{B^*}(\chi_i)$

$\varphi_{A^*}(\chi_i) \preccurlyeq \varphi_{B^*}(\chi_i)$, $\phi_{A^*}(\chi_i) \succcurlyeq \phi_{B^*}(\chi_i)$, for $\varphi_{B^*}(\chi_i) \preccurlyeq \phi_{B^*}(\chi_i)$

$\varphi_{A^*}(\chi_i) \succcurlyeq \varphi_{B^*}(\chi_i)$, $\phi_{A^*}(\chi_i) \preccurlyeq \phi_{B^*}(\chi_i)$, for $\varphi_{B^*}(\chi_i) \succcurlyeq \phi_{B^*}(\chi_i)$

(I4)$E(A^*) = E((A^*)^C)$

Definition 5[15] A real function E:$IVPF(X) \to [0,1]$ is named as fuzzy entropy on IVPFS, and E has to satisfy following properties:

(P1)$E(A^{P^*}) = 0$ iff $A^{P^*}$is a crisp set;

(P2)$E(A^{P^*}) = 1$ iff $\varphi_{A^{P^*}}(\chi_i) = \phi_{A^{P^*}}(\chi_i), \forall \chi_i \in X$

(P3)$E(A^{P^*}) \leq E(B^{P^*})$ iff

$A^{P^*} \subseteq B^{P^*}$when $\varphi^-_{B^{P^*}}(\chi_i) \leq \phi^-_{B^{P^*}}(\chi_i)$ and $\varphi^+_{B^{P^*}}(\chi_i) \leq \phi^+_{B^{P^*}}(\chi_i), \forall \chi_i \in X$

$B^{P^*} \subseteq A^{P^*}$when $\varphi^-_{B^{P^*}}(\chi_i) \geq \phi^-_{B^{P^*}}(\chi_i)$ and $\varphi^+_{B^{P^*}}(\chi_i) \geq \phi^+_{B^{P^*}}(\chi_i), \forall \chi_i \in X$

(P4) $E(A^{P^*}) = E((A^{P^*})^C)$.

3 Interval-valued fuzzy entropy on IVq-ROFSs

Definition 4 and Definition 5 are the current axiomatic definitions of fuzzy entropy on IVIFS and IVPFS, and they have some defects. First of all the relations between interval values then include four relations of distance, adjacency, intersection and inclusion. In the case of both MD and NMD, defining the interval relationship between the MD and NMD of two IVq-ROFNs is a very complicated process. The (I3) condition of Definition 4 only analyzes the MD and NMD of one of the IVPFS. The (I3) condition of Definition 4 only analyzes the comparison of the fuzzy entropy values of two IVPFSs under the four interval values of MD and NMD of one IVPFS, and does not analyze the interval values of MD and NMD of the other IVPFS under the four interval values of MD and NMD of one IVPFS; the (P3) condition of Definition 5 only takes into account the existence of both MD and NMD inclusion relationship. In addition, these two fuzzy entropy formulas both transform interval values into specific values to carry out the fuzzy degree of a certain fuzzy set, this transformation process itself will bring some information loss, so in order to solve the above defects, this paper proposes a new IVFE on IVq-ROFSs, which directly expresses the fuzzy degree using IVFE, and judges the size of fuzzy degree by the defined interval-valued dominance relationship, solving the interval relationship problem between MD and NMD from the mathematical point of view, while reducing the information loss.

Consequently, the axiomatic definition of IVFE is proposed.

Definition 6 An interval-valued function E: $IVq-ROF(\mathrm{X}) \rightarrow [0,1]$ is named as IVFE on IVq-ROFSs, and E has to satisfy following properties:

(I-Q1) $[0,0] \leq E_{\mathrm{IV}}\left(A^{Q^*}\right) \leq [1,1]$

(I-Q2) $E_{\mathrm{IV}}\left(A^{Q^*}\right) = [0,0]$ if A^{Q^*} is a crisp set.

(I-Q3) $E_{\mathrm{IV}}\left(A^{Q^*}\right) = [1,1]$ if $\widetilde{\varphi_Q}(\chi) = \widetilde{\phi_Q}(\chi)$.

(I-Q4) A^{Q^*} less fuzzy than B^{Q^*} if $E_{\mathrm{IV}}\left(A^{Q^*}\right) \precsim E_{\mathrm{IV}}\left(B^{Q^*}\right)$

(I-Q5) $E_{\mathrm{IV}}\left(A^{Q^*}\right) = E_{\mathrm{IV}}\left(\left(A^{Q^*}\right)^C\right)$

Also based on the axiomatic definition of the IVFE we can give the graph of the function corresponding to the fuzzy entropy, here for ease of illustration is given when q = 1, i.e., the graph of the fuzzy entropy function of the IVIFS, as shown in Figure 1:

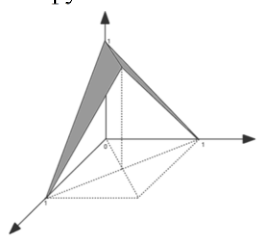

Figure 1. Interval-valued fuzzy entropy function graph on IVq-ROFSs (q=1)

From Figure 1, the x-axis is $\varphi_I(\chi)$, the y-axis is $\phi_I(\chi)$, the z-axis is fuzzy entropy value. Straight line in the plane xOy $\varphi_I(\chi) + \nu_I(\chi) = 1$,where $\varphi_I(\chi)$ and $\phi_I(\chi)$ is the MD and NMD of IFS. When $\varphi_I(\chi) = \phi_I(\chi)$, $E\left(A^{I^*}\right)$=1, according to the axiomatic definition of fuzzy entropy, the image of the function will be symmetrical along $F(\varphi_I(\chi), \phi_I(\chi))$.

The fuzzy elements of IVIFS are $\varphi_I(\chi) + \phi_I(\chi) = 1$ and the subplanes of the plane formed by x-axis, y-axis. So, on the basis of the constant fuzzy entropy function image, the geometric representation of the IVFE is the intercept of the projection of this subplane on the function image on the z-axis. The reasonableness of the (I-Q3) condition can be seen more intuitively through the geometric expression.

Example: Interval-valued fuzzy entropy on IVq-ROFSs:

$$E_{\mathrm{IV}}\left(A^{Q^*}\right) = [\frac{1}{n}\sum_{i=1}^{n} \sqrt[q]{\frac{1-\left|\left(\widetilde{\varphi_Q}^{+}(\chi)\right)^q - \left(\widetilde{\phi_Q}^{+}(\chi)\right)^q\right|}{1+\left|\left(\widetilde{\varphi_Q}^{+}(\chi)\right)^q - \left(\widetilde{\phi_Q}^{+}(\chi)\right)^q\right|}}, \frac{1}{n}\sum_{i=1}^{n} \sqrt[q]{\frac{1-\left|\left(\widetilde{\varphi_Q}^{-}(\chi)\right)^q - \left(\widetilde{\phi_Q}^{-}(\chi)\right)^q\right|}{1+\left|\left(\widetilde{\varphi_Q}^{-}(\chi)\right)^q - \left(\widetilde{\phi_Q}^{-}(\chi)\right)^q\right|}}]$$

Proof:

(I-Q1) Easy to proof.

(I-Q2) A^{Q^*} is a crisp set means that $\widetilde{\varphi_Q}(\chi) = [0,0]$ and $\widetilde{\phi_Q}(\chi) = [1,1]$ or $\widetilde{\varphi_Q}(\chi) = [1,1]$ and $\widetilde{\phi_Q}(\chi) = [0,0]$.

When $\widetilde{\varphi_Q}(\chi) = [0,0]$ and $\widetilde{\phi_Q}(\chi) = [1,1]$,

$$E_{\mathrm{IV}}\left(A^{Q^*}\right) = [\frac{1-1}{1+1}, \frac{1-1}{1+1}] = [0,0]$$

when $\widetilde{\varphi_Q}(\chi) = [1,1]$ and $\widetilde{\phi_Q}(\chi) = [0,0]$,

$$E_{\mathrm{IV}}\left(A^{Q^*}\right) = [\frac{1-1}{1+1}, \frac{1-1}{1+1}] = [0,0]$$

(I-Q3) $\widetilde{\varphi_Q}(\chi) = \widetilde{\phi_Q}(\chi)$ means that $\left[\widetilde{\varphi_Q}^{-}(\chi), \widetilde{\varphi_Q}^{+}(\chi)\right] = \left[\widetilde{\phi_Q}^{-}(\chi), \widetilde{\phi_Q}^{+}(\chi)\right]$

$$E_{\mathrm{IV}}\left(A^{Q^*}\right) = [\frac{1-0}{1+0}, \frac{1-0}{1+0}] = [1,1]$$

(I-Q4) $E_{\mathrm{IV}}\left(A^{Q^*}\right) \precsim E_{\mathrm{IV}}\left(B^{Q^*}\right)$ means that

$\left[E_{\mathrm{IV}}^{-}\left(A^{Q^*}\right), E_{\mathrm{IV}}^{+}\left(A^{Q^*}\right)\right] \leq \left[E_{\mathrm{IV}}^{-}\left(B^{Q^*}\right), E_{IV}^{+}\left(B^{Q^*}\right)\right]$

$$\sqrt[q]{\frac{1-\left|\left(\widetilde{\varphi_{AQ^*}}^{-}(\chi)\right)^q - \left(\widetilde{\phi_{AQ^*}}^{-}(\chi)\right)^q\right|}{1+\left|\left(\widetilde{\varphi_{AQ^*}}^{-}(\chi)\right)^q - \left(\widetilde{\phi_{AQ^*}}^{-}(\chi)\right)^q\right|}} \leq \sqrt[q]{\frac{1-\left|\left(\widetilde{\varphi_{BQ^*}}^{-}(\chi)\right)^q - \left(\widetilde{\phi_{BQ^*}}^{-}(\chi)\right)^q\right|}{1+\left|\left(\widetilde{\varphi_{BQ^*}}^{-}(\chi)\right)^q - \left(\widetilde{\phi_{BQ^*}}^{-}(\chi)\right)^q\right|}}$$

and

$$\sqrt[q]{\frac{1-\left|\left(\widetilde{\varphi_{AQ^*}}^{+}(\chi)\right)^q - \left(\widetilde{\phi_{AQ^*}}^{+}(\chi)\right)^q\right|}{1+\left|\left(\widetilde{\varphi_{AQ^*}}^{+}(\chi)\right)^q - \left(\widetilde{\phi_{AQ^*}}^{+}(\chi)\right)^q\right|}} \leq \sqrt[q]{\frac{1-\left|\left(\widetilde{\varphi_{BQ^*}}^{+}(\chi)\right)^q - \left(\widetilde{\phi_{BQ^*}}^{+}(\chi)\right)^q\right|}{1+\left|\left(\widetilde{\varphi_{BQ^*}}^{+}(\chi)\right)^q - \left(\widetilde{\phi_{BQ^*}}^{+}(\chi)\right)^q\right|}}$$

let $\left|\left(\widetilde{\varphi_{AQ^*}}^{+}(\chi)\right)^q - \left(\widetilde{\phi_{AQ^*}}^{+}(\chi)\right)^q\right| = M, \left|\left(\widetilde{\varphi_{BQ^*}}^{+}(\chi)\right)^q - \left(\widetilde{\phi_{BQ^*}}^{+}(\chi)\right)^q\right| = N$

then we have

$$\frac{1-M}{1+M} \leq \frac{1-N}{1+N}$$
$$(1-M)(1+N) \leq (1-N)(1+M)$$
$$1 - M + N - MN \leq 1 - N + M - MN$$
$$-M + N \leq -N + M$$
$$2N \leq 2M$$
$$N \leq M$$

So $\left|\left(\widetilde{\varphi_{AQ^*}}^{-}(\chi)\right)^q - \left(\widetilde{\phi_{AQ^*}}^{-}(\chi)\right)^q\right| \geq \left|\left(\widetilde{\varphi_{BQ^*}}^{-}(\chi)\right)^q - \left(\widetilde{\phi_{BQ^*}}^{-}(\chi)\right)^q\right|$

and $\left|\left(\widetilde{\varphi_{AQ^*}}^{+}(\chi)\right)^q - \left(\widetilde{\phi_{AQ^*}}^{+}(\chi)\right)^q\right| \geq \left|\left(\widetilde{\varphi_{BQ^*}}^{+}(\chi)\right)^q - \left(\widetilde{\phi_{BQ^*}}^{+}(\chi)\right)^q\right|$

So A^{Q^*} is less fuzzy than B^{Q^*}.

or $\left[E_{\mathrm{IV}}^{-}\left(A^{Q^*}\right), E_{\mathrm{IV}}^{+}\left(A^{Q^*}\right)\right] \succcurlyeq \left[E_{\mathrm{IV}}^{-}\left(B^{Q^*}\right), E_{\mathrm{IV}}^{+}\left(B^{Q^*}\right)\right]$

$$\sqrt[q]{\frac{1-\left|\left(\widetilde{\varphi_{AQ^*}}^{-}(\chi)\right)^q - \left(\widetilde{\phi_{AQ^*}}^{-}(\chi)\right)^q\right|}{1+\left|\left(\widetilde{\varphi_{AQ^*}}^{-}(\chi)\right)^q - \left(\widetilde{\phi_{AQ^*}}^{-}(\chi)\right)^q\right|}} \geq \sqrt[q]{\frac{1-\left|\left(\widetilde{\varphi_{BQ^*}}^{-}(\chi)\right)^q - \left(\widetilde{\phi_{BQ^*}}^{-}(\chi)\right)^q\right|}{1+\left|\left(\widetilde{\varphi_{BQ^*}}^{-}(\chi)\right)^q - \left(\widetilde{\phi_{BQ^*}}^{-}(\chi)\right)^q\right|}}$$

and

$$\sqrt[q]{\frac{1-\left|\left(\widetilde{\varphi_{A^{Q^*}}}^{+}(\chi)\right)^q-\left(\widetilde{\phi_{A^{Q^*}}}^{+}(\chi)\right)^q\right|}{1+\left|\left(\widetilde{\varphi_{A^{Q^*}}}^{+}(\chi)\right)^q-\left(\widetilde{\phi_{A^{Q^*}}}^{+}(\chi)\right)^q\right|}} \le \sqrt[q]{\frac{1-\left|\left(\widetilde{\varphi_{B^{Q^*}}}^{+}(\chi)\right)^q-\left(\widetilde{\phi_{B^{Q^*}}}^{+}(\chi)\right)^q\right|}{1+\left|\left(\widetilde{\varphi_{B^{Q^*}}}^{+}(\chi)\right)^q-\left(\widetilde{\phi_{B^{Q^*}}}^{+}(\chi)\right)^q\right|}}$$

As above, we can obtain

$$\left|\left(\widetilde{\varphi_{A^{Q^*}}}^{-}(\chi)\right)^q-\left(\widetilde{\phi_{A^{Q^*}}}^{-}(\chi)\right)^q\right| \le \left|\left(\widetilde{\varphi_{B^{Q^*}}}^{-}(\chi)\right)^q-\left(\widetilde{\phi_{B^{Q^*}}}^{-}(\chi)\right)^q\right|$$

and $\left|\left(\widetilde{\varphi_{A^{Q^*}}}^{+}(\chi)\right)^q-\left(\widetilde{\phi_{A^{Q^*}}}^{+}(\chi)\right)^q\right| \ge \left|\left(\widetilde{\varphi_{B^{Q^*}}}^{+}(\chi)\right)^q-\left(\widetilde{\phi_{B^{Q^*}}}^{+}(\chi)\right)^q\right|$

So $A^{Q^*} \subseteq B^{Q^*}$

A^{Q^*} is less fuzzy than B^{Q^*}.

(I-Q5) Easy to proof.

The function valued graph is shown as figure 2.

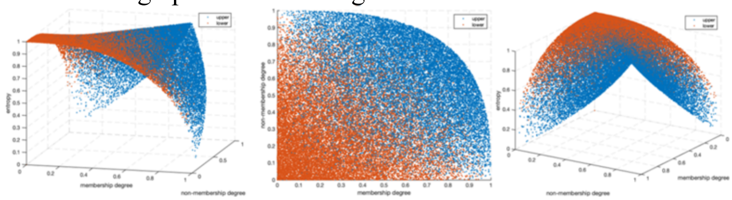

Figure 2. The function valued graph of $E_{\mathrm{IV}}(A^{Q^*})$

4 Examples and Comparation

In this section, the proposed IVFE will be compared with Bu[17]'s IVIFE to verify the rationality of the proposed IVFE in this paper.

The fuzzy entropy formula of Bu[17]'s on IVIFSs as follow:

$$E(A^{I^*}) = \frac{1}{n}\sum_{i=1}^{n}\frac{\min\{\varphi_{I^*}^{-}(\chi_i),\phi_{I^*}^{-}(\chi_i)\}+\min\{\varphi_{I^*}^{+}(\chi_i),\phi_{I^*}^{+}(\chi_i)\}+\pi_{I^*}^{-}(\chi_i)+\pi_{I^*}^{+}(\chi_i)}{\max\{\varphi_{I^*}^{-}(\chi_i),\phi_{I^*}^{-}(\chi_i)\}+\max\{\varphi_{I^*}^{+}(\chi_i),\phi_{I^*}^{+}(\chi_i)\}+\pi_{I^*}^{-}(\chi_i)+\pi_{I^*}^{+}(\chi_i)}$$

To verify the validity as well as the reasonableness of the fuzzy entropy formula proposed in this paper, we set q to 1 for q-RIVOFSs and use the formula to compare with the fuzzy entropy of Bu[17]'s on IVPFS as follows:

Table 1. The comparison of proposed fuzzy entropy with traditional fuzzy entropy

A^{I^*}	**(x, [0.3,0.6]) [0.1,0.2])**	**(x, [0.1,0.2]) [0.3,0.5])**	**(x, [0.1,0.1]) [0.1,0.1])**
$E(A^{I^*})$ [17]	0.65	0.71	1
$E_{IV}(A^{Q^*})$	[0.43,0.67]	[0.54,0.67]	[1, 1]

As can be seen from Table 1, the order of the fuzzy entropy values obtained from the fuzzy entropies is consistent for the three different IVIFNs, which can prove the correctness as well as the validity of the proposed formula. However, the form of the data is different, and for interval values, the information loss that translated to interval values is less than it is converted to exact values.

5 Conclusion

In this paper, according to the disadvantages of existing IVq-ROFSs fuzzy entropy, the IVFE on IVq-ROFSs axiomatic definitions is proposed, and the corresponding fuzzy entropy function image is drawn and the fuzzy entropy values are interpreted accordingly by the graphs, and finally the corresponding formulas are given according to the axiomatic definition, and the rationality, validity and less information loss than traditional fuzzy entropy of the proposed fuzzy entropy are proved by a simple example.

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