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Interval-Valued Fermatean Fuzzy Multi-Attribute Group Decision-Making Method with a Consensus Mechanism

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Abstract. Interval-valued Fermatean fuzzy sets are a new powerful tool for dealing with uncertainty problems. However, group decision-making methods under this new model are rarely studied. Therefore, this paper proposes a group decision-making method with a consensus mechanism. First, this paper proposes a new score function. The advantages of the score function are proved through comparative analysis. Secondly, a consensus-reaching method is proposed based on the new score function, and in order to shorten the consensus-reaching time, the preference approval structure is combined with the consensus-reaching process. Finally, this paper proposes a multi-attribute group decision-making framework with a consensus process in interval-valued Fermatean fuzzy environment.

Keywords. Interval-valued Fermatean fuzzy sets, Consensus, Score Function.

1. Introduction

It is a significant challenge to deal with uncertain and fuzzy data in real-world applications [1, 2]. Q-rung orthopair fuzzy sets [3, 4] which can handle uncertainty. The area of the acceptable orthopairs will increase as q rises. There are two examples when q = 1 and q = 2, respectively: intuitionistic fuzzy sets [5] and Pythagorean fuzzy sets [6]. Q-rung orthopair fuzzy sets when q=3 were also referred to as Fermatean fuzzy sets (FFSs) and were suggested by Senapati and Yager [7]. Jeevaraj et al. proposed the notion of interval-valued Fermatean fuzzy sets (IVFFSs) [5] by extending FFSs. When the upper bounds and lower bounds of membership degrees of (IVFFSs) are equal and the upper bounds and lower bounds of non-membership degrees are equal, IVFFSs can be transformed into FFSs. Thus, FFSs are a special case of IVFFSs, which are a crucial type of q-rung orthopair fuzzy sets. The difference between the IVFFSs and FFSs is that the FFSs are a set of membership degrees and non-membership degrees, while IVFFSs are a set of membership degree intervals and non-membership degree intervals. The same is true that FFSs satisfy the sum of the power of membership and the power of non-membership belonging to [0, 1], and the IVFFSs also satisfy the sum of the upper bounds power of membership degrees and the upper bounds power of non-membership degrees belonging to [0, 1]. the IVFFSs involve the advantages of FFSs and have greater flexibility in dealing with fuzzy and imprecise information. As a result, IVFFSs-based multi-

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attribute decision-making techniques were developed. The multi-attribute group decision-making (MAGDM) problem has drawn a growing amount of interest due to the decision-making environment's complexity. Consensus building is one of the key research areas of MAGDM challenges because group decision-making has the benefit of being more comprehensive and compelling, but it also presents possible conflicts and inconsistencies among experts. Many consensus models have been put forth in the area of multi-attribute group decision-making lately [8, 9]. The current consensus research process (CRP) focuses on the five aspects listed below.

Models of consensus based on diverse preference structures. For instance, Herrera-Viedma et al.[10] offered a consensus model based on four assessment structures. Models of consensus with minimal adjustment, such as a two-stage consensus-building strategy based on minimal adjustment was presented by Long et al. [11]. Models of consensus that take non-cooperative behavior into account. For instance, a large-scale group decision-making model based on trust consensus was proposed by Xu et al. [12] in 2019. Large-scale group decision-making consensus model. Wu et al. [13] propose a community detection-based clustering method to manage different network setups with dynamic consensus thresholds. Social network consensus models. Wu et al. [14] proposed a distributed linguistic social network group decision-making consensus model based on the minimum adjustment cost.

There are some problems with the existing research: 1) The existing score function does not effectively compare any two IVFFNs. 2) In a multi-attribute group decision-making environment, consensus should measure the level of agreement among experts based on the outcomes of the decision-making process in accordance with the characteristics of the multi-attribute decision-making problem rather than just taking into account the consistency of the assessments made directly by the experts. 3) The decision-makers in the group should comprehend at least some of the solutions. The experts should have faith in the alternatives they are familiar with, allowing the process of consensus feedback to focus solely on adjusting the evaluation of the alternatives they are unfamiliar with.

This essay proposed three crucial contributions: 2) A new consensus-reaching approach based on IVFFSs is proposed. 1) A new scoring function based on IVFFSs is proposed. 3) Based on the consensus method, a multi-attribute group decision-making approach with interval-valued Fermat fuzzy is developed. The remainder is structured as follows: A few concepts are introduced in Section 2, the proposed score function is introduced and proven in Section 3, the suggested consensus framework is introduced in Section 4, this paper is conclusion in Section 5.

2. Preliminaries

Some basic concepts are introduced in this section.

2.1. Interval-valued Fermatean Fuzzy Sets

Definition 2.1.[15] Let $\mathrm{S}[0,1]$ be a set of all closed sub-intervals of the interval $[0,1]$. An interval-valued Fermatean fuzzy sets (IVFFSs) on a set $\mathrm{X} \neq \emptyset$ is an expression given by $\mathrm{F} = \{\langle x, \mu(x), \nu(x)\rangle : x \in X\}$ where $\mu(x): X \to S[0,1]$ and $\mu(x): X \to S[0,1]$ are closed intervals, F can also be expressed as follows: $\mathrm{F} = \{\langle x, [\mu_F^L, \mu_F^U], [\nu_F^L, \nu_F^U]\rangle : x \in X\}$, where $[\mu_F^L, \mu_F^U]$denote the lower and upper bounds of the membership respectively,and $[\nu_F^L, \nu_F^U]$ denote the lower and upper bounds of the non-membership $0 \leq {\mu_F^U}^3 + {\nu_F^U}^3 \leq 1$. For each element $x \in X$, $F1 = [\mu_{F1}^L, \mu_{F1}^U], [\nu_{F1}^L, \nu_{F1}^U]$ is also called an interval-valued Fermatean fuzzy number (IVFFN).

Definition 2.2. Let $F_1 = \left(\left[\mu_{F_1}^L, \mu_{F_1}^U\right], \left[\nu_{F_1}^L, \nu_{F_1}^U\right]\right)$ and $F_2 = \left(\left[\mu_{F_2}^L, \mu_{F_2}^U\right], \left[\nu_{F_2}^L, \nu_{F_2}^U\right]\right)$ be any two IVFFNs, The definition of a subset $\mathrm{S}_1 \subset \mathrm{IVFFN}$ with relation $\subseteq$ is $\mathrm{F}_1 \subseteq \mathrm{F}_2$ if $\mu_{F_1}^L \leq \mu_{F_2}^L$, $\mu_{F_1}^U \leq \mu_{F_2}^L$, $\nu_{F_1}^L \geq \nu_{F_2}^L$ and $\nu_{F_1}^U \geq \nu_{F_2}^U$.

Definition 2.3. [16] Assuming $\mathrm{F_i} = \left([\mu_{F_i}^L, \mu_{F_i}^U], [\nu_{F_i}^L, \nu_{F_i}^U]\right), (i = 1,2, \dots, n)$ is a set of IVFFNs,w_i is the weight of F_i,and $\sum_{i=1}^n w_i = 1$, then the IVFFWG operator can be given as follow:

$$IVFFWG(F_1, F_2, \dots, F_n) = \oplus_{i=1}^n {F_i}^{w_i} \tag{1}$$

2.2. The preference-approval structures

The preference-approval structures were initially proposed by Brams and Sanver [17]. Assume that the people give the alternatives in a precise positional order, such as $x_2 > x_1$ or $x_1 > x_2$, for alternativesx_1 and x_2. Each person uses a bar to discern between the approved and non-approved alternatives, depending on the positional ordering of the alternatives.

3. The newly proposed score function

This section proposes a brand-new score function.

3.1. Score Function

Definition 3.1. Let $\mathrm{F}_1 = \left([\mu_{F_1}^L, \mu_{F_1}^U], [\nu_{F_1}^L, \nu_{F_1}^U]\right) \in \mathrm{IVFFN}$. Then the score function S for F_1 is defined as follow:

$$S(\mathrm{F}_1) = \frac{-1 + 2{\mu_{F_1}^L}^3 + 2{\mu_{F_1}^U}^3 - {\nu_{F_1}^L}^3 - {\nu_{F_1}^U}^3 - {\nu_{F_1}^L}^3 \times {\nu_{F_1}^U}^3}{3} \tag{2}$$

Theorem 3.1. Let $\mathrm{F}_1, \mathrm{F}_2 \in \mathrm{IVFFN}$. if $\mathrm{F}_1 \subseteq \mathrm{F}_2$, then $\mathrm{S}(\mathrm{F}_1) \leq \mathrm{S}(\mathrm{F}_2)$.

Proof. If $\mathrm{F}_1 \subseteq \mathrm{F}_2 \Rightarrow \mu_{F_1}^L \leq \mu_{F_2}^L, \mu_{F_1}^U \leq \mu_{F_2}^U, \nu_{F_1}^L \geq \nu_{F_2}^L, \nu_{F_1}^U \geq \nu_{F_2}^U$, we have

$$S(F_1) - S(F_2) = \frac{2\left({\mu_{F_1}^L}^3 - {\mu_{F_2}^L}^3\right) + 2\left({\mu_{F_1}^U}^3 - {\mu_{F_2}^U}^3\right) + \left({\nu_{F_2}^L}^3 - {\nu_{F_1}^L}^3\right) + \left({\nu_{F_2}^U}^3 - {\nu_{F_1}^U}^3\right) + \left({\nu_{F_2}^L}^3 \times {\nu_{F_2}^U}^3 - {\nu_{F_1}^L}^3 \times {\nu_{F_1}^U}^3\right)}{3} \leq 0$$

Thus $S(F_1) \leq S(F_2)$.

Proposition 3.1. For any $\mathrm{F} = \left([\mu_{F_1}^L, \mu_{F_1}^U], [\nu_{F_1}^L, \nu_{F_1}^U]\right) \in \mathrm{IVFFN}, \mathrm{S}(\mathrm{F}) \in \left[-\frac{4}{3}, 1\right]$.

Proof. Because $0 \leq \mu_{F_1}^L \leq \mu_{F_1}^U \leq 1$, $0 \leq \nu_{F_1}^L \leq \nu_{F_1}^U \leq 1$, we have $2{\mu_{F_1}^L}^3 + 2{\mu_{F_1}^U}^3 \in [0,4]$, ${\nu_{F_1}^L}^3 + {\nu_{F_1}^U}^3 + {\nu_{F_1}^L}^3 \times {\nu_{F_1}^U}^3 \in [0,3]$, then it can be seen that $S(\mathrm{F}_1) \in \left[-\frac{4}{3}, 1\right]$.

3.2. Comparing existing score functions

This section will compare the proposed score function with the existing scoring function. We compared the three score functions in [15, 18, 19], defined as follows, respectively. The comparison results are shown in Table 1.

$$J_M(F_1) = \frac{1}{2}\left({\mu_{F_1}^L}^3 + {\mu_{F_1}^U}^3 - {\nu_{F_1}^L}^3 - {\nu_{F_1}^U}^3\right) \tag{3}$$

$$R_S(F_1) = \frac{1}{2}\left(\left({\mu_{F_1}^L}^3 - {\nu_{F_1}^L}^3\right)\left(1 + \sqrt[3]{1 - {\mu_{F_1}^L}^3 - {\nu_{F_1}^L}^3}\right) + \left({\mu_{F_1}^U}^3 - {\nu_{F_1}^U}^3\right)\left(1 + \sqrt[3]{1 - {\mu_{F_1}^U}^3 - {\nu_{F_1}^U}^3}\right)\right) \tag{4}$$

$$P_S(\mathrm{F}_1) = \frac{1}{2}\left({\mu_{F_1}^L}^3 + {\mu_{F_1}^U}^3 + {\mu_{F_1}^L}^3\sqrt{1 - {\nu_{F_1}^L}^3} + {\mu_{F_1}^U}^3\sqrt{1 - {\nu_{F_1}^U}^3}\right) \tag{5}$$

Table 1. Comparison with existing score functions

	$\mathbf{A_1 = ([0.2, 0.3], [0.2, 0.3])}$ $\mathbf{A_2 = ([0.3, 0.4], [0.3, 0.4])}$	$\mathbf{A_1 = ([0, 0], [0.2, 0.3])}$, $\mathbf{A_2 = ([0, 0], [0.3, 0.4])}$	**comparability**
[15]	$J_M(A_1) = 0$， $J_M(A_2) = 0$	$J_M(A_1) = -0.017, J_M(A_2) = -0.046$	No
[18]	$R_S(A_1) = 0, R_S(A_2) = 0$	$R_S(A_1) = -0.035, R_S(A_2) - 0.09$	No
[19]	$P_S(A_1) = 0.2659, P_S(A_2) = 0.3898$	$P_S(A_1) = 0, P_S(A_2) = 0$	No
Proposed	$S(A_1) = -0.322, S(A_2) = -0.304$	$S(A_1) = -0.345, S(A_2) = -0.364$	Yes

4. New Consensus Framework

This section proposes a consensus framework to resolve the MAGDM problem.

4.1. Proposed consensus framework

MAGDM problem description: $E = \{e_1, e_2, e_3, \ldots, e_l\}$ is a set of experts, $\varpi = (\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_l)$is a weight vector of associated experts, $X = \{x_1, x_2, x_3, \ldots, x_n\}$ is a set of alternatives, $C = \{c_1, c_2, c_3, \ldots, c_m\}$ is a set of attributes, $W = (w_1, w_2, w_3, \ldots, w_m)$ is a weight vector of associated attributes, $D^{tk} = \left(x_{ij}^{tk}\right)$ is the decision matrix provided by the expert e^k at state t, and $D^t = \left(x_{ij}^t\right)_{n\times m}$ is the collective decision matrix of all experts.

The consensus index is defined as follow:

$$\mathrm{GIL}^t = \phi \frac{\sum_{k=1}^{l}\sum_{i=1}^{n}\sum_{j=1}^{m} S_{GE}\left(x_{ij}^{0k}, x_{ij}^{0}\right)}{n+m+l} + (1-\phi)\frac{\sum_{k=1}^{l}\sum_{i=1}^{n}\sum_{j=1}^{m} S_{GE}\left(x_{ij}^{tk}, x_{ij}^{t}\right)}{n+m+l} \tag{6}$$

The $S_{GE}\left(x_{ij}^{0k}, x_{ij}^{0}\right)$ represents the level of consensus among experts in the initial state. $S_{GE}\left(x_{ij}^{tk}, x_{ij}^{t}\right)$represents the level of consensus among experts after the t- time adjustment. A higher value GIL^t denotes higher agreement among the expert group. Ideally, when all experts reach absolute agreement, the consensus index GIL^t equals 1, but it is almost impossible for independent experts to reach complete agreement. Therefore, given an acceptable consensus threshold θ, if GIL^t is greater than θ, the expert group reaches an acceptable consensus level, which is also referred to as a "soft consensus".

- The selection process

The selection stage is the first phase of the CRP. The expert who needs to be changed is chosen when the expert group's consensus falls short of an acceptable consensus threshold. The expert who has to be changed should be the one who makes the least overall contribution to the expert group's consensus. According to the ranking results, an expert whose ranking results deviate too much from the expert group's overall rankings has made the least contribution to the expert general consensus, and their evaluation should be revised.

The selection alternatives are similar methods to selecting experts; i.e. the alternatives that deviate the most from the overall ranking contribute the least to the overall consensus and need to be adjusted. The two selection methods differ in that the experts who need to be adjusted are chosen based on the set of all experts, while the alternatives who need to be adjusted are chosen based on a subset of the alternatives set. The reason for this is that each expert chosen to make a decision is more or less competent, and at least some of the assessments given are credible, so only untrustworthy assessments need to be adjusted. For instance, if there are four alternatives, x_1, x_2, x_3, and x_4, and expert e^2 has enough knowledge of alternative x1 and alternative x_2, but little to no knowledge of x_3 and x_4, if expert e^2 is determined to adjust, only alternative x_3 and alternative x_4 evaluation need to be adjusted. This study introduces the concept of the preference approval structure based on that idea. This paper's approval structure denotes that experts provide unreliable assessments since they lack adequate knowledge of the alternatives; hence, it needs to be further adjusted. In the non-approval structure, the expert's assessment is reliable and doesn't need adjustment. The adoption of the preference approval structure cuts down on CRP time, a benefit that will become more apparent when there are more than enough alternatives.

- The feedback adjustment process

The feedback stage is the second stage of CRP. The second stage of CRP is the feedback stage. The experts who need to be adjusted and the associated alternatives are identified after the first step. To improve the level of consensus, the evaluation of these experts needs to be modified to a certain extent. Therefore, the first issue that needs to be addressed in the feedback stage is how to determine the direction of the adjustment.

4.2. Group Multi-attribute Decision Making Method based on Consensus

The specific consensus process is as follows:
Step1: Calculation of individual ranking and collective ranking results.

Aggregation to obtain a comprehensive decision matrix $D^t = \left(x_{ij}^t\right)_{n\times m}$. where x_{ij}^t denotes collective evaluation of alternative x_i under attribute c_j at time t.

Calculation of the score matrix. The decision matrix of individual experts and the collective decision matrix of all experts are transformed into score matrices, which are denoted as $P^{tk} = \left(p_{ij}^{tk}\right)_{n\times m}$ and $P^t = \left(p_{ij}^t\right)_{n\times m}$, where $p_{ij}^{tk} = S\left(x_{ij}^{tk}\right)$, $p_{ij}^t = S\left(x_{ij}^t\right)$.

Performing the individual ranking and collective ranking. Separately calculate each expert's score for each alternative.$p_i^{tk} = \sum_{j=1}^{m} w_j S\left(x_{ij}^{tk}\right), (i = 1,2,\dots,n)$. where p_i^{tk}denotes the expert e^k's score on alternative x_i.

For each alternative, determine a collective score.$p_i^t = \sum_{j=1}^{m} w_j S\left(x_{ij}^t\right), (i = 1,2,\dots,n)$. where p_i^t denotes the collective score on alternative x_i.

The ranking results of each expert and the group of experts as a whole are then generated by descending sorting according to score, which is denoted by $PO^{tk} = \left\{o_1^{tk}, o_2^{tk}, o_3^{tk}, \dots, o_r^{tk}\right\}$, (k = 1,2,…l)and $CO^t = \{o_1^t, o_2^t, o_3^t, \dots, o_r^t\}$, respectively, where r represents the ranking place.

Step2: Identification of adjustment experts. Comparing PO^{tk}, CO^t determines the difference matrix DG^t.

$$DG^t = (g_{kr}^t)_{l\times n} = m - r, if\, o_r^{tk} = o_m^{tc}, (k = 1,2,\dots l, r = 1,2,\dots n) \tag{7}$$

then chooses the experts who need to be adjusted. g_{kr}^t represents the difference in the position of the alternative in the ranking of experts e^k and the ranking of the expert group.

According to the difference matrix, experts NS^t are chosen who provide insufficient contributions to the collective ranking.

$$NS^t = \left\{e_k | \max \sum_{r=1}^{n} \left|g_{kr}^t\right|\right\} \tag{8}$$

Step3: Selecting the structure for preference approval.

Based on the previously mentioned concepts, the following division method is used to divide the alternatives into two groups: those whose ranking difference between the experts and the expert group exceeds the overall mean value of the difference matrix and those who are divided into a approval structure, indicating that their evaluation does require further adjustment.

$$AP_k^t = \left\{x_i \middle| \left|g_{ki}^t\right| \geq \frac{\sum_{i=1}^{n} \left|g_{ki}^t\right|}{n}\right\}, NAP_k^t = \left\{x_i \middle| \left|g_{ki}^t\right| < \frac{\sum_{i=1}^{n} \left|g_{ki}^t\right|}{n}\right\} \tag{9}$$

Where AP_k^t denotes the approved structure of expert e^k, and NAP_k^t denotes the disapproved structure of expert e^k.

Step4: Feedback adjustment

The selection stage is completed and the feedback adjustment stage is entered. In order to continually close the gap between the adjusted evaluation and the collective evaluation, this study applies the distance measurement approach. Furthermore, as the number of adjustments rises, the adjustment variables gradually close toward 0. We connected adjustment variables with the frequency of the consensus process. Assuming that e^k is the expert who needs to adjust, alternative x_i is separated into a set of approved structures of expert e^k. the feedback regulation is proposed as follows:

$$x_{ij}^{(t+1)k} = \left(\begin{bmatrix} \mu^L_{x_{ij}^{(t+1)k}} = \mu^L_{x_{ij}^{tk}} \pm \frac{D_{GE}(x_{ij}^{tk}, x_{ij}^{tc})}{t+1} \\ \mu^U_{x_{ij}^{(t+1)k}} = \mu^U_{x_{ij}^{tk}} \pm \frac{D_{GE}(x_{ij}^{tk}, x_{ij}^{tc})}{t+1} \end{bmatrix}, \begin{bmatrix} \nu^L_{x_{ij}^{(t+1)k}} = \nu^L_{x_{ij}^{tk}} \pm \frac{D_{GE}(x_{ij}^{tk}, x_{ij}^{tc})}{t+1} \\ \nu^U_{x_{ij}^{(t+1)k}} = \nu^U_{x_{ij}^{tk}} \pm \frac{D_{GE}(x_{ij}^{tk}, x_{ij}^{tc})}{t+1} \end{bmatrix} \right) \tag{10}$$

It is important to keep in mind that the adjusted evaluation value must satisfied to IVFFN limits during the feedback adjustment procedure. If the consensus level is not satisfied, the consensus process will continue until the acceptable consensus threshold or the Max_circles is reached, whichever occurs first, and then the consensus process is concluded.

5. Conclusion

The consensus method proposed in this paper can save the consensus reaching process time quickly and effectively, and retain the initial information of the expert group to the greatest extent. The method proposed in this paper has certain advantages in small-group decision-making, but it needs to be further expanded if it is to be applied to large-group decision-making. Therefore, the future work direction is to apply the method proposed in this paper to large-group decision-making.

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