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The Similarity Measurement of Normal Cloud Concept Based on Hellinger Distance and Expectation Curve with Entropy

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Abstract. Uncertainty exists widely in nature and human society. The handling and analysis of these uncertain phenomena have long been a focal point of research in natural sciences, representing a hot and crucial topic. The cloud model uses three numerical characteristics to realize qualitative and quantitative transformation of uncertain knowledge. Normal cloud is one of the more important and common cloud models. The similarity measurement of cloud model is used to judge the similarity of two cloud concepts, and the purpose is to understand the correlation between different cloud concepts, so as to improve the efficiency of classification and clustering. As a deformation extension of f divergence, Hellinger distance has good properties and is suitable for describing the difference between two probability distributions. HECMk and HCCMk algorithms are proposed by combining Hellinger distance and expectation curve cluster with entropy. Compared with previous algorithms, these two algorithms have higher distinguishing ability. Among them, HCCMk algorithm has better comprehensive performance and is feasible in actual similarity measurement.

Keywords. normal cloud, similarity measure, Hellinger distance, expectation curve cluster with entropy

1. Introduction

Uncertainty exists widely in nature and human society. How to deal with and analyze these uncertain things has been widely concerned by natural science research. To address the existence of fuzziness and randomness, the theory of probability and fuzzy mathematics are often combined. Using three numerical characteristics to quantitatively describe a qualitative concept, Li [1] proposed a mathematical model for qualitative and quantitative transformation of uncertain knowledge using cloud model. Normal cloud is one of the most important and commonly used cloud models, as many real-world phenomena approximate normal distribution, and it has good mathematical properties.

Similarity measurement is used to judge the degree of similarity between two research objects. In practical applications, cloud models often need to compare and

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classify different cloud concepts in order to understand the correlation between different cloud concepts, so as to improve efficiency in classification and clustering. Therefore, constructing an effective similarity measurement method can not only reduce the computational complexity, but also improve the running efficiency of the cloud model.

There are many methods to measure normal cloud similarity. Zhang [2] proposed to use the included Angle cosine method to obtain the similarity measure of cloud concept, which has a good feature of low computational complexity. However, the differentiation ability of this method becomes weak. Li [3] proposed that the similarity is constructed by the area of the overlapping part of the expected curve of the cloud concept, which has strong differentiation ability, but the calculation complexity is high, and the influence of the cloud model hyper entropy is not considered. Wang [4] proposed a comprehensive similarity calculation framework, which combined the numerical characteristics and shape similarity calculation formulas of the cloud model, but also resulted in high computational complexity. Xu [5] proposed using Hellinger distance to calculate the similarity between cloud concepts, and using expectation curve, Internal and external envelope curves to measure the distance between normal cloud concepts. F-divergence is a function widely used in probability statistics to measure the difference of probability distribution. Hellinger distance is the deformation extension of such divergence, which has good properties and can be used to describe the difference between two probability distributions. However, the expectation curve, Internal and external envelope curves are only a few special cases of the expected curve with entropy, and there is no general study. the influence of hyper entropy is not taken into account when calculating the similarity of expected curve.

2. Normal Cloud Concept and Similarity

2.1. Cloud Model and Normal Cloud

Let U be a quantitative discourse domain represented by an exact number, and A is a qualitative concept on U. If the quantitative value $x \in U$, and x is a one-time random implementation of the qualitative concept A, and the certainty of x to A $\mu_A(x) \in [0,1]$ is a random number with stable tendency, then the distribution of $(x, \mu_A(x))$ on the discourse domain U is called a cloud, and each $(x, \mu_A(x))$ is called a cloud droplet [6].

The numerical characteristics of the cloud are represented by three parameters (Ex, En, He) , called the cloud model (Ex, En, He) . Ex is the expectation of the cloud model, which describes the expected value of the spatial distribution of cloud droplets in the discourse domain, represents the central position of qualitative concepts, and reflects the basic metric value of a certain concept. En is the entropy of the cloud model, which describes the span of the cloud, represents the acceptance range of a qualitative concept, and reflects the fuzziness and randomness of the concept. He is the hyper entropy of the cloud model, which is the entropy of entropy. It describes the thickness of the cloud and reflects the uncertainty of entropy [7,8].

Definition 2.1([1]) If the random variable x satisfies: $x \sim N(Ex, En'^2)$, where $En' \sim N(En, He^2)$ the certainty of the qualitative concept A satisfies:

$$\mu_A(x) = e^{-\frac{(x-Ex)^2}{2En'^2}} , \tag{1}$$

then the distribution of x over the domain U is called a normal cloud, where En' is a normal random number with En as the expectation and He as the standard deviation. Normal cloud mainly realizes the mutual conversion between qualitative concepts and quantitative values through normal cloud transformation, in which the forward normal cloud transformation converts the numerical characteristics $C(Ex, En, He)$, which represents the connotation of concepts, into quantitative values.

2.2. Expectation Curves with Entropy of Normal Cloud

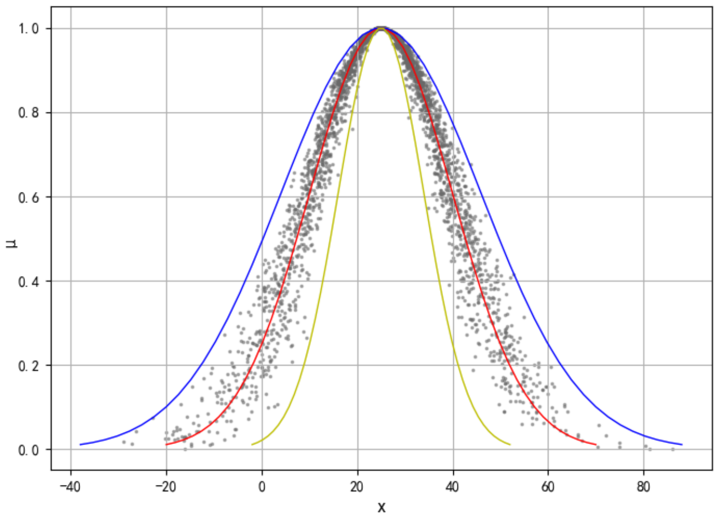

Figure 1. Graph of expectation curve with entropy for the normal cloud model

Definition 2.2([9]) If cloud droplet x is satisfied $x \sim N(Ex, En'^2)$, $En \neq 0$ and $En' \sim N(En, He^2)$, then

$$y_k = \exp[-(x-Ex)^2 / (2(En+kHe)^2)], \quad (2)$$

is a cluster of expected curves with entropy for cloud (Ex, En, He). Where k is an adjustable parameter, and the conventional value range is [-3, 3]. If $k=0$, the curve degenerates into the expected curve of the cloud mode $y=\exp[-(x-Ex)^2/(2En^2)]$. The inner envelope curve and the outer envelope curve are two cases of $k=-3$ and $k=3$, respectively. According to the 3σ rule, almost all cloud droplets are between the inner envelope curve and the outer envelope curve, and fluctuate around the expected curve. Therefore, the geometric characteristics of the cloud can be directly reflected through the entropy expected curve cluster of the cloud model [10,11].

2.3. Normalization of Expectation Curves with Entropy

According to literature [12], there is no analytic solution to the probability density of second-order normal clouds, so the analytic expression of distance cannot be obtained directly by using the probability density. The normal cloud can also be characterized by the feature curve, and the geometric properties of the original cloud concept will not be changed when the feature curve is scaled according to its uncertainty characteristics (entropy and hyper entropy), so the entropy expected curve cluster of the normal cloud is used to calculate the distance. First, the characteristic curves are normalized by the corresponding coefficients respectively, and the general formula of the expectation curve cluster with entropy is obtained:

$$p_k(x) = \frac{1}{\sqrt{2\pi}\left|En + kHe\right|}\exp\left\{-\frac{(x-Ex)^2}{2(En + k\cdot He)^2}\right\}, \tag{3}$$

the value range of k is [-3, 3]. The Hellinger distance of the expected curve with entropy under different k values can be obtained easily by this general formula.

2.4. The Hellinger Distance between Two Normal Distributions

Hellinger distance is a measure of the amount of overlap between two statistical samples or populations. In probability statistical theory, Hellinger distance is often used to measure the similarity of two probability distributions [13].

Definition 2.3([5]) For two normal distributions $P \sim N(\mu_1, \sigma_1^2)$ and $Q \sim N(\mu_2, \sigma_2^2)$, their Hellinger distance is :

$$D_H(P,Q) = \sqrt{1 - \sqrt{\frac{2\sigma_1\cdot\sigma_2}{\sigma_1^2 + \sigma_2^2}}\cdot\exp\left\{-\frac{(\mu_1 - \mu_2)^2}{4(\sigma_1^2 + \sigma_2^2)}\right\}}, \tag{4}$$

it can be seen from the above that for any two normal distributions, the Hellinger distance can be converted into an algebraic operation of expectation and variance without the need for integral operation, which will greatly reduce the computational complexity.

3. Similarity Algorithm Based on Hellinger Distance and Expected Curve Clusters with Entropy

3.1. Similarity of Expected Curves with Entropy at Hellinger Distance

Definition 3.1 Suppose U is a quantitative theoretical domain represented by an exact numerical value, $C_1(Ex_1, En_1, He_1)$ and $C_2(Ex_2, En_2, He_2)$ are two normal cloud concepts on U, then the Hellinger distance based on the expectation curve cluster with entropy is:

$$D_{Hk}(C_1,C_2) = \sqrt{1 - \sqrt{\frac{2\sigma_1\cdot\sigma_2}{\sigma_1^2 + \sigma_2^2}}\cdot\exp\left\{-\frac{(Ex_1 - Ex_2)^2}{4(\sigma_1^2 + \sigma_2^2)}\right\}}, \tag{5}$$

where $\sigma_1 = \left|En_1 + kHe_1\right|$, $\sigma_2 = \left|En_2 + k\cdot He_2\right|$, $k \in [-3,3]$ further according to the transformation relationship between distance and similarity, the similarity measure of the second order normal cloud concept can be obtained. It can be seen from the above that the value range of Hellinger distance is between [0, 1], and in the calculation, the distance often approaches 1, which makes the similarity basically 0, which makes it difficult to compare the similarity difference between different concepts. Therefore, the similarity measurement based on the expected curve cluster containing entropy is as follows:

$$Sim_{Hk}(C_1,C_2) = 1 - D_{Hk}(C_1,C_2)^2, \tag{6}$$

the similarity measure satisfies the following properties, here, the similarity is simply referred to as $Sim_H(C_1,C_2)$:

(1) $Sim_H(C_1,C_2) = Sim_H(C_2,C_1)$,

(2) $0 < Sim_H(C_1, C_2) < 1$,

(3) when $C_1 = C_2$, $Sim_H(C_1, C_2) = 1$.

Proof :(1) $Sim_H(C_1,C_2) = 1 - D_H(C_1,C_2)^2 = 1 - D_H(C_2,C_1)^2 = Sim_H(C_2,C_1)$.

(2) Since $0 \le 2\sigma_1 \cdot \sigma_2 / (\sigma_1{}^2 + \sigma_1{}^2) \le 1$ where $(\sigma_1 > 0, \sigma_2 > 0)$,and when $x \in [0, +\infty)$, exponential function $0 < e^{-x} \le 1$,there is $0 \le D_{Hk}(C_1, C_2) \le 1$. Therefore the similarity is satisfied $0 < Sim_H(C_1, C_2) \le 1$.

(3) $Ex_1 = Ex_2$, $\sigma_1 = \sigma_2$ is satisfied if and only if $C_1 = C_2$,in which case $D_{Hk}(C_1, C_2) = 0$, there is $Sim_H(C_1, C_2) = 1$.

3.2. Similarity Algorithm

Based on the above theory, different values of *k* are set to obtain the similarity of the expected curve clusters containing entropy, and the similarity of the combination is calculated by weighted summation, which reflects the distribution characteristics of the whole cloud concept. Based on this, the following similarity algorithm is designed:

Algorithm 1: HECMk algorithm

Input: numerical characteristics $C_1(Ex_1, En_1, He_1)$, $C_2(Ex_2, En_2, He_2)$ and distance *k* value.

Output: similarity: $Sim_{HECMk}(C_1, C_2)$.

(1) Calculate the expected curve with entropy based on Hellinger distance $D_H(C_1, C_2)$ at corresponding *k* value.

(2) Computational similarity: $Sim_{HECMk}(C_1, C_2) = 1 - D_H(C_1, C_2)^2$.

Algorithm 2: HCCMk algorithm

Input: numerical characteristics $C_1(Ex_1, En_1, He_1)$, $C_2(Ex_2, En_2, He_2)$ and distance *k* value.

Output: similarity $Sim_{HCCMk}(C_1, C_2)$.

(1) For different *k* values, the Hellinger distance $D_{Hk}(C_1, C_2)$ of the expected curve with entropy is calculated respectively, where the value of *k* is between [-*t*, *t*], *t* ∈ [0, 3], and:

$$D_{HCCMk}(C_1, C_2) = \frac{1}{2t}\int_{-t}^{t}(C_1, C_2)dk \,. \tag{7}$$

(2) Computational similarity $Sim_{HCCMk}(C_1, C_2) = 1 - D_{HCCMk}(C_1, C_2)^2$.

These two algorithms retain the good properties of Hellinger distance, and fully consider the numerical characteristics of the cloud model. At the same time, through the derivation of the formula, it can be seen that they have less computational complexity. In addition, on the basis of calculating the Hellinger distance of the expectation curve cluster with entropy, the HCCMk algorithm finally obtains the similarity by weighted average, fully considering the randomness of the cloud concept caused by hyper entropy He in the cloud model, and can also measure the similarity between cloud concepts from a geometric point of view, which has a better comprehensiveness.

4. The Similarity Measurement of Normal Cloud Concept Based on Hellinger Distance and Expectation Curve with Entropy

4.1. Discriminative Ability of Cloud Concepts

For cloud concepts, if they belong to the same category, the distance will be smaller and the similarity will be higher. If they do not belong to the same class, then the distance will be greater and the similarity will be smaller [14]. The difference degree is used to measure the effective differentiation ability of different algorithms.

Definition 4.1 The difference degree of cloud concept C_i is defined as:

$$\delta_{C_i} = \sum_{s,t} \left| Sim(C_i, C_s) - Sim(C_i, C_t) \right| , \tag{8}$$

C_s represents a cloud concept of the same class as C_i, and C_t represents a cloud concept of a different class from C_i.

4.2. The Selection of k Value

This method mainly studies the feasibility and differentiation ability of HECMk and HCCMk algorithms. Four classical normal cloud concepts given in literature [2, 3] are used for numerical simulation experiments, and compared with LICM, ECM, MCM and PDCM algorithms. The four normal cloud concepts are: $C_1(1.5, 0.62666, 0.339)$, $C_2(4.6, 0.60159, 0.30862)$, $C_3(4.4, 0.75199, 0.27676)$ and $C_4(1.6, 0.60159, 0.30862)$, the corresponding cloud model diagram is shown in the Figure 2.

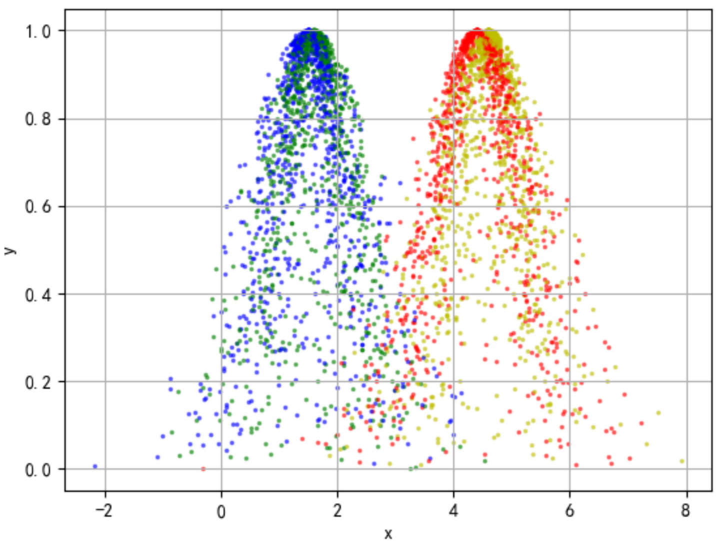

Figure 2. Cloud concept distribution map

The *k* value is [-3, 3], but only part of the value may be more appropriate in practice. Therefore, the experiment first takes a rough value to find out the *k* value with better results, and then takes a fine value to study the similarity measurement capability of expectation curve with entropy of the cloud model under this interval. First of all, in the experiment, the *k* value is [-3, 3], and the step length is 0.25. 25 values are roughly selected and substituted into the algorithm, and the results are shown in the Table 1.

Combined with the results of the comparison algorithm, the results obtained by HECMk algorithm for different k values are all similar to C_1 and C_4, C_2 and C_3, and similarity is $Sim(C_1,C_4)=Sim(C_2,C_3)$. In the experiment, with the change of k value, En and $-k \cdot He$ will be very close to each other, and then the similarity will be close to 0. Such results will be regarded as outliers in the experiment.

Table 1. Cloud concept similarity of HECMk algorithm under different k values

k value	$Sim(C_1,C_2)$	$Sim(C_1,C_3)$	$Sim(C_1,C_4)$	$Sim(C_2,C_3)$	$Sim(C_2,C_4)$	$Sim(C_3,C_4)$
-3	0.0001	0.0000	0.9819	0.6174	0.0000	0.0000
…	…	…	…	…	…	…
-1.75	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-1.5	0.0000	0.0000	**0.9215**	**0.7781**	0.0000	0.0000
-1.25	0.0000	0.0000	**0.9710**	**0.8684**	0.0000	0.0001
-1	0.0000	0.0010	**0.9852**	**0.9154**	0.0000	0.0018
-0.75	0.0002	0.0077	**0.9910**	**0.9423**	0.0003	0.0105
-0.5	0.0028	0.0270	**0.9938**	**0.9589**	0.0036	0.0326
-0.25	0.0146	0.0620	**0.9953**	**0.9698**	0.0167	0.0699
0	0.0414	0.1105	**0.9963**	**0.9771**	0.0447	0.1193
0.25	0.0833	0.1676	**0.9969**	**0.9823**	0.0870	0.1763
0.5	0.1361	0.2284	**0.9973**	**0.9861**	0.1396	0.2361
0.75	0.1949	0.2891	**0.9975**	**0.9889**	0.1977	0.2956
1	0.2553	0.3475	**0.9977**	**0.9910**	0.2572	0.3526
1.25	0.3144	0.4021	**0.9978**	**0.9926**	0.3154	0.4059
…	…	…	…	…	…	…
3	0.6197	0.6675	0.9982	0.9976	0.6174	0.6666

As can be seen from Table 1, the similarity distribution may have certain rules. In order to find a more suitable k value, the similarity results were continued to be substituted into formula 7 to calculate the difference degrees of the four cloud concepts under different k values, and the results were drawn into a line chart to obtain the results as shown in Figure 3.

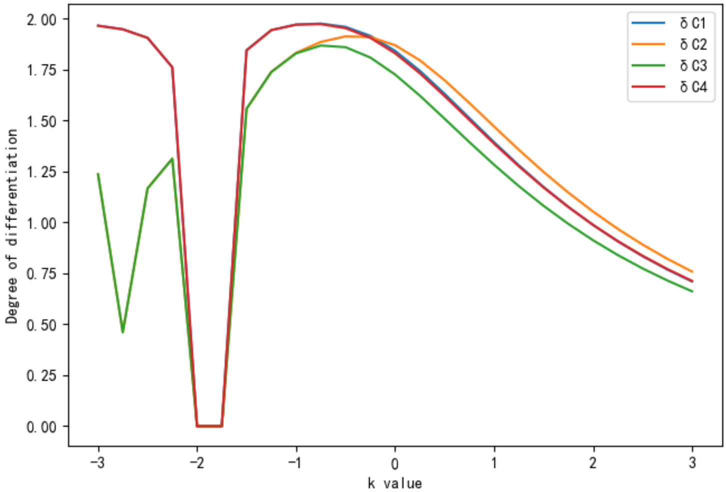

Figure 3. Difference degree curve of HECMk algorithm

The difference degree of HECMk algorithm under different k values is calculated, and the obtained results are drawn into a line chart to get the results as shown in the Figure 3. There are outliers of similarity in the above experiments, so 0 value also appears in the difference degree result Figure 3. It can be seen from the table that the difference degree curve as a whole approximately follows a normal distribution without considering the outliers. Therefore, we strive for the principle of symmetry, take a relatively stable interval, and find that the cluster of entropy-containing expected curves with k value between [-1,1] is better distinguished.

4.3. Distinguishing Ability of HECMk Algorithm and HCCMk Algorithm

In order to further study the expectation curve with entropy, the k value is again selected as [-1,1], and the value is taken as a step of 0.1. 21 values are selected to calculate the similarity of expectation curve with entropy. By changing the k value, a more refined cluster of expected curves containing entropy is obtained, and the corresponding difference degree is calculated according to the obtained similarity results. The results are shown in the Table 2

Table 2. Cloud concept difference degree of HECMk algorithm with different k values

k value	δ_{C_1}	δ_{C_2}	δ_{C_3}	δ_{C_4}
-1	1.9694	1.8307	1.8279	1.9686
-0.9	1.9735	1.8556	1.8491	1.9721
…	…	…	…	…
-0.1	1.8733	1.8886	1.7608	1.8619
0	1.8406	1.8682	1.7244	1.8285
0.1	1.8040	1.8423	1.6847	1.7915
…	…	…	…	…
1	1.4961	1.5724	1.3975	1.4872

Meanwhile, ECM, MCM, PDCM has the same result as HECMk algorithm, while the results of LICM show that the similarity of the four clouds is close, all above 0.95.

Table 3. Cloud concept similarity under different contrast algorithms

Similarity	$Sim(C_1,C_2)$	$Sim(C_1,C_3)$	$Sim(C_1,C_4)$	$Sim(C_2,C_3)$	$Sim(C_2,C_4)$	$Sim(C_3,C_4)$
LICM	0.9561	0.9648	**0.9990**	**0.9992**	0.9679	0.9755
ECM	0.0116	0.0356	**0.9370**	**0.8868**	0.0127	0.0389
MCM	0.3286	0.3688	**0.9714**	**0.9504**	0.3261	0.3680
PDCM	0.0110	0.0306	**0.8858**	**0.8000**	0.0126	0.0319
HCCMk	**0.0698**	**0.1275**	**0.9951**	**0.9714**	**0.0719**	**0.1339**

As can be seen from the Table 2, for the four cloud concepts, HECM has a high concept difference degree under different values, which can better highlight the difference degree among cloud concepts, indicating that the algorithm has a good differentiation ability. In combination with the Table 3, it can be seen that in the cloud concept difference degree results of the comparison algorithm, the concept difference degree of LICM is less than 0.8, which indicates that the method has poor differentiation ability. Both the ECM algorithm and the HECMk algorithm when $k=0$ adopt the expectation curve as the characteristic curve for studying cloud similarity, that is, the expectation curve with entropy if k value is 0, and the uncertainty effect of hyper entropy He on cloud concept is not taken into account. In the case of ignoring the hyper entropy, the HECMk algorithm has higher differentiation ability. In the case of hyper entropy, PDCM has better differentiation ability than MCM algorithm. HECMk algorithm with k

not equal to 0 has stronger differentiation ability and is better than PDCM and MCM algorithm as a whole.

Table 4. Difference degree of cloud concept under different similarity algorithms

Difference degree	δ_{C_1}	δ_{C_2}	δ_{C_3}	δ_{C_4}
LICM	0.0772	0.0744	0.0582	0.0546
ECM	1.8267	1.7492	1.6991	1.8224
MCM	1.2454	1.2462	1.1641	1.2487
PDCM	1.7301	1.5764	1.5376	1.7272
HCCMk	**1.8126**	**1.8174**	**1.7011**	**1.8043**

In contrast algorithm, ECM has the best distinction ability, but it does not take into account the case of hyper entropy, which is equivalent to comparing the similarity of normal distribution. When the hyper entropy is relatively large, that is, when the entropy En is less than three times the hyper entropy He or is close to it, the similarity of the comparison cloud model has its limitations. In addition, HECMk only carries out simple numerical calculation, without using integral operation, and the calculation complexity is far less than that of the comparison algorithm. Therefore, it can be considered that the expectation curve cluster with entropy with k value [-1, 1] has better distinguishing ability and comprehensive ability when a part of the distinguishing ability is abandoned in order to consider the hyper entropy.

After explaining the comprehensive ability of the expectation curve cluster with entropy, the HCCMk algorithm is further used to calculate the similarity. The k value is selected as [-1, 1], and the value is taken as a step of 0.1. 21 values are selected to calculate the Hellinger distance of the expected curve containing entropy. Further, the differentiation ability of HCCMk algorithm was calculated by combining the formula 8, and the results as shown in Table 4 were obtained.

As shown in the Table 4, the difference degree of HCCMk algorithm for the four cloud concepts is greater than that of PDCM algorithm , and is very close to that of ECM algorithm without considering overentropy. It shows that HCCMk has better distinguishing ability and comprehensive performance compared with ECM, MCM and PDCM. By comprehensive comparison, both HECMk algorithm and HCCMk algorithm have better performance, are feasible in measuring cloud concept similarity, and have less computational complexity than other methods.

5. Conclusions

HECMk algorithm and HCCMk algorithm were used to measure the similarity of four classical clouds, and LICM, ECM, MCM and PDCM algorithms were set as comparison experiments. The results show that in the normal cloud expectation curve cluster with entropy, when k is [-1, 1], the similarity measurement results are better and the overall differentiation ability is better. Without considering the influence of hyper entropy, HECMk algorithm ($k = 0$) is superior to ECM algorithm in distinguishing ability, and is suitable for cloud concept similarity measurement with low hyper entropy. Considering the influence of hyper entropy, the HECMk algorithm ($k \neq 0$) The overall differentiation ability is better than the MCM and PDCM algorithms in the comparison experiment, and the computational complexity is smaller than the two algorithms; The HCCMk algorithm synthesizes the expected curve clusters with entropy whose value is [-1, 1]. Compared with the ECM algorithm that does not take cloud concept uncertainty into account, the HCCMk algorithm has the same differentiation ability and fully considers the influence of overentropy, and the computational complexity is smaller. Therefore, in summary, HECMk algorithm and HCCMk algorithm have good distinguishing ability, feasibility of cloud concept similarity measurement, and low computational complexity.

Acknowledgments

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