Fuzzy Systems and Data Mining IX
A.J. Tallón-Ballesteros and R. Beltrán-Barba (Eds.)

doi:10.3233/FAIA231058

Restricted δ Lie Triple Systems

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Abstract. This article mainly introduces some structural properties of confined δ Lie triple systems are studied by means of the relation among the restricted Lie three systems, modular Lie algebras and δ Lie triple systems. Firstly, we verify by induction that the set of all linear transformations that restricted δ Lie triple systems satisfies the condition of restricted δ Lie triple systems. Next,the first mathematical induction method is used to verify that the generalized derivations and the quasiderivations are p-subsystems of the set of all linear transformations that restricted δ Lie three systems. Finally, it is proved that if the restricted δ Lie system has a trivial center, then the center is the commutator system of the generalized derivations.

Keywords. restricted δ Lie triple systems; δ Lie triple systems; generalized derivations; centroids

1. Introduction

The Lie triple systems first appeared in Cardan's research on riemannian geometry. As a generalization of the Lie triple systems, the concept of a δ Lie triple system is introduced in reference [1]. With the development of the Lie triple systems, the restricted Lie triple systems has also had a certain development [2],[3],[4]. In [5], they present some basic properties of a Lie triple system T, with the relationship $\mathrm{Der(T)} \subseteq \mathrm{QDer(T)} \subseteq \mathrm{GDer(T)} \subseteq \mathrm{End(T)}$. They show that the quasiderivations of T can be embedded as derivations in a larger Lie triple system. [6] mainly studies that T is a necessary condition for decomposability. Then, they study the structure theory of the center of form of Lie super triple system, and give some important properties of tensor products of Lie super triple system and associative algebras with identity elements. [7] gives some basic properties of the generalized derivation derivations, quasi-derivation derivations, core, quasi-core and central derivation derivations of the Jordan-Lie algebra. Peng Jianrong gives the generalized derivation of δ Lie three systems [8]. Cohomological characterizations of δ -Jordan Lie triple systems are established, then deformations, Nijenhuis operators, Abelian extensions and T^*-extensions of δ -Jordan Lie triple systems are studied using cohomology [9]. Liu Yanpei gives the structural properties limiting the Lie three systems [10].

In this paper, we extend the results of Peng Jianrong to restricted δ Lie triple systems, and mainly study some important properties of derivation algebras, quasiderivation algebras, generalized derivation algebras, type centers and central derivations. Assuming that the characteristic of the base field $\mathbb{F}$ is p, where p is a prime number greater than 2.

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2. Related works

Okubo S first proposed the concept of δ-lie triple systems. On the basis of [1], Peng Jianrong got the concept of δ-lie triple systems and the properties of their derivates [8]. Liu Yanpei got the related properties of limiting the derivations of Lie triple systems [9]. By referring to references [8], [9] and [10], I came to the conclusion that the focus of this paper is the concept of limiting δ-Lie triple systems and the properties of their derivations, which is also my innovation point.

3. Basic Conceptions

Definition 3.1. *[1] Supposing T is a vector space with ternary linear operations on the field $\mathbb{F}$ and satisfies:*
(1)$[z,h,r]=-\delta[h,z,r];\delta=\pm 1$
(2)$[z,h,r]+[h,r,z]+[r,z,h]=0;$
(3)$[z,h,[r,u,v]]=[[z,h,r],u,v]+[r,[z,h,u],v]+\delta[r,u,[z,h,v]],$
then T is called δ Lie three systems and T still known as Jordan Lie three systems. Note that δ is a Lie three system when $\delta=1$.

Definition 3.2. *Supposing T is the δ Lie three system on the field $\mathbb{F}$, if exists a mapping $[p]:T\to T,\forall d,k,c\in T,\alpha\in F$, the next conditions are satisfies:*
(1)$(\alpha d)^{[p]}=\alpha^p d^{[p]}$;
(2)$(d+k)^{[p]}=d^{[p]}+k^{[p]}+\sum_{i=1}^{p-1}s_i(d,k)$;
(3)$[d,k^{[p]},c]=(d,k,\cdots,k,c)$.
where is$s_i(d,k)$ is the coefficient of λ^{i-1} in $(\mathrm{ad}(\lambda d+k))^{p-1}(d)\in L_s(T)$, then $(T,[p])$ is called the Restricted δ Lie triple system, here $(d,k,\cdots,k,c)=[[[[d,k,k],k,k],\cdots],k,c]$.

A subspace ψ of a Restricted δ Lie triple systems T, if satisfied $[\psi,\psi,\psi]\subseteq\psi$ and for $\forall x\in\psi, x^{[p]}\in\psi$, then ψ is called a $p-$subsystem of T .

A subspace ω of a restricted δ lie triple systems T, if satisfied $[\omega,T,T]\subseteq\omega$,and for $\forall x\in\omega, x^{[p]}\in\omega$, then ω is called a $p-$ideal of T.

Definition 3.3. *Supposing T is the restricted δ lie triple system on the field $\mathbb{F}$. If exists a linear map $D:T\to T$ satisfies: $\delta^k([D(s),u,n])+\delta^k([s,D(u),n])+\delta^k([s,u,D(n)])=D([s,u,n]),\forall s,u,n\in T,\forall k\in Z$, thus $D:T\to T$ is called a $k-$step derivation of T, so we denote a set of the whole $k-$step derivations by* $\mathrm{Der_k(T)}$.

Definition 3.4. *Let T be the Restricted δ Lie triple system on the field $\mathbb{F}$. If exists $\mathrm{D}',\mathrm{D}'',\mathrm{D}'''\in\mathrm{End(T)}$ satisfies $\delta^k[D(a),b,c]+\delta^k[a,D'(b),c]+\delta^k[a,b,D''(c)]=D'''([a,b,c]),\forall a,b,c\in T$, thus $D\in\mathrm{End(T)}$ is called a $k-$step generalized derivation of T, then we denote the gather of overall $k-$step generalized derivations by* $\mathrm{GDer(T)}=\bigoplus_{k\geq 0}\mathrm{GDer(T)}$.

Definition 3.5. *Supposing T be the restricted δ lie triple system on the field $\mathbb{F}$. If exists $\mathrm{D}'\in\mathrm{End(T)}$ satisfies $\delta^k[D(f),k,o]+\delta^k[f,D(k),o]+\delta^k[f,k,D(o)]=D'([f,k,o])$, $\forall f,k,o\in T$, then $D\in\mathrm{End(T)}$ is called the $k-$step quasiderivation of T, we denote a gather of all $k-$step quasiderivations by* $\mathrm{QDer(T)}=\bigoplus_{k\geq 0}\mathrm{QDer(T)}$.

Definition 3.6. *Establish T be a Restricted δ Lie triple system on the field $\mathbb{F}$. If* $\mathrm{C_k(T)} = \{D \in \mathrm{End(T)} | \delta^k[D(f),k,o] = \delta^k[f,D(k),o] = \delta^k[f,k,D(o)] = D([f,k,o])\}, \forall f,k,o \in T$, *so* $\mathrm{C_k(T)}$ *is called the $k-$step centroid of T. Then we can denote the set of all $k-$step centroids by* $\mathrm{C(T)} = \bigoplus_{k\geq 0}\mathrm{C(T)}$.

Definition 3.7. *Establish a T be the restricted δ lie triple system on the field $\mathbb{F}$. If* $\mathrm{ZDer(T)} = \{D \in \mathrm{End(T)} | [D(e),g,p] = D([e,g,p]) = 0\}, \forall e,g,p \in T$, *so* $\mathrm{ZDer(T)}$ *can be called a central derivation of T.*

4. Main Results

Theorem 4.1. *Supposing T be a restricted δ Lie triple system, if* $\mathrm{End(T)}$ *is the set of all linear transformations of T, then* $\mathrm{End(T)}$ *with respect to the ternary linear operation* $[\cdot,\cdot,\cdot]$ *is a δ lie triple system.*

Proof. $\mathrm{End(T)}$ satisfies the three conditions of the δ lie triple system.
$\forall \mathrm{D_1,D_2,D_3,D_4,D_5} \in \mathrm{End(T)}$,
$(1)[D_1,D_2,D_3] = [[D_1,D_2],D_3] = -\delta[[D_2,D_1],D_3] = -\delta[D_2,D_1,D_3]$.

$$\begin{aligned}(2)&[D_1,D_2,D_3]+[D_2,D_3,D_1]+[D_3,D_1,D_2]\\ &= D_1D_2D_3 - D_2D_1D_3 - D_3D_1D_2 + D_3D_2D_1 + D_2D_3D_1 - D_3D_2D_1 - D_1D_2D_3\\ &\quad + D_1D_3D_2 + D_3D_1D_2 - D_1D_3D_2 - D_2D_3D_1 + D_2D_1D_3 = 0\end{aligned}$$

$$\begin{aligned}(3)&[D_1,D_2,[D_3,D_4,D_5]]+[D_4,D_3,[D_1,D_2,D_5]]\\ &= D_1D_2D_3D_4D_5 - D_1D_2D_4D_3D_5 - D_2D_1D_3D_4D_5 + D_2D_1D_4D_3D_5 + D_5D_3D_4D_1D_2\\ &- D_5D_3D_4D_2D_1 - D_5D_4D_3D_1D_2 + D_5D_4D_3D_2D_1 + D_4D_3D_1D_2D_5 - D_4D_3D_2D_1D_5\\ &- D_3D_4D_1D_2D_5 + D_3D_4D_2D_1D_5 + D_5D_1D_2D_4D_3 - D_5D_1D_2D_3D_4 - D_5D_2D_1D_4D_3\\ &+ D_5D_2D_1D_3D_4\\ &= [[D_1,D_2,D_3],D_4,D_5] + [D_3,[D_1,D_2,D_4],D_5]\end{aligned}$$

In summary, End(T) is a δ Lie triple system composed of a ternary linear operation$[\cdot,\cdot,\cdot]$. □

Theorem 4.2. *Supposing T be the restricted δ Lie triple system on the field $\mathbb{F}$, if* $\mathrm{End(T)}$ *is the set of total linear transformations of T. We assume $[p]$:$D \to D^p, \forall D \in$* $\mathrm{End(T)}$*, then* $\mathrm{End(T)}$ *is the restricted δ lie triple system.*

Proof. From theorem 3.1 we know that End(T) is a δ Lie triple system. $\forall D, D_1, D_2 \in \mathrm{End(T)}, \alpha \in \mathbb{F}$ $(\alpha D)^p = (\alpha D)(\alpha D)\cdots(\alpha D) = \alpha^p D^p$. Because the characteristic of $\mathbb{F}$ is p, $(D_1+D_2)^p = \sum_{k=0}^{p} C_p^k D_1^{p-k} D_2^k = (D_1)^p + (D_2)^p$. Next we use first induction to prove that $(D_1,D,\cdots,D,D_2) = [\sum_{i=0}^{n}(-1)^i C_n^i D^i D_1 D^{n-i}, D_2]$ is valid when $n \geq 3$ is true. When $n=3$, conclusion $(D_1,D,D,D,D_2) = [D_1D^3 - 3DD_1D^2 + 3D^2D_1D - D^3D, D_2] = [\sum_{i=0}^{3}(-1)^i C_3^i D^i D_1 D^{3-i}, D_2]$ is valid. Assuming when $n=m$ the conclusion is valid, that is $(D_1,D,\cdots,D,D_2)=[\sum_{i=0}^{m}(-1)^iC_m^iD^iD_1D^{m-i},D_2]$.When $n=m+1$, $(D_1,D,\cdots,D,D_2)=[\sum_{i=0}^{m}(-1)^iC_m^iD^iD_1D^{m-i},D,D_2]=[\sum_{i=1}^{m}(-1)^iC_{m+1}^iD^iD_1D^{m+1-i}+D_1D^{m+1}+(-1)^{m+1}D^{m+1}D_1,D_2]=[\sum_{i=0}^{m+1}(-1)^iC_{m+1}^iD^iD_1D^{m+1-i},D_2]$, the conclusion is valid. In particular, let $n=p$, $(D_1,D,\cdots,D,D_2)=[[[[D_1,D,D],D,D],\cdots],D,D_2]=[\sum_{k=0}^{p}(-1)^kC_p^kD^kD_1D^{p-k},D_2]=[D_1D^p-D^pD_1,D_2]=[D_1,D^{[p]},D_2]$. So End(T) is the Restricted δ lie triple system. □

Theorem 4.3. *Let T be the Restricted δ lie triple system on the field $\mathbb{F}$, then:*
*(1)*GDer(T) *is a $p-$subsystem of* End(T).
*(2)*QDer(T) *is a $p-$subsystem of* End(T).

Proof. (1)From theorem 3.2 we know that End(T) is a Restricted δ Lie triple system. $\forall D\in GDer_k(T),D_4\in GDer_k(T),D_5\in GDer_s(T),D_6\in GDer_q(T),t,v,w\in T$.
By reason of

$$\begin{aligned}
&[D_4D_5D_6(t),v,w]\\
&=\delta^{k+s+q}D_4^{'''}D_5^{'''}D_6^{'''}[t,v,w]-\delta^{k+s}D_4^{'''}D_5^{'''}[t,D_6^{'}(v),w]-\delta^{k+s}D_4^{'''}D_5^{'''}[t,v,D_6^{''}(w)]\\
&\quad-\delta^{k+s}D_4^{'''}D_6^{'''}[t,D_5^{'}(v),w]+\delta^{k}D_4^{'''}[t,D_6^{'}D_5^{'}(v),w]+\delta^{k}D_4^{'''}[t,D_5^{'}(v),D_6^{''}(w)]\\
&\quad-\delta^{k+s}D_4^{'''}D_6^{'''}[t,v,D_5^{''}(w)]+\delta^{k}D_4^{'''}[t,D_6^{'}(v),D_5^{''}(w)]+\delta^{k}D_4^{'''}[t,v,D_6^{''}D_5^{''}(w)]\\
&\quad-\delta^{s+q}D_5^{'''}D_6^{'''}[t,D_4^{'}(v),w]+\delta^{s}D_5^{'''}[t,D_6^{'}D_4^{'}(v),w]+\delta^{s}D_5^{'''}[t,D_4^{'}(v),D_6^{''}(w)]\\
&\quad+\delta^{q}D_6^{'''}[t,D_5^{'}D_4^{'}(v),w]-[t,D_6^{'}D_5^{'}D_4^{'}(v),w]-[t,D_5^{'}D_4^{'}(v),D_6^{''}(w)]\\
&\quad+\delta^{q}D_6^{'''}[t,D_4^{'}(v),D_5^{''}(w)]-[t,D_6^{'}D_4^{'}(v),D_5^{''}(w)]-[t,D_4^{'}(v),D_6^{''}D_5^{''}(w)]\\
&\quad-\delta^{s+q}D_5^{'''}D_6^{'''}[t,v,D_4^{''}(w)]+\delta^{s}D_5^{'''}[t,D_6^{'}(v),D_4^{''}(w)]+\delta^{s}D_5^{'''}[t,v,D_6^{''}D_4^{''}(w)]\\
&\quad+\delta^{q}D_6^{'''}[t,D_5^{'}(v),D_4^{''}(w)]-[t,D_6^{'}D_5^{'}(v),D_4^{''}(w)]-[t,D_5^{'}(v),D_6^{''}D_4^{''}(w)]\\
&\quad+\delta^{q}D_6^{'''}[t,v,D_5^{''}D_4^{''}(w)]-[t,D_6^{'}(v),D_5^{''}D_4^{''}(w)]-[t,v,D_6^{''}D_5^{''}D_4^{''}(w)].
\end{aligned}$$

The same can be obtained that $[D_5D_4D_6(t),v,w]$,$[D_6D_4D_5(t),v,w]$ and $[D_6D_5D_4(t),v,w]$. So can get$[[D_4,D_5,D_6](t),v,w]=\delta^{k+s+q}[D_4^{'''},D_5^{'''},D_6^{'''}]([t,v,w])-[t,[D_4^{'},D_5^{'},D_6^{'}](v),w]-[t,v,[D_4^{''}D_5^{''}D_6^{''}](w)]$. Hence GDer(T) is a subsystem of End(T). The following is to prove that GDer(T) is a $p-$subsystem of End(T).

Above all we use the first mathematical induction to prove that $[D^n(t),v,w]=\sum_{0\le l,b,r\le n,l+b+r=n}(\frac{n!}{l!b!r!})(-1)^{n-l}\delta^{lk}(D^{'''})^l[t,(D^{'})^b(v),(D^{''})^r(w)]$ is true when $n\ge 3$. If $n=3$, $[D^3(t),v,w]=\sum_{0\le l,b,r\le 3,l+b+r=3}(\frac{3!}{l!b!r!})(-1)^{3-l}\delta^{lk}(D^{'''})^l[t,(D^{'})^b(v),(D^{''})^r(w)]$ conclusion is valid. Supposing when $n=m$ the conclusion is valid, that is $[D^m(t),v,w]=\sum_{0\le l,b,r\le m,l+b+r=m}(\frac{m!}{l!b!r!})(-1)^{m-l}\delta^{lk}(D^{'''})^l[t,(D^{'})^b(v),(D^{''})^r(w)]$. While $n=m+1$,

$$[D^{m+1}(t),v,w] = \sum_{0\le l,b,r\le m, l+b+r=m} (\frac{m!}{l!b!r!})(-1)^{m-l}\delta^{(l+1)k}(D^{'''})^{l+1}[t,(D^{'})^{b}(v),(D^{''})^{r}(w)]$$
$$+\sum_{0\le l,b,r\le m, l+b+r=m} (\frac{m!}{l!b!r!})(-1)^{m-l}\delta^{lk}(D^{'''})^{l}[t,(D^{'})^{b+1}(v),(D^{''})^{r}(w)]$$
$$+\sum_{0\le l,b,r\le m, l+b+r=m} (\frac{m!}{l!b!r!})(-1)^{m-l}\delta^{lk}(D^{'''})^{l}[t,(D^{'})^{b}(v),(D^{''})^{r+1}(w)].$$

Define the case where there have a negative number in l,b,r, then $(\frac{m!}{l!b!r!}) = 0$. So in the above equation, the coefficient of $(D^{'''})^{e}[t,(D^{'})^{f}(v),(D^{''})^{g}(w)]$ is $(-1)^{m+1-l}(\frac{m!}{e-1!f!g!}) + (\frac{m!}{e!f-1!g!}) + (\frac{m!}{e!f!g-1!}) = (-1)^{m+1-l}(\frac{m+1!}{e!f!g!})$, that is

$$[D^{m+1}(t),v,w] = \sum_{0\le l,j,r\le m+1, l+b+r=m+1} (\frac{m+1!}{l!b!r!})(-1)^{m+1-l}\delta^{lk}(D^{'''})l[t,(D^{'})b(v),(D^{''})^{r}(w)],$$

then the conclusion is valid when $n = m+1$. In particular, when $n = p$,

$$[D^{p}(t),v,w] = \sum_{0\le l,b,r\le p, l+b+r=p} (\frac{p!}{l!b!r!})(-1)^{p-l}\delta^{lk}(D^{'''})^{l}[t,(D^{'})^{b}(v),(D^{''})^{r}(w)].$$

Because the characteristic of $\mathbb{F}$ is p, so $[D^{p}(t),v,w] = \delta^{pk}(D^{'''})^{p}[t,v,w] - [t,(D^{'})^{p}(v),w] - [t,v,(D^{''})^{p}(w)]$. That is, GDer(T) is a $p-$subsystem of End(T). □

(2) Similarly, it can be proved that QDer(T) is the $p-$subsystem of End(T).

Theorem 4.4. *Let T be the restricted δ lie triple system on the field $\mathbb{F}$, then:*
(1) ZDer(T) *is the $p-$ideal of* Der(T).
(2) C(T) *is the $p-$subsystem of* End(T).
(3) ZDer(T) *is the $p-$subsystem of* GDer(T).

Proof. (1) $\forall D_1 \in \mathrm{ZDer(T)}, \forall D_2 \in \mathrm{Der_k(T)}, D_3 \in \mathrm{Der_s(T)}, \forall d,g,o \in T$.
$[[D_1,D_2,D_3](d),g,o] = \delta^{k+s}D_3D_2D_1[d,g,o] - D_1[d,D_2(g),o] - D_1[d,g,D_2(o)]$
$-\delta^{k}D_2D_1[d,D_3(g),o] + D_1[d,D_2D_3(g),o] + D_1[d,D_3(g),D_2(o)] - \delta^{k}D_2D_1[d,g,D_3(o)] + D_1[d,D_2g,D_3(o)] + D_1[d,g,D_2D_3(o)] = 0.$

The same can be obtained that $[D_1,D_2,D_3]([d,g,o])$. Due to the definition of the central derivation $[D_1,D_2,D_3] \in \mathrm{ZDer(T)}$, so ZDer(T) is the ideal of Der(T). $\forall D \in \mathrm{ZDer(T)}, c,z,f \in T. D^{[p]}([c,z,f]) = D^{p}([c,z,f]) = D^{p-1}(D([c,z,f])) = 0$, $[D^{[p]}(c),z,f] = [D^{p}(c),z,f] = D([D^{p-1}(c),z,f]) = \cdots = D^{p-1}[D(c),z,f] = 0$, consequently, $D^{[p]} \in \mathrm{ZDer(T)}$, so by the definition of $p-$ideal, ZDer(T) is the $p-$ideal of Der(T). □

The proof methods of (2) and (3) are similar to those of (1).

Proposition 4.1. *Supposing T be a restricted δ Lie triple system on field $\mathbb{F}$, if T has a trivial center, then* C(T) *is a commutative subsystem of* GDer(T).

Proof. From previous studies we know that C(T) is a subspace of GDer(T).

$\forall D_4 \in \mathrm{C_k}(\mathrm{T})$, $\forall D_5 \in \mathrm{C_s}(\mathrm{T})$,$\forall D_6 \in \mathrm{C_q}(\mathrm{T})$, $\forall z,e,r \in \mathrm{T}$. $[D_4,D_5,D_6]([z,e,r]) = \delta^q D_4 D_5([D_6(z),e,r]) - \delta^q D_5 D_4([D_6(z),e,r]) - \delta^s D_6 D_4([D_5(z),e,r]) + \delta^k D_6 D_5([D_4(z),e,r]) = \delta^{k+s+q}[[D_4,D_5,D_6](z),e,r]$. In a similar way, $[D_4,D_5,D_6]([z,e,r]) = \delta^{k+s+q}[z,[D_4,D_5,D_6](e),r]) = \delta^{k+s+q}[z,e,[D_4,D_5,D_6](r)])$. Hence $[\mathrm{C(T)},\mathrm{C(T)},\mathrm{C(T)}] \subseteq \mathrm{C(T)}$. Next we prove the exchange. $[[D_4,D_5,D_6](z),e,r] = [D_6(z),D_5(e),D_4(r)] - [D_6(z),D_5(e),D_4(r)] - [D_5(z),D_4(e),D_6(r)] + [D_5(z),D_4(e),D_6(r)] = 0$. Because e,r is arbitrary and the center of T is 0,so we get $[D_4,D_5,D_6](z) = 0$. And since w is arbitrary, so$[D_4,D_5,D_6] = 0$.

So C(T) is a commutative subsystem of GDer(T) . □

5. Conclusion

This article mainly introduces some structural properties of restricted δ Lie triple systems. First, the basic concepts of restricted δ Lie triple system derivation, generalized derivation and quasi-derivation are given. Then we verify that End(T) is a restricted δ Lie triple system. On this basis, it is proved that GDer(T) and QDer(T) are P-subsystems of End(T). And then proved that ZDer(T) is the $p-$ideal of Der(T), C(T) is the $p-$subsystem of End(T), ZDer(T) is the $p-$subsystem of GDer(T). Finally, it is proved that if the restricted δ Lie system has a trivial center, then the center is the commutator system of the generalized derivations.

6. Future Direction

I will continue to study whether Ker(D) and Im(D) are P-ideals limiting δ Lie triple systems under certain conditions and construct a new restriction δ Lie triple systems τ, and whether the derivations of τ have direct sum decomposition when Z(T) is zero.

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