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A Portfolio Adjusting Model with Triangular Intuitionistic Fuzzy Return

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Abstract. To fit the changes in the investment process, a portfolio adjusting method with triangular intuitionistic fuzzy return is put forward. The expected return rate and risk of the portfolio are characterized by mean value and variance of triangular intuitionistic fuzzy number. Then, an intuitionistic fuzzy portfolio adjusting model is established by minimizing the variance risk of portfolio and ensuring the expected return greater than some aspired return levels. Finally, an application example of stock portfolio is given to demonstrate the practicability of intuitionistic fuzzy portfolio adjusting model.

Keywords. Intuitionistic fuzzy portfolio, weighted mean, variance

1. Introduction

Due to the incomplete disclosure of company's financial information, there is a large amount of fuzzy uncertainty in the expected return of the stock asset. To deal with the asset portfolio problem involved fuzzy returns, a lot of scholars have put forward many optimization models [1-5]. However, in the uncertain scenario the expected return of invested asset can be more conveniently estimated by triangular intuitionistic fuzzy number (TrIFN) than fuzzy numbers. Since TrIFN is more powerful and flexible for representing uncertain return data than ordinary fuzzy number because TrIFN can comprehensively consider both membership and nonmembership of return. In fact, the existing fuzzy portfolio decision models only consider the true membership or satisfaction degree of uncertain return without considering the nonmembership of uncertain return. So, it is more valuable to investigate intuitionistic fuzzy portfolio problem than ordinary fuzzy portfolio.

Till now, there are few studies on intuitionistic fuzzy portfolio decision problem. Although some intuitionistic fuzzy optimization methods [6-9] of portfolio selection problem have been investigated, the existing intuitionistic fuzzy portfolio models (IFPMs) only transform fuzzy portfolio objectives to intuitionistic fuzzy objectives. The existing IFPMs are generally constructed by maximizing the membership and

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minimizing nonmembership or hesitation degree of all the portfolio objectives. Recently, Zhou [10] proposed an IFPM based on score-hesitation of IFN. Deng [11] investigated portfolio programming model based on distance measure of IFN. However, the existing IFPM are still unable to directly handle the asset selection problems with TrIFN returns.

Motivated by the above limitations, we try to establish a new IFPM involved with TrIFN returns of the assets and transaction costs. Since TrIFN is widely used in real-world application fields [12-15], we employ TrIFN to assess uncertain return of each security and construct an intuitionistic fuzzy portfolio adjusting model.

2. The weighted possibility mean and variance of TrIFN

A TrIFN $\tilde{a}=(a_1',a_1,a_2,a_3,a_3')$ is a special kind of intuitionistic fuzzy set on R, whose membership and nonmembership functions are as the following forms [16].

$$\mu_{\tilde{a}}(x)=\begin{cases}(x-a_1)/(a_2-a_1), if & a_1\le x<a_2,\\ 1\ , & if\quad x=a_2,\\ (a_3-x)/(a_3-a_2), & if\quad a_2<x\le a_3,\\ 0\ , & otherwise.\end{cases}\qquad \nu_{\tilde{a}}(x)=\begin{cases}(a_2-x)/(a_2-a_1'), if & a_1'\le x<a_2,\\ 0\ , & if\quad x=a_2,\\ (x-a_2)/(a_3'-a_2), & if\quad a_2<x\le a_3',\\ 1\ , & otherwise.\end{cases}$$

where $-\infty<a_1'\le a_1\le a_2\le a_3\le a_3'<+\infty$.

Definition 1[16]. Let $\tilde{a}=(a_1',a_1,a_2,a_3,a_3')$ and $\tilde{b}=(b_1',b_1,b_2,b_3,b_3')$ be two TrIFNs, some basic operations of them are defined by

(1) $\tilde{a}+\tilde{b}=(a_1'+b_1',a_1+b_1,a_2+b_2,a_3+b_3,a_3'+b_3')$.

(2) $\tilde{a}-\tilde{b}=(a_1'-b_3',a_1-b_3,a_2-b_2,a_3-b_1,a_3'-b_1')$

(3) $x\tilde{a}=(xa_1',xa_1,xa_2,xa_3,xa_3'),\forall x\ge 0; x\tilde{a}=(xa_3',xa_3,xa_2,xa_1,xa_1'),\forall x<0.$

Definition 2[17]. Let $\tilde{a}=(a_1',a_1,a_2,a_3,a_3')$ be a TrIFN, the λ -cut set of membership and γ -cut set of nonmembership function of $\tilde{a}$ are respectively defined as

$$\tilde{a}^{[\lambda]}=[a^-(\lambda),a^+(\lambda)]=[a_2-(a_2-a_1)(1-\lambda),a_2+(a_3-a_2)(1-\lambda)];$$

$$\tilde{a}_{[\gamma]}=[a^-(\gamma),a^+(\gamma)]=[a_2-(a_2-a_1')\gamma,a_2+(a_3'-a_2)\gamma];\qquad \forall\lambda,\gamma\in(0,1).$$

Definition 3[17]. The expected mean of membership and nonmembership of TrIFN $\tilde{a}$ are respectively defined as

$$M_\mu(\tilde{a})=\int_0^1[a^-(\lambda)+a^+(\lambda)]\lambda d\lambda=\frac{4a_2+(a_1+a_3)}{6};$$

$$M_\nu(\tilde{a})=\int_0^1[a^-(\gamma)+a^+(\gamma)](1-\gamma)d\gamma=\frac{4a_2+(a_1'+a_3')}{6}.$$

Definition 4. Let $\tilde{a}=(a_1',a_1,a_2,a_3,a_3')$ be a TrIFN, the weighted expected mean value of $\tilde{a}$ is defined as

$$M(\tilde{a}) = (1-t)M_\mu(\tilde{a}) + tM_\nu(\tilde{a}) = \frac{4a_2}{6} + \frac{(1-t)(a_1+a_3)+t(a_1'+a_3')}{6}, \quad \forall t \in [0,1]. \quad (1)$$

Definition 5. Assume $\tilde{a} = (a_1', a_1, a_2, a_3, a_3')$ is a TrIFN, the variance of membership and nonmembership of $\tilde{a}$ are, respectively, defined as

$$Var_\mu(\tilde{a}) = \frac{1}{2}\int_0^1 \lambda[a^+(\lambda) - a^-(\lambda)]^2 d\lambda = \frac{1}{24}(a_3 - a_1)^2.$$

$$Var_\nu(\tilde{a}) = \frac{1}{2}\int_0^1 (1-\gamma)[a^+(\gamma) - a^-(\gamma)]^2 d\gamma = \frac{1}{24}(a_3' - a_1')^2.$$

Definition 6. Let $\tilde{a} = (a_1', a_1, a_2, a_3, a_3')$ be a TrIFN, the variance of $\tilde{a}$ is defined as

$$\text{var}(\tilde{a}) = \frac{1}{2}[\text{var}_\mu(\tilde{a}) + \text{var}_\nu(\tilde{a})] = \frac{1}{48}[(a_3 - a_1)^2 + (a_3' - a_1')^2]. \quad (2)$$

Definition 7. Let $\tilde{a} = (a_1', a_1, a_2, a_3, a_3')$, $\tilde{b} = (b_1', b_1, b_2, b_3, b_3')$ be two TrIFNs, the covariance of membership and nonmembership of $\tilde{a}, \tilde{b}$ are, respectively, defined as

$$\text{cov}_\mu(\tilde{a}, \tilde{b}) = \frac{1}{2}\int_0^1 [a^-(\lambda) - a^+(\lambda)][b^-(\lambda) - b^+(\lambda)]\lambda d\lambda ;$$

$$\text{cov}_\nu(\tilde{a}, \tilde{b}) = \frac{1}{2}\int_0^1 [a^-(\gamma) - a^+(\gamma)][b^-(\gamma) - b^+(\gamma)](1-\gamma) d\gamma .$$

Definition 8. Let $\tilde{a} = (a_1', a_1, a_2, a_3, a_3')$, $\tilde{b} = (b_1', b_1, b_2, b_3, b_3')$ be two TrIFNs, the covariance of $\tilde{a}, \tilde{b}$ is defined as

$$\text{cov}(\tilde{a}, \tilde{b}) = \frac{1}{2}[\text{cov}_\mu(\tilde{a}, \tilde{b}) + \text{cov}_\nu(\tilde{a}, \tilde{b})] = \frac{(a_1 - a_3)(b_1 - b_3) + (a_1' - a_3')(b_1' - b_3')}{48}. \quad (3)$$

Property 1. Assume $\tilde{a} = (a_1', a_1, a_2, a_3, a_3')$, $\tilde{b} = (b_1', b_1, b_2, b_3, b_3')$ are two TrIFNs, for any x, y, we have

$$M(x\tilde{a} + y\tilde{b}) = xM(\tilde{a}) + yM(\tilde{b}). \quad (4)$$

It can be easily verified by Definition 1, 4 and formula (1). (Omitted)

Property 2. Let $\tilde{a} = (a_1', a_1, a_2, a_3, a_3')$, $\tilde{b} = (b_1', b_1, b_2, b_3, b_3')$ be two TrIFNs. Then for any $x, y \geq 0$, we have

$$\text{var}(x\tilde{a} + y\tilde{b}) = x^2 \text{var}(\tilde{a}) + 2xy\text{cov}(\tilde{a}, \tilde{b}) + y^2 \text{var}(\tilde{b}).$$

It can be easily proved with Definition 1,5,7 and formulas (2), (3) .

3. Intuitionistic fuzzy portfolio adjusting model

3.1 Intuitionistic fuzzy return mean and risk of portfolio

Let us consider a portfolio includes n risky securities $\{S_1, \Lambda, S_n\}$. Since the expected returns of assets $\{S_i\}(i = 1,2,\Lambda\ n)$ are imprecise, we use the historical return data of financial assets and extend Vercher's method [18] to evaluate the intuitionistic fuzzy return of S_i as a TrIFN $\widetilde{r}_i = (r'_{i1}, r_{i1}, r_{i2}, r_{i3}, r'_{i3})$, where $r'_{i1}, r_{i1}, r_{i2}, r_{i3}, r'_{i3}$ are the 3-th percentile, 5-th percentile, 50-th percentile, 95-th percentile, 97-th percentile of the historical return rates of security S_i, respectively.

Assume the investor holds an existing portfolio $X^0 = (x_1^0, x_2^0, \Lambda, x_n^0)$ and plans to adjust his capital on each asset. Suppose the whole investment process is self-financing and the cost rate of buying or selling security i is $c_i, (1 \le i \le n)$. After adjusting capital the optimal portfolio changes to $x = (x_1, x_2, \Lambda, x_n)$, and the IFP return is $\sum_{i=1}^n x_i \widetilde{r}_i = (\sum_{i=1}^n x_i r'_{i1}, \sum_{i=1}^n x_i r_{i1}, \sum_{i=1}^n x_i r_{i2}, \sum_{i=1}^n x_i r_{i3}, \sum_{i=1}^n x_i r'_{i3})$.
Then, with formula (4) we calculate the weighted return mean of the portfolio as

$$r_p = M(\sum_{i=1}^n x_i \widetilde{r}_i) = \sum_{i=1}^n x_i [\tfrac{4r_{i2}}{6} + \tfrac{(1-t)(r_{i1}+r_{i3})+t(r'_{i1}+r'_{i3})}{6}].$$

By utilizing formula (2) we compute the variance risk of portfolio as

$$\mathrm{var}(\sum_{i=1}^n x_i \widetilde{R}_i) = \tfrac{1}{48}\{[\sum_{i=1}^n x_i (r_{i3} - r_{i1})]^2 + [\sum_{i=1}^n x_i (r'_{i3} - r'_{i1})]^2\}. \quad (5)$$

The total transaction cost of portfolio in the adjusting process is calculated by

$$Cost(X) = \sum_{i=1}^n c_i \left| x_i - x_i^0 \right|.$$

And the net return of this portfolio is

$$r_{p,N} = \sum_{i=1}^n x_i [\tfrac{4r_{i2}}{6} + \tfrac{(1-t)(r_{i1}+r_{i3})+t(r'_{i1}+r'_{i3})}{6}] - \sum_{i=1}^n c_i \left| x_i - x_i^0 \right|. \quad (6)$$

3.2 Construction of intuitionistic fuzzy portfolio model

Assume an investor now holds the existing portfolio $X^0 = (x_1^0, x_2^0, \Lambda, x_n^0)$ and he/she try to reallocate n assets by minimizing the risk of portfolio under some uncertain constraints. Thus, the intuitionistic fuzzy portfolio adjusting method is formulated as programming model (P1) by minimizing variance risk formula (5) and ensuring net return formula (6) of portfolio greater than a given aspiration level.

$$\text{(P1)} \quad \min \tfrac{1}{48}\{[\sum_{i=1}^n x_i (r_{i3} - r_{i1})]^2 + [\sum_{i=1}^n x_i (r'_{i3} - r'_{i1})]^2\}$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^n [\tfrac{2r_{i2}}{3} + \tfrac{(1-t)(r_{i1}+r_{i3})+t(r'_{i1}+r'_{i3})}{6}] x_i - \sum_{i=1}^n c_i \left| x_i - x_i^0 \right| \ge \eta \\ \sum_{i=1}^n x_i + \sum_{i=1}^n c_i \left| x_i - x_i^0 \right| = 1 \\ l_i \le x_i \le u_i, i = 1, 2 \Lambda, n; \end{cases}$$

where η denotes the minimum aspired net return level determined by the risky investor. $l_i, u_i \in [0,1]$ denote lower bound and upper bound of capital invested on security i.

Note that there is an absolute value function in the constraint of the above portfolio model, it results in complexity of solving this model. In order to simplify the optimization model, we let $d_i^+ = \frac{\left| x_i - x_i^0 \right| + (x_i - x_i^0)}{2}, d_i^- = \frac{\left| x_i - x_i^0 \right| - (x_i - x_i^0)}{2}$, then the above complex portfolio adjusting model (P1) can be transformed to the following simple quadratic programming model.

$$\text{(P2)} \quad \min \tfrac{1}{48}\{[\sum_{i=1}^{n} x_i (r_{i3} - r_{i1})]^2 + [\sum_{i=1}^{n} x_i (r'_{i3} - r'_{i1})]^2\}$$

$$\text{s.t.} \quad \begin{cases} \sum_{i=1}^{n} [\frac{2r_{i2}}{3} + \frac{(1-t)(r_{i1}+r_{i3})+t(r'_{i1}+r'_{i3})}{6}]x_i - \sum_{i=1}^{n} c_i (d_i^+ + d_i^-) \geq \eta \\ \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} c_i (d_i^+ + d_i^-) = 1 \\ l_i \leq x_i \leq u_i, \\ d_i^+, d_i^- \geq 0, \forall i = 1,2\Lambda \ ,n; \end{cases}$$

Thus, one can easily solve the optimal strategy of model (P2) by taking different return aspiration η and using Lingo nonlinear optimization software. The portfolio solutions of model (P2) vary according to the different aspired return value η .

4. Application example

Suppose an investor already holds an existing portfolio comprising of the following five stocks: S_1 (code 600192), S_2 (code 600537), S_3 (code 603256), S_4 (code 603223), S_5 (code 603628), which are selected from Shanghai Stock Exchange in China. And the adjusting investment process is self-financing.

We collect the alternative corporations' financial statement information and the assets' monthly prices from September 2018 to September 2022. By utilizing the historical monthly return data and Vercher's percentile method [18] we estimate the uncertain return of each stock Si as a TrIFN $\tilde{r}_i = (r'_{i1}, r_{i1}, r_{i2}, r_{i3}, r'_{i3})$. The distribution parameters of the evaluated TrIFN return rates of Stock i are listed in Table 1. Some coefficients of the proposed adjusting model are listed in Table 2.

Table 1. TrIFN return assessment of the five stocks

Parameter of TrIFN	S1	S2	S3	S4	S5
r'_{i1}	-0.215	-0.156	-0.193	-0.151	-0.231
r_{i1}	-0.165	-0.134	-0.186	-0.131	-0.21
r_{i2}	0.030	0 .022	0.004	0.049	0.0264
r_{i3}	0.249	0.282	0.275	0.286	0.2976
r'_{i3}	0.271	0.339	0.411	0.301	0.3314
Variance risk	0.0085	0.0087	0.012	0.0079	0.012

Table 2. Some coefficients of the portfolio adjusting model

Coefficients	S1	S2	S3	S4	S5
$r_{i3} - r_{i1}$	0.4140	0.4160	0.4610	0.4170	0.5076
$r'_{i3} - r'_{i1}$	0.4860	0.4950	0.6040	0.4520	0.5624
$r'_{i1} + r'_{i3}$	0.0560	0.1830	0.2180	0.150	0.1004

$r_{i1}+r_{i3}$	0.0840	0.1480	0.0890	0.1550	0.0876
$\frac{4r_{i2}+(1-t)(r_{i1}+r_{i3})+t(r'_{i1}+r'_{i3})}{6}$	0.034-0.0047t	0.039+0.0058t	0.0175+0.0215t	0.0585-0.0008t	0.0322+0.002t

Let the existing portfolio be $x^0=(x_1^0,x_2^0,x_3^0,x_4^0,x_5^0)=(0.15,0.2,0.25,0.2,0.2)$.

If taking t=1, $M(r_{x^0})=0.0416;\quad \mathrm{var}(r_{x^0})=0.4099.$

If taking t=0, $M(r_{x^0})=0.0355;\quad \mathrm{var}(r_{x^0})=0.4099.$

Now we use the portfolio adjusting model to reallocate the assets. Assume the capital upper bounds of five stocks is $(u_1,\Lambda,u_5)=(0.3, 0.3, 0.4, 0.4, 0.5)$ and the cost rates of purchasing and selling stocks are the same as $c_i=0.001,\forall i=1,2,\Lambda,5.$

In order to obtain the optimal adjusting strategy, we substitute the return mean, variance and coefficients of each stock into the constructed portfolio model (P2) and reformulate the IFP adjusting problem as the following programming model (P3).

$$(\mathrm{P3})\ \min \tfrac{1}{48}\{[\sum_{i=1}^{5}x_i(r_{i3}-r_{i1})]^2+[\sum_{i=1}^{5}x_i(r'_{i3}-r'_{i1})]^2\}$$

$$\text{s.t.}\ \begin{cases}\sum_{i=1}^{5}[\frac{4r_{i2}+(1-t)(r_{i1}+r_{i3})+t(r'_{i1}+r'_{i3})}{6}]x_i-\sum_{i=1}^{5}c_i(d_i^++d_i^-)\ge\eta\\ \sum_{i=1}^{5}x_i+\sum_{i=1}^{5}0.001(d_i^++d_i^-)=1\\ 0\le x_1,x_2\le 0.3,\\ 0\le x_3,x_4\le 0.4,\\ 0\le x_5\le 0.5,\\ d_i^+,d_i^-\ge 0,i=1,2,\Lambda,5.\end{cases}$$

By solving the above model (P3), some optimal portfolio results are listed in Table 3 when the investor is optimistic and take t=1. If the investor is pessimistic and take t=0, then the solved optimal portfolio strategies are displayed in Table 4. If the investor is neutral and take t= 0.5, then he can obtain the optimal portfolio adjusting strategies in Table 5 by taking different aspired return level η.

Table 3. Some optimal portfolios when taking t=1

η	**0.01**	**0.02**	**0.03**
Min var	0.008	0.0082	0.0083
X1	0.3	0.3	0.3
X2	0.2	0.229	0.2991
X3	0	0	0
X4	0.38	0.39	0.4
X5	0	0	0

Table 4. Some optimal portfolios when taking t=0

η	**0.015**	**0.025**	**0.04**
Min var	0.0006	0.0017	0.0062
X1	0	0	0.16394

X2	0.0012	0.0645	0.3
X3	0	0	0
X4	0.2713	0.4	0.4
X5	0	0	0

Table 5. Some optimal portfolios when taking t=0.5

η	**0.031**	**0.042**	**0.045**
Min var	0.00296	0.00696	0.00874
X1	0	0.21824	0.1639
X2	0.2029	0.3	0.3
X3	0	0	0
X4	0.4	0.4	0.4
X5	0	0	0.1361

Table 6. Some optimal portfolios when taking t=1 with the proportion lower bound

η	**0.01**	**0.02**	**0.03**
Min var	0.0084	0.00848	0.0085
X1	0.3	0.3	0.3
X2	0.22	0.228	0.23
X3	0.03	0.03	0.03
X4	0.4	0.4	0.4
X5	0.04	0.04	0.04

From Table 3, 4, 5 one can see that the investor should adjust the existing portfolio to obtain the optimal portfolios. Especially in Table 3, for $\eta = 1\%$, the investor should buy 0.15 of stock 1, sell 0.25 of stock 3, buy 0.18 of stock 4, and sell 0.2 of stock 5 to get the optimal portfolio $x^* = (0.3,0.2,0,0.38,0)$. If the investor is optimistic and plans to hold five assets in the portfolio, he need set the lower bound constraints such as l_1=0.02, l_2 =0.02, l_3=0.03, l_4 =0.03, l_5=0.04. Then by solving the portfolio adjusting model (P3) he can get some optimal portfolios as shown in Table 6. Comparing Table 3 and Table 6, we find that the optimal portfolio in Table 3 is more efficient than the corresponding one in Table 6.

5. Comparative analysis

In this section, we will compare our proposed intuitionistic fuzzy adjusting portfolio model with the existing fuzzy adjusting portfolio model.

When $r'_{i1} = r_{i1}, r'_{i3} = r_{i3}$, the estimated TrIFN return $\widetilde{r}_i = (r'_{i1}, r_{i1}, r_{i2}, r_{i3}, r'_{i3})$ of asset i in this paper is degenerated into triangular fuzzy return $\widetilde{r}_i = (r_{i1}, r_{i2}, r_{i3})$, which is fuzzy possibility distribution $\widetilde{r}_i = (r_{i2}; \alpha, \beta)$, where $\alpha = r_{i2} - r_{i1}, \beta = (r_{i3} - r_{i2})$ are respectively the left width and right width of TrFN in literature [1]. The proposed IFPM is reduced to the corresponding FPM. So, the existing FPM [1] is a special case of our proposed IFPM. Hence, our presented intuitionistic fuzzy portfolio adjusting model is more extensive than the existing fuzzy adjusting portfolio method [1, 19].

Moreover, if $r'_{i1} = r_{i1}, r'_{i3} = r_{i3}$, the presented intuitionistic fuzzy adjusting model is transformed into the following fuzzy portfolio form.

$$\min \frac{1}{24}\{[\sum_{i=1}^{n} x_i(r_{i3} - r_{i1})]^2\}$$

s.t.

$$\sum_{i=1}^{n}[\frac{2r_{i2}}{3} + \frac{(r_{i1}+r_{i3}))}{6}]x_i - \sum_{i=1}^{n} c_i\left|x_i - x_i^0\right| \geq \eta$$

$$\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} c_i\left|x_i - x_i^0\right| = 1$$

$$l_i \leq x_i \leq u_i, i = 1,2\Lambda\ ,n;$$

We compare our portfolio adjusting model with the following known fuzzy portfolio adjusting model proposed in Zhang's work [1].

$$\min \quad \frac{1}{24}[\sum_{i=1}^{n} x_i(\alpha_i + \beta_i)]^2 + \frac{1}{72}[\sum_{i=1}^{n} x_i(\alpha_i - \beta_i)]^2$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{n}[\frac{2r_{i2}}{3} + \frac{(r_{i1}+r_{i3}))}{6}]x_i - \sum_{i=1}^{n} c_i\left|x_i - x_i^0\right| \geq \eta \\ \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} c_i\left|x_i - x_i^0\right| = 1 \\ l_i \leq x_i \leq u_i, i = 1,2\Lambda\ ,n; \end{cases}$$

In the above-mentioned two adjusting portfolio models, the objective function is different but the constraints are setting same for comparing the portfolio efficiency.

Here, we set the lower bound and upper bound vector on holding five assets as $(l_1,l_2,l_3,l_4,l_5) = (0.02,0.02,0.03,0.03,0.04), (u_1,u_2,u_3,u_4,u_5) = (0.3,0.3,0.4,0.4,0.5)$. By utilizing optimization software, we get the portfolio strategy results as in Table 7, 8.

Table 7. The optimal portfolio result obtained from our model

η	**0.01**	**0.02**	**0.03**
Min var	**0.00736**	**0.007364**	**0.007365**
X1	0.3	0.3	0.3
X2	0.3	0.29	0.292
X3	0.03	0.028	0.029
X4	0.3292	0.393	0.394
X5	0.04	0.041	0.0415

Table 8. The optimal portfolio result obtained from Zhang's fuzzy model [1]

η	**0.01**	**0.02**	**0.03**
Min var	**0.00741**	**0.007416**	**0.00742**
X1	0.3	0.3	0.3
X2	0.3	0.3	0.3
X3	0.03	0.03	0.03
X4	0.32924	0.32925	0.32926
X5	0.04	0.0415	0.042

From the above optimal strategy Tables 7, 8 one can see that the risk objective of our intuitionistic fuzzy adjusting portfolio model is smaller than that obtained from zhang's adjusting portfolio model [1] under the same aspired return level η . So, our proposed intuitionistic fuzzy portfolio adjusting model is better than the known fuzzy portfolio adjusting model of Zhang [1].

6. Conclusion

We study the portfolio adjusting problem when transaction cost is considered. The presented portfolio adjusting model can deal with the TrIFN returns of stocks and the bounded holding proportions of assets. With the portfolio adjusting model, the investors can obtain the appropriate portfolio according to their risk preference.

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