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Arguing About Choosing a Normative System: Conflict of Laws

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Abstract. This paper presents a formal model of specific reasoning patterns in conflict of laws (CoL). CoL arises when multiple countries have jurisdiction due to the diverse nationalities of the involved factors. When initiating legal action in one country, the question of which country's substantial law to apply emerges, possibly involving the CoL regulations of other countries (in cases of transmission and renvoi). Moreover, parties contemplating legal action in a case falling under CoL often engage in a deliberation process known as forum shopping: determining which country's CoL regulations would result in the most favorable outcome for them. Our model integrates deontic logic (specifically Input/Output logic) with proof theory and formal argumentation techniques to model both types of reasoning.

Keywords. conflict of laws, forum shopping, renvoi, deontic logic, formal argumentation, input/output logic, proof theory

1. Introduction

Conflict of Laws (CoL) governs cases in private international law. These cases concern private law matters involving some substantial international element(s), such as the parties being from different nations, a contract being created in another country, etc. Each nation has its own CoL regulation stipulating under which conditions which nation's legal system should be applied. Various scenarios may arise: the lex fori[2] may appoint a nation's law whose respective CoL regulation redirects the case to the lex fori (referred to as *renvoi*), or the lex fori may appoint a nation whose CoL regulation appoints yet another nation's law and so on (referred to as *transmission*). Such referral gives rise to challenges: e.g., how to deal with the possibility of infinite transmission caused by a transferal loop? To handle such challenges each nation's CoL regulation has specific meta-rules that allow for, limit, or forbid transmission and renvoi (see [1]).

This work provides a formal model for analyzing specific aspects of CoL reasoning. First, when starting a legal procedure from a specific country, the model represents the

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[2]The lex fori is the law of the forum/jurisdiction where a legal action is being heard and decided.

reasoning underlying the determination of the country whose substantive laws are applicable to a case, based on the allocation rules of the involved countries. Unlike other CoL models (e.g., [2,3]), our model allows for reasoning with sequential and cyclic allocations of cases. Second, the model determines the obligations of the country whose substantive laws are applicable. Third, the model represents the perspective of a party planning where to initiate legal action in cases that involve foreign elements and so where more than one country has jurisdiction. This deliberation process is known as *forum shopping*.

To reach these aims, we develop a class of Conflict of Laws Calculi (CoLC) that extends the proof theoretic approach to Input/Output Logic [4] by [5], using formal argumentation [6] (Sect. 2). CoLC distinguishes reasoning with and about norms, enabling reasoning about the inapplicability of substantive norms and CoL transmissions. By embedding CoLC within formal argumentation (Sect. 3), the notion of inapplicability gives naturally rise to argumentative attacks in the three aforementioned layers of reasoning.

We illustrate our formalism in terms of a CoL running example of [1] that features both transmission and renvoi (see Example 1 below). We stress three points: (1) Each nation has CoL regulation that uniquely determines the nation whose law applies to the case. This may lead to *chains of transmissions* of a given case as governed by the respective CoL regulations. (2) Some nations *impose a limit* on the number of transmissions. (3) An exception to a limit is a *renvoi regulation*: given a limit of n, if a transmission chain of length at most $n+1$ leads back to the lex fori, the lex fori's laws apply. ([7] take renvoi as referral in general. We follow [1] and use renvoi for redirection to the lex fori.)

Example 1 (Conflict of Laws) *Sándor Farkas (SF), a Hungarian citizen, moved to France leaving his house behind. He settled in France after finding love and an agreeable life, choosing the French citizenship over the Hungarian one. Following his death, his sibling initiated a probate action at a Hungarian public notary in the hope of inheriting the house. The Hungarian CoL regulation states that in case of an inheritance, the applicable law is the one based on the personal law (citizenship) of the testator SF. The French one in this case. The Hungarian CoL regulation does not allow for transmission. Therefore, the public notary must apply the substantial rules of the French law (without dealing with France's CoL regulation). However, the Hungarian CoL regulation does allow for renvoi. In other words, if France's CoL regulation would appoint the Hungarian one as applicable, the forum has to apply the substantial rules of its own law. This is exactly what happened in the case of SF. The French CoL regulation orders to apply the law based on the to-be-inherited real estate's location, which is Hungary. As a result, the public notary decided who inherits the house based on the Hungarian inheritance law.*

2. Conflict of Laws Calculi: CoLC

We develop a general formalism for CoL reasoning by extending the deontic argumentation calculi (DAC) from [5]. We extend the calculi by reasoning about multiple national legal systems and develop various rules for restricted CoL reasoning such as reasoning with renvoi. The resulting calculi are referred to as Conflict of Laws Calculi (CoLC).

The Formal Language of CoL. We extend the classical propositional language $\mathscr{L}$ defined by the BNF grammar: $\varphi ::= p \mid \top \mid \bot \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \rightarrow \varphi$, where $p, q, \ldots$ are used to denote propositional atoms and $\varphi, \psi, \ldots$ for arbitrary formulas. In particular, we adopt labellings of $\mathscr{L}$ to differentiate between formulas that denote *facts*, *obligations*, and *constraints* [5]. Since we aim at reasoning with legal systems of various nations, we index obligations with nation labels $i \in \mathsf{nations} = \{i, \ldots, n\}$ (where $n \in \mathbb{N}$):

- $\mathscr{L}^f$ for facts φ^f;
- $\mathscr{L}^o$ for obligations φ_i^o of various nations $i \in \mathsf{nations}$;
- $\mathscr{L}^c$ for constraints φ^c with which obligations must be consistent.

Moreover, for each nation $i \in \mathsf{nations}$ we differentiate in a legal system between norms of the *substantive* norm code and those beloning to the *conflict of laws* norm code:

- $\mathscr{L}_i^{\mathsf{sbt}}$ for substative norms $(\varphi, \psi)_i$ of nation i (where $\varphi, \psi \in \mathscr{L}$);
- $\mathscr{L}_i^{\mathsf{col}}$ for conflict of laws norms $(\varphi, j)_i$ for nation i (where $\varphi \in \mathscr{L}$ and $j \in \mathsf{nations}$).

For a nation i, a substantive norm $(\varphi, \psi)_i$ reads "Given φ, it is obligatory that ψ," and a CoL norm $(\varphi, j)_i$ reads "Given φ, consult the legal system of nation j." The two norm types are easily differentiated since only CoL norms use nations as their consequent.

Normative conflicts may arise when reasoning with substantive norms. In such cases, we want to express which norms cannot be consistently asserted given a certain context. To express this, we adopt formulas that represent the *inapplicability of a norm* [5]:

- $\overline{\mathscr{L}_i^{\mathsf{sbt}}}$ for inapplicable norms $\neg(\varphi, \psi)_i$, where $(\varphi, \psi)_i \in \mathscr{L}_i^{\mathsf{sbt}}$.

A formula $\neg(\varphi, \psi)_i$ reads "Nation i's substantive norm (φ, ψ) is inapplicable." Notice that negated CoL norms are not required since every nation's CoL regulation is unique and conflict-free. We let $\mathscr{L}^{\mathsf{sbt}} = \bigcup_{j=1}^{n} \mathscr{L}_j^{\mathsf{sbt}}$, $\mathscr{L}^{\mathsf{col}} = \bigcup_{j=1}^{n} \mathscr{L}_j^{\mathsf{col}}$ and $\overline{\mathscr{L}^{\mathsf{sbt}}} = \bigcup_{j=1}^{n} \overline{\mathscr{L}_j^{\mathsf{sbt}}}$.

Example 2 (Ex. 1 continued) *For our running example, we adopt the following atoms:*
$\mathtt{hou}$:= *"The house is in Hungary."* $\mathtt{spo}$:= *"The spouse inherits the property."*
$\mathtt{dec}$:= *"The deceased is French."* $\mathtt{sib}$:= *"The siblings inherit the house."*

The facts of this case are $\mathtt{hou}^f$ *and* $\mathtt{dec}^f$*. The relevant substantial norms of Hungary and France are* $(\mathtt{hou}, \mathtt{sib})_{hun}$*, respectively* $(\mathtt{dec}, \mathtt{spo})_{fra}$*, which read "Given the house is in Hungary, the siblings should inherit the house," respectively "Given the deceased is French, the spouse should inherit." The relevant CoL norms of Hungary and France are* $(\mathtt{dec}, fra)_{hun}$*, respectively* $(\mathtt{hou}, hun)_{fra}$ *and read "Given the deceased is French, consult French law," respectively "Given the house is in Hungary, consult Hungarian law."*

The chains of transmissions resulting from CoL regulations are expressed by:

- $\mathscr{L}^{\mathsf{ctrns}}$ for finite sequences $\langle i, \ldots, j \rangle_{\mathsf{col}}$ of nation labels denoting *CoL transitions* leading (indirectly) from nation i to j.

Furthermore, meta-rules ensure CoL reasoning is deterministic. This means that eventually we end up with a nation whose substantive norms will be applied. We use

- $\mathscr{L}^{\mathsf{strns}}$ for finite sequences $\langle i, \ldots, j \rangle_{\mathsf{sbt}}$ of nation labels denoting CoL transitions from nation i that support *substantive norm application* of nation j.

One may think of the object $\langle i, \ldots, j \rangle_{\mathsf{sbt}}$ as an inference license issued by nation i to apply the substantive norms of nation j. Furthermore, these sequences represent from whose perspective we are reasoning, listing all nations passed in the chain of transmission.

Last, renvoi expresses a limit on CoL transmissions, *unless* the last transmission refers back to the lex fori. Hence, the limit is *defeasible* upon the possibility of a renvoi. For this, we extend our language to reason about *inapplicable* CoL sequences:

- $\overline{\mathscr{L}^{\mathsf{strns}}}$ containing $\neg\langle i, \ldots, j \rangle_{\mathsf{sbt}}$ denoting the inapplicability of $\langle i, \ldots, j \rangle_{\mathsf{sbt}}$.

$$\textbf{Ax}\ \frac{}{\vdash_{\mathsf{LK}} \Delta^j \Rightarrow \Gamma^j} \qquad \textbf{L-CT}_i^a\ \frac{\varphi^f, \Delta \Rightarrow \Theta}{\varphi_i^o, \Delta \Rightarrow \Theta} \qquad \textbf{Cut}\ \frac{\Delta \Rightarrow \varphi \qquad \varphi, \Delta' \Rightarrow \Theta}{\Delta, \Delta' \Rightarrow \Theta}$$

$$\textbf{Sub-Det}_i\ \frac{}{\varphi^f, (\varphi, \psi)_i \Rightarrow \psi_i^o} \qquad \textbf{R-C}_i\ \frac{\Delta \Rightarrow \varphi_i^o}{\Delta, \neg\varphi^c \Rightarrow} \qquad \textbf{R-N}_i\ \frac{\Delta^f, \Delta^c, \Delta_i^{\mathsf{sbt}}, (\varphi, \psi)_i \Rightarrow}{\Delta^f, \Delta^c, \Delta_i^{\mathsf{sbt}} \Rightarrow \neg(\varphi, \psi)_i}$$

Figure 1. Rules of DAC for CoL reasoning (Def. 2). We have **Ax** for each language $\mathscr{L}^j$, $j \in \{f, o, c\}$. For each $i \in \mathsf{nations}$ there is an instance of **L-CT**$_i$, **CoL-Det**$_i$, **R-C**$_i$, and **R-N**$_i$. Condition (a) expresses $\Delta \cap \mathscr{L}_i^{\mathsf{sbt}} \neq \emptyset$.

Example 3 (Ex. 2 continued) *The object $\langle hun, fra, hun\rangle_{\mathsf{col}}$ expresses that we reason from the perspective of Hungary's legal system (hun) whose CoL norms refer to the legal system of France (fra), whose CoL norms refer back to Hungary's legal system (hun).*

Hungarian CoL regulation does not allow for additional transmission, except for renvoi. Hence, a limit of 1 is imposed, that warrants the application of the substantive norms of the appointed nation $\langle hun, fra\rangle_{\mathsf{sbt}}$, unless a renvoi ensues that provides an inference license for Hungary to apply its own law $\langle hun, fra, hun\rangle_{\mathsf{sbt}}$. In the latter case, the license to invoke French law is revoked, this is expressed by $\neg\langle hun, fra\rangle_{\mathsf{sbt}}$. In other words, $\overline{\mathscr{L}^{\mathsf{strns}}}$ expresses the inapplicability of previously warranted CoL transitions.

Definition 1 *The full* CoLC *language $\mathscr{L}^{\mathsf{CoLC}}$ is defined as:*

$$\mathscr{L}^{\mathsf{CoLC}} := \mathscr{L}^f \cup \mathscr{L}^o \cup \mathscr{L}^c \cup \mathscr{L}^{\mathsf{sbt}} \cup \overline{\mathscr{L}^{\mathsf{sbt}}} \cup \mathscr{L}^{\mathsf{col}} \cup \mathscr{L}^{\mathsf{ctrns}} \cup \mathscr{L}^{\mathsf{strns}} \cup \overline{\mathscr{L}^{\mathsf{strns}}}$$

For each $i \in \mathsf{nations}$, a normative system $\mathsf{NS}_i = (\mathscr{N}_i^{\mathsf{sbt}}, \mathscr{N}_i^{\mathsf{col}})$ *comprises a set of substantive norms $\mathscr{N}_i^{\mathsf{sbt}} \subseteq \mathscr{L}_i^{\mathsf{sbt}}$ and a set of CoL norms $\mathscr{N}_i^{\mathsf{col}} \subseteq \mathscr{L}_i^{\mathsf{col}}$. A* knowledge base *$\mathscr{K} = (\mathscr{F}, \{\mathsf{NS}_i \mid i \in \mathsf{nations}\}, \mathscr{C})$ consists of facts $\mathscr{F} \subseteq \mathscr{L}^f$, normative systems NS_i for all $i \in \mathsf{nations}$, and constraints $\mathscr{C} \subseteq \mathscr{L}^c$. We assume $\mathscr{F}$ and $\mathscr{C}$ to each be consistent.*

Example 4 (Ex. 2 continued) *The normative systems of Hungary and France are given by $\mathsf{NS}_{hun} = (\mathscr{N}_{hun}^{\mathsf{sbt}}, \mathscr{N}_{hun}^{\mathsf{col}})$ and $\mathsf{NS}_{fra} = (\mathscr{N}_{fra}^{\mathsf{sbt}}, \mathscr{N}_{fra}^{\mathsf{col}})$, where $\mathscr{N}_{hun}^{\mathsf{sbt}} = \{(\mathtt{hou}, \mathtt{sib})_{hun}\}$, $\mathscr{N}_{hun}^{\mathsf{col}} = \{(\mathtt{dec}, fra)_{hun}\}$, $\mathscr{N}_{fra}^{\mathsf{sbt}} = \{(\mathtt{dec}, \mathtt{spo})_{fra}\}$, and $\mathscr{N}_{fra}^{\mathsf{col}} = \{(\mathtt{hou}, hun)_{fra}\}$. The set of facts is $\mathscr{F} = \{\mathtt{dec}^f, \mathtt{hou}^f\}$. In sum, our knowledge base is $\mathscr{K} = (\mathscr{F}, \{\mathsf{NS}_{hun}, \mathsf{NS}_{fra}\}, \emptyset)$.*

Deontic Argumentation Calculi: Substantive Norms. CoLC are defined as an extension of deontic argumentation calculi (DAC) [5]. DAC are Gentzen-style sequent calculi [8] whose rules manipulate sequents. Sequents, or *arguments* [9], are of the form $\Gamma \Rightarrow \Delta$, where Γ and Δ are finite sets of formulas where Γ (read conjunctively) provides reasons for concluding Δ (read disjunctively). We are interested in two types of argument:

$$\underbrace{\mathtt{hou}^f, (\mathtt{hou}, \mathtt{sib})_{hun}}_{\textit{reasons...}} \Rightarrow \underbrace{\mathtt{sib}^o}_{\textit{for}} \qquad \underbrace{\mathtt{hou}^f, \neg\mathtt{sib}^c}_{\textit{reasons...}} \Rightarrow \underbrace{\neg(\mathtt{hou}, \mathtt{sib})_{hun}}_{\textit{for}}$$

The argument on the left provides reasons for an obligation and the one on the right gives reasons for the inapplicability of a norm. We interpret the two arguments as "given that the house is in Hungary and the norm $(\mathtt{hou}, \mathtt{sib})_{hun}$, the siblings should inherit the house," respectively "given the fact that the house is in Hungary and the constraint that the obligations must be consistent with the siblings not inheriting the house, the norm $(\mathtt{hou}, \mathtt{sib})_{hun}$ is inapplicable." We now discuss the rules that generate such arguments.

Since DAC enables reasoning with the substantive norms of a single normative system, DAC-rules must be generalized to accommodate multiple normative systems. The resulting rules are depicted in Figure 1. Henceforth, we assume a classical logic sequent calculus LK as the underlying base logic. Furthermore, we only present one generalized DAC-system (Figure 1) but stress that the formalism presented in this paper extends to the entire class of calculi in [5]. Here, we briefly discuss the adapted rules.

The rule **Ax** stipulates that a labelled version of each LK-derivable sequent $\Delta \Rightarrow \Gamma$ may be taken as an initial sequent in a CoLC derivation (LK-rules can be shown admissible [5]). Hence, we can use classical logic to reason about facts, obligations and constraints. The rule **Sub-Det**$_i$ introduces initial sequents expressing detachment for reasoning with substantive norms: given fact φ^f and norm $(\varphi, \psi)_i$, we detach the obligation ψ_i^o (facts are referred to as the input and obligations as the output). The rule **L-CT**$_i$ corresponds to successive detachment [4], expressing that the obligations detached from one norm may serve as additional input for detaching obligations from other norms. The side condition on **L-CT**$_i$ is important: we only allow for successive detachment from norms of the same nation. **Cut** is the only structural rule of the calculus ([8]).

Of particular interest are the rules **R-C**$_i$ and **R-N**$_i$ for reasoning with constraints and the inapplicability of norms [5]. **R-C**$_i$ expresses that if some set Δ provides reasons for the obligation φ_i^o (i.e., the rule's premise), then Δ is inconsistent with a constraint stipulating that the obligations must be consistent with $\neg\varphi^c$ (i.e., the rule's conclusion). Namely, an empty right hand side of a sequent expresses the inconsistency of the sequent's left hand side [8]. Considering **R-N**$_i$, when $\Delta^f \subseteq \mathscr{L}^f$, $\Delta^c \subseteq \mathscr{L}^c$, and $\Delta_i^{\mathsf{sbt}} \cup \{(\varphi, \psi)_i\} \subseteq \mathscr{L}^{\mathsf{sbt}}$, are jointly inconsistent (i.e., the rule's premise), we know at least one of the used norms is inapplicable, and so, Δ^f, Δ^c, and Δ_i^{sbt} provide reasons for the inapplicability of $(\varphi, \psi)_i$. These two rules, **R-C**$_i$ and **R-N**$_i$, enable argumentative reasoning with conflicts among substantive norms in Section 3.

Example 5 *Consider* $\varphi^f, (\varphi, \psi)_i \Rightarrow \psi_i^o$ *and* $\theta^f, (\theta, \neg\psi)_i \Rightarrow \neg\psi_i^o$. *The two conclusions are inconsistent, and we can derive* $\varphi^f, \theta^f, (\varphi, \psi)_i, (\theta, \neg\psi)_i \Rightarrow$ *(see [5] for example derivations). By consecutive application of* ***R-C****$_i$,* ***R-N****$_i$, and* ***Cut*** *we obtain* $\varphi^f, \theta^f, (\varphi, \psi)_i \Rightarrow \neg(\theta, \neg\psi)_i$*, giving reasons for the inapplicability of the norm* $(\theta, \neg\psi)_i$.

CoL Calculi: Conflict of Laws Norms. So far, we modeled reasoning with each nation's substantive norms individually. We now turn to CoL norms. Nations may have different mechanisms at play, but share two minimal requirements:

- **Function.** For each context for each nation, there is exactly one CoL norm.
- **Loop.** Once a nation is encountered whose CoL norms were already considered during the CoL reasoning, the process terminates.

The first condition signifies the deterministic aspect of CoL reasoning, whereas the second avoids transmission cycles. Hence, since there are finitely many nations, CoL reasoning always terminates. In addition, each nation i may employ other mechanisms:

- **Limit.** The upper limit of CoL transitions is lim_i, for some $\mathsf{lim}_i \geq 1$.
- **Renvoi.** The upper limit of CoL transitions is lim_i unless the last nation's CoL norms refer back to the Lex Fori (i.e., the law of the original nation).

These four CoL conditions are formalized by the sequent rules in Figure 2. The rule **CoL-Det**$_i$ introduces initial sequents $\varphi^f, (\varphi, j)_i \Rightarrow \langle i, j \rangle_{\mathsf{col}}$ that express that under the fact φ and nation i's CoL norm $(\varphi, j)_i$ we are redirected to nation j's legal system $\langle i, j \rangle_{\mathsf{col}}$. In other words, such initial sequents express the first transmission in the CoL reasoning.

$$\textbf{CoL-Det}_i \quad \varphi^f, (\varphi, j)_i \Rightarrow \langle i, j \rangle_{\mathsf{col}}$$

$$\textbf{Col-Trans}_i^a \; \frac{\Delta_1 \Rightarrow \langle i, \dots, j \rangle_{\mathsf{col}} \qquad \Delta_2 \Rightarrow \langle j, k \rangle_{\mathsf{col}}}{\Delta_1, \Delta_2 \Rightarrow \langle i, \dots, j, k \rangle_{\mathsf{col}}}$$

Side-condition *a*: (read conjunctively)

Loop* There is no loop in $\langle i, \dots, j \rangle_{\mathsf{col}}$;
Limit† $|\langle i, \dots, j \rangle_{\mathsf{col}}| < \mathsf{lim}_i$; **or**‡ **Renvoi**† $|\langle i, \dots, j \rangle_{\mathsf{col}}| < \mathsf{lim}_i$ or $(|\langle i, \dots, j \rangle_{\mathsf{col}}| = \mathsf{lim}_i \;\&\; k = i)$.

$$\textbf{Sub-Trans}_i^b \; \frac{\Delta \Rightarrow \langle i, \dots, j \rangle_{\mathsf{col}}}{\Delta \Rightarrow \langle i, \dots, j \rangle_{\mathsf{sbt}}}$$

Side-condition *b*: (read disjunctively)

Loop* j establishes a loop with some nation label in $\langle i, \dots, j \rangle_{\mathsf{col}}$ prior to j;
Limit† $|\langle i, \dots, j \rangle_{\mathsf{col}}| < \mathsf{lim}_i$; **or**‡ **Renvoi**† $|\langle i, \dots, j \rangle_{\mathsf{col}}| = \mathsf{lim}_i$ or $|\langle i, \dots, j \rangle_{\mathsf{col}}| = \mathsf{lim}_i + 1$.

$$\textbf{Renvoi-Def}_i^c \; \frac{\Delta \Rightarrow \langle i, \dots, l, \dots, j, k \rangle_{\mathsf{sbt}}}{\Delta \Rightarrow \neg \langle i, \dots, l \rangle_{\mathsf{sbt}}}$$

Side-condition *c*:

Renvoi $|\langle i, \dots, j, k \rangle_{\mathsf{sbt}}| = \mathsf{lim}_i + 1$.

Figure 2. The CoL rules extending the rules of Fig. 1 with $|\langle i, \dots, j \rangle_{\mathsf{col}}| = \mathsf{card}(\{i, \dots, j\}) - 1$. * Loop is always imposed, † Limit and Renvoi only if adopted by nation i, ‡ no nation adopts both Limit and Renvoi.

The rule **Col-Trans**$_i$ stipulates how we may continue the CoL reasoning. Namely, if Δ_1 provides reasons for the (indirect) CoL transition from i to j and Δ_2 provides reasons for a transition from j to k, then Δ_1 and Δ_2 together provide the reasons for an (indirect) CoL transition from i to k. The side-condition on **Col-Trans**$_i$ expresses when we may make these transitions. This side-condition must be read conjunctively and adopting Limit or Renvoi is mutually exclusive. The **Loop** condition ensures that we may only continue the CoL reasoning provided we are not in a loop. A loop is defined as the double occurrence of a nation in the sequence. The **Limit** condition limits the amount of transmissions. Here $|\langle i, \dots, j \rangle_{\mathsf{col}}|$ stands for the number of transmissions underlying the sequence: $\mathsf{card}(\langle i, \dots, j \rangle_{\mathsf{col}}) - 1$. The **Renvoi** condition allows for an additional transmission after the limit has been reached, provided the transition redirects to the lex fori.

Once the CoL reasoning terminates (either through a loop or a limit), we may start reasoning with the substantive norms of the appointed nation in the CoL sequence. This is expressed by the rule **Sub-Trans**$_i$, which stipulates the conditions under which we may move from a CoL-sequence $\langle i, \dots, j \rangle_{\mathsf{col}}$ and start reasoning with the substantive norms of nation j. This license is expressed by $\langle i, \dots, j \rangle_{\mathsf{sbt}}$. The side-condition of **Sub-Trans**$_i$ corresponds to the three conditions of **Col-Trans**$_i$ but must be read disjunctively: namely, either a loop has been obtained, or the limit has been reached, or a case of renvoi ensues.

Renvoi takes the limit of CoL transitions for nation i to be lim_i, *unless* the next CoL transition refers back to i. Hence, the CoL transition initiated by the limit is *defeasible* upon the existence of a renvoi. The rule **Renvoi-Def**$_i$ expresses this: if a CoL sequence of length $\mathsf{lim}_i + 1$ is derived that refers back to i (renvoi), then we infer the annulation of the previous license expressed by $\neg\langle i, \dots, j \rangle_{\mathsf{sbt}}$ (this mechanism will be formally discussed in Section 3). The side-condition on **Renvoi-Def**$_i$ ensures that only the presence of a renvoi revokes the substantive norm reasoning licenses by the limit. That $i = k$ in the CoL sequence of this rule, is guaranteed by the **Renvoi** side-condition imposed on **Col-Trans**$_i$.

The class of Conflict of Laws Calculi (CoLC) is defined in Definition 2.

Definition 2 *Let* $\mathsf{CoLC}_{\emptyset}$ *be the base system containing for each* $i \in \mathsf{nations}$ *the rules* ***Ax****,* ***Sub-Det****$_i$,* ***L-CT****$_i$,* ***Cut****,* ***R-C****$_i$, and* ***R-N****$_i$ from Figure 1 and the rules* ***CoL-Det****$_i$,* ***Col-Trans****$_i$, and* ***Sub-Trans****$_i$ from Figure 2. A calculus* $\mathsf{CoLC}_{\mathscr{S}}$ *extends* $\mathsf{CoLC}_{\emptyset}$ *with the rules* $\mathscr{S} \subseteq \{$***Renvoi-Def****$_i \mid i \in \mathsf{nations}\}$. The* ***Loop*** *condition is always active. The adaptation of* ***Limit*** *varies from nation to nation. The* ***Renvoi*** *condition is imposed iff* ***Renvoi-Def****$_i \in \mathscr{S}$.*

A $\mathsf{CoLC}_{\mathscr{S}}$*-derivation of* $\Delta \Rightarrow \Gamma$ *is a tree whose leafs are initial sequents of* $\mathsf{CoLC}_{\mathscr{S}}$*, whose root is* $\Delta \Rightarrow \Gamma$*, and whose rule-applications are instances of the rules of* $\mathsf{CoLC}_{\mathscr{S}}$*.*

Example 6 (Ex. 4 continued) *Consider the following* CoLC*-derivable arguments from the knowledge base* $\mathscr{K}$ *of Example 4:*

$$A := \mathtt{dec}^f, (\mathtt{dec}, \mathit{fra})_{\mathit{hun}} \Rightarrow \langle \mathit{hun}, \mathit{fra}\rangle_{\mathsf{col}} \qquad B := \mathtt{hou}^f, (\mathtt{hou}, \mathit{hun})_{\mathit{fra}} \Rightarrow \langle \mathit{fra}, \mathit{hun}\rangle_{\mathsf{col}}$$
$$C := \mathtt{dec}^f, \mathtt{hou}^f, (\mathtt{dec}, \mathit{fra})_{\mathit{hun}}, (\mathtt{hou}, \mathit{hun})_{\mathit{fra}} \Rightarrow \langle \mathit{hun}, \mathit{fra}, \mathit{hun}\rangle_{\mathsf{col}}$$
$$D := \mathtt{dec}^f, (\mathtt{dec}, \mathit{fra})_{\mathit{hun}} \Rightarrow \langle \mathit{hun}, \mathit{fra}\rangle_{\mathsf{sbt}}$$
$$E := \mathtt{dec}^f, (\mathtt{dec}, \mathit{fra})_{\mathit{hun}}, \mathtt{hou}^f, (\mathtt{hou}, \mathit{hun})_{\mathit{fra}} \Rightarrow \langle \mathit{hun}, \mathit{fra}, \mathit{hun}\rangle_{\mathsf{sbt}}$$
$$F := \mathtt{dec}^f, \mathtt{hou}^f, (\mathtt{dec}, \mathit{fra})_{\mathit{hun}}, (\mathtt{hou}, \mathit{hun})_{\mathit{fra}} \Rightarrow \neg\langle \mathit{hun}, \mathit{fra}\rangle_{\mathsf{sbt}}$$
$$G := \mathtt{hou}^f, (\mathtt{hou}, \mathtt{sib})_{\mathit{hun}} \Rightarrow \mathtt{sib}^o_{\mathit{hun}} \qquad H := \mathtt{dec}^f, (\mathtt{dec}, \mathtt{spo})_{\mathit{fra}} \Rightarrow \mathtt{spo}^o_{\mathit{fra}}$$

CoL-Det*$_i$ gives the arguments A and B, which via an application of* ***CoL-Trans****$_{hun}$, yield C. Although the limit* $\mathsf{lim}_{hun} = 1$ *is reached with* $\langle hun, fra\rangle_{\mathsf{col}}$ *in B, due to renvoi the transition* $\langle hun, fra, hun\rangle_{\mathsf{col}}$ *in C is warranted. Applying* ***Sub-Trans****$_{hun}$ to A, respectively C, we derive D and E licensing the application of substantive norms for France and Hungary. Here, renvoi defeasibility enters by applying* ***Renvoi-Def****$_{hun}$ to E, generating F concluding* $\neg\langle hun, fra\rangle_{\mathsf{sbt}}$*. No further CoL reasoning is possible from the perspective of Hungary. So, by substantive norm reasoning using* ***Sub-Det****$_i$ (for* $i \in \{hun, fra\}$*) we obtain arguments G and H. However, in H, the French substantive norm is applied, but F* objects*: substantive norm reasoning from the perspective of France is not justified. In contrast, G is* supported *by the CoL argument E. In the next section we make the notions of objection/attack and support precise by embedding* CoLC *in formal argumentation.*

3. CoLC and Logical Argumentation

We saw that CoLC reflects reasoning with substantive norms and reasoning with CoL norms. Arguments generated by the latter support arguments of the former. Namely, $\Delta \Rightarrow \langle i, \ldots, j\rangle_{\mathsf{sbt}}$ provides an inference license to nation i for applying the substantive norms of nation j, e.g., as in the argument $\psi^f, (\psi, \varphi)_j \Rightarrow \varphi^o_j$. Such inference licenses we call *support*. This interaction is captured in CoLC-induced *argumentation frameworks*.

An argumentation framework (AF) consists of a set of arguments and an attack relation between arguments [6], where defeasibility is captured by argumentative attacks. We now formalize CoL reasoning using formal argumentation. At the end of this section, we illustrate our approach with two examples formalizing renvoi and forum shopping. We assume an arbitrary $\mathsf{CoLC}_{\mathscr{S}}$ and $\mathscr{K} = (\mathscr{F}, \{\mathscr{N}_i^{\mathsf{sbt}} \mid i \in \mathsf{nations}\}, \mathscr{C})$.

Definition 3 $\Delta \Rightarrow \Gamma$ *is* $\mathsf{CoLC}_{\mathscr{S}}$-derivable *from* $\mathscr{K}$ *iff it is* $\mathsf{CoLC}_{\mathscr{S}}$*-derivable and* $\Delta \subseteq \mathscr{K}$*. The set of all* $\mathsf{CoLC}_{\mathscr{S}}$*-derivable sequents from* $\mathscr{K}$ *is* $\mathsf{Deriv}(\mathscr{K})$*. Henceforth, we omit reference to* $\mathscr{K}$ *where the context disambiguates. We distinguish the following* arguments*:*

- *The set* ArgCol *[resp.* $\overline{\mathsf{ArgCol}}$*] contains all arguments* $\Delta \Rightarrow \langle i, \ldots, j\rangle_{\mathsf{sbt}} \in \mathsf{Deriv}$ *[resp.* $\Delta \Rightarrow \neg\langle i, \ldots, j\rangle_{\mathsf{sbt}} \in \mathsf{Deriv}$*]. These arguments state that in the current situation, starting from nation i the laws of j are [resp. not] binding.*

- *The set* ArgSbt_i *contains all arguments* $\Delta_f, \Delta_i, \Delta_c \Rightarrow \Gamma \in \mathsf{Deriv}$ *stating that for nation i given facts* Δ_f *and constraints* Δ_c*,* Γ *follows. Let* $\mathsf{ArgSbt} = \bigcup_{i \in \mathsf{nations}} \mathsf{ArgSbt}_i$*.*
- *We let* $\mathsf{ArgSup} \subseteq (\mathsf{ArgCol} \times \mathsf{ArgSbt})$ *be pairs* $\langle \Delta \Rightarrow \langle i, \ldots, j \rangle_{\mathsf{sbt}} \,;\, \Delta' \Rightarrow \Gamma \rangle$ *of arguments where* $\Delta' \Rightarrow \Gamma \in \mathsf{ArgSbt}_j$*. The idea is that* $\Delta \Rightarrow \langle i, \ldots, j \rangle_{\mathsf{sbt}}$ *provides the justification (support) for applying the norms of nation j expressed in* $\Delta' \Rightarrow \Gamma$*. (For instance* $\langle \Delta \Rightarrow \langle 1,3,2 \rangle_{\mathsf{sbt}} \,;\, p_f, (p,q)_2 \Rightarrow q_2^o \rangle$ *is an argument of type* ArgSup*.)*
- *Finally, we let* $\mathsf{Arg} = \mathsf{ArgCol} \cup \overline{\mathsf{ArgCol}} \cup \mathsf{ArgSbt} \cup \mathsf{ArgSup}$*.*

Definition 4 *We define four types of attacks* $\mathsf{Att} = \mathsf{AttRen} \cup \mathsf{AttFS} \cup \mathsf{AttSbt}$*, where*

- *Renvoi-Attacks* $\mathsf{AttRen} \subseteq (\overline{\mathsf{ArgCol}} \times (\mathsf{ArgCol} \cup \mathsf{ArgSup}))$*:*
  - $(\Delta \Rightarrow \neg\langle i, \ldots, j \rangle_{\mathsf{sbt}},\ \Delta' \Rightarrow \langle i, \ldots, j \rangle_{\mathsf{sbt}}) \in \mathsf{AttRen}$*, and*
  - $(a,\ (b,c)) \in \mathsf{AttRen}$ *iff* $(a,b) \in \mathsf{AttRen}$*.*
- *Forum Shopping Attacks* $\mathsf{AttFS} \subseteq (\mathsf{ArgCol} \times (\mathsf{ArgCol} \cup \mathsf{ArgSup}))$*:*
  - $(\Delta \Rightarrow \langle i, \ldots, j \rangle_{\mathsf{sbt}},\ \Delta' \Rightarrow \langle k, \ldots, m \rangle_{\mathsf{sbt}}) \in \mathsf{AttFS}$ *iff* $i \neq k$*, and*
  - $(a,\ (b,c)) \in \mathsf{AttFS}$ *iff* $(a,b) \in \mathsf{AttFS}$*.*
- *Simple Substantive Attacks* $\mathsf{AttSbt}_i \subseteq (\mathsf{ArgSbt}_i \times \mathsf{ArgSbt}_i)$*:* $(\Delta \Rightarrow \neg(\varphi,\psi)_i,\ \Delta', (\varphi,\psi)_i \Rightarrow \Gamma') \in \mathsf{AttSbt}_i$*.*
- *Substantive Attacks* $\mathsf{AttSbt} \subseteq (\mathsf{ArgSup} \times \mathsf{ArgSup})$*:* $((a,b),\ (a',b')) \in \mathsf{AttSbt}$ *iff* $(b,b') \in \mathsf{ArgSbt}_i$ *for some* $i \in \mathsf{nations}$*.*

Definition 5 *Let a* $\mathsf{CoLC}_{\mathscr{S}}$*-induced AF over* $\mathscr{K}$ *be* $AF_{\mathscr{S}}(\mathscr{K}) = \langle \mathsf{Arg}(\mathscr{K}), \mathsf{Att}(\mathscr{K}) \rangle$*.*

Definition 6 *Let* $(\mathsf{Arg}, \mathsf{Att})$ *be an AF and let* $\mathscr{E} \subseteq \mathsf{Arg}$*. We say that* $\mathscr{E}$ *is* conflict-free *if for all* $a, b \in \mathscr{E}$*,* $(a,b) \notin \mathsf{Att}$*; and say that* $\mathscr{E}$ *is* stable *if it is conflict-free and* $(\forall a \in \mathsf{Arg} \setminus \mathscr{E})(\exists b \in \mathscr{E})$ *such that* $(b,a) \in \mathsf{Att}$ *([6]). Let* `Stable` *be the set of stable extensions of AF. A skeptical nonmonotonic consequence relations can then be defined, e.g., by (see [10] for variants):* $AF \mid\!\sim \varphi^o$ *iff* $(\forall \mathscr{E} \in \mathtt{Stable})\ (\exists i \in \mathsf{nations})\ (\exists a \in \mathscr{E})$ *concluding* φ_i^o*.*

Our approach follows the logical argumentation tradition [9] to structured argumentation, using sequent calculi to generate the arguments of an AF and defining attacks in terms of the arguments' syntax. Renvoi and (simple) substantive attacks are called rebuttals, respectively direct defeats in logical argumentation. The use of ArgSup-pairs (Def. 3) is idiosyncratic but resembles techniques from bipolar argumentation [11], which extends AFs with support relations. We also note that argumentative representations of normative reasoning are receiving increasing interest [12,13,14,15].

We state two central properties of CoLC. Prop. 1 expresses that CoLC incorporate the reasoning with substantive norms based on Input/Output logic [4]. Prop. 2 shows that stable extensions adequately model forum shopping since their arguments are based on a unique CoL-sequence and (only) apply substantive laws from the target nation.

Proposition 1 $\mathsf{CoLC}_{\mathscr{S}}$ *is a conservative extension of the* DAC*-system [5] and so characterizes (the constrained version of) Input/Output logic* Out_3 *[4].*

Proposition 2 *Let* $\mathscr{E} \in \mathtt{stable}(\mathscr{K})$ *for a knowledge base* $\mathscr{K}$*.*

1. *There is a* $\langle i, \ldots, j \rangle_{\mathsf{sbt}}$ *such that for all* $\Delta \Rightarrow \varphi \in (\mathscr{E} \cap \mathsf{ArgCol})$ *and for all* $(\Delta \Rightarrow \varphi; B) \in (\mathscr{E} \cap \mathsf{ArgSup})$*,* $\varphi = \langle i, \ldots, j \rangle_{\mathsf{sbt}}$*.*
2. *There is an* $i \in \mathsf{nations}$ *s.t. for all* $\Delta \Rightarrow \Gamma \in \mathscr{E}$ *and all* $(\Delta \Rightarrow \Gamma; \Delta' \Rightarrow \varphi) \in \mathscr{E}$*,* $\Delta \subseteq \mathscr{F} \cup \mathscr{C} \cup \mathscr{N}_i^{\mathsf{col}}$*,* $\Delta' \subseteq \mathscr{F} \cup \mathscr{C} \cup \mathscr{N}_i^{\mathsf{sbt}}$ *and* $\varphi \in \mathscr{L}_i^o \cup \mathscr{L}_i^p \cup \overline{\mathscr{N}_i^{\mathsf{sbt}}}$*.*

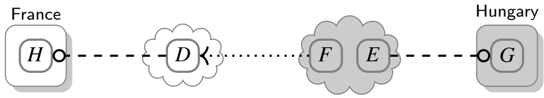

Figure 3. The AF based on the CoL-reasoning with starting point Hungary. Simple substantive arguments are in boxes to the left (France) and right (Hungary) in Ex. 7. Other arguments in clouds based on the col-sequences $\langle hun, fra\rangle_{\mathsf{col}}$ (left) and $\langle hun, fra, hun\rangle_{\mathsf{col}}$ (right). Supported arguments in ArgSup are represented by, for instance $E \multimap G$ for (E,G). Due to the renvoi attack (dotted arrow from F to D) we end up with Hungarian substantive laws (supported argument (E,G)).

Example 7 (Renvoi, Ex. 6 cont.) *The reasoning in our renvoi example is captured in the* CoLC*-induced AF in Figure 3. It represents the defeasible nature of CoL reasoning, and renvoi in particular. Due to renvoi, the justification* $\langle hun, fra\rangle_{\mathsf{sbt}}$ *provided by argument D for applying French law is defeated by F concluding* $\neg\langle hun, fra\rangle_{\mathsf{sbt}}$*. Acceptable is the argument* $(E,G) \in \mathsf{ArgSup}$ *that applies Hungerian law based on the justification* $\langle hun, fra, hun\rangle_{\mathsf{sbt}}$ *provided by E. The AF illustrates that the application of French substantive norms is defeated by the presence of a renvoi, initiating a redirection to the lex fori, applying Hungarian law instead. Last,* E, F*, and G form a stable extension.*

Example 8 (Forum Shopping.) *We illustrate the reasoning underlying forum shopping. Our agent (possibly a lawyer or a client) has the choice between three nations to start a legal precedure. Our knowledge base is given by* $\mathscr{K} = (\mathscr{F}, \{(\mathscr{N}_i^{\mathsf{sbt}}, \mathscr{N}_i^{\mathsf{col}}) \mid i \in \{1,2,3\},\}, \emptyset)$ *with* $\mathscr{F} = \{p\}$, $\mathscr{N}_1^{\mathsf{sbt}} = \{(p,\neg s),(p,u)\}$, $\mathscr{N}_2^{\mathsf{sbt}} = \{(p,s),(p,u)\}$, $\mathscr{N}_3^{\mathsf{sbt}} = \{(p,q),(p,\neg q),(p,u)\}$, $\mathscr{N}_1^{\mathsf{col}} = \{(p,2)\}$, $\mathscr{N}_2^{\mathsf{col}} = \{(p,3)\}$ *and* $\mathscr{N}_3^{\mathsf{col}} = \{(p,1)\}$*. We abbreviate* $\Delta_{12}^{\mathsf{col}} = \{p_f,(p,2)_1,(p,3)_2\}$, $\Delta_{312}^{\mathsf{col}} = \{(p,1)_3\} \cup \Delta_{12}^{\mathsf{col}}$, $\Delta_{31}^{\mathsf{col}} = \{p_f,(p,1)_3,(p,2)_1\}$.

We assume that nation 1 and 3 work with a limit of 2 CoL-transitions and all nations impose renvoi. Figure 4 illustrates the CoLC_1*-induced AF based on* $\mathscr{K}$*. Highlighted is one stable extension providing one choice for forum shopping, namely to start the legal precedure from nation 3. There are two other choices, to start from nation 2 (left) which leads via renvoi to the application of substantive laws from nation 2, or to start from nation 1, leading to the applications of the substational laws of nation 3. We notice that* u^o *is a skeptical consequence, unlike* s^o *which is not concluded in all stable extensions.*

Related Work and Conclusion. Despite various studies, several formal aspects of CoL remain understudied. [1] uses an extended language and semantics of Input/Output logic to characterize CoL rules and renvoi, [2] present a model based on modular assumption-based argumentation, and [16,3] based on defeasible logic. Unlike ours, the latter approaches do not formally study the meta-constraints (limits, renvoi, etc.) governing CoL-referral sequences. Similar to [2], our AFs are modular in that the substantive law applications of a single given nation j form a strongly connected component within an AF which is called upon by a CoL sequence $\langle i,\ldots,j\rangle_{\mathsf{sbt}}$. Similar to [16,3], CoL-norms serve the role of meta-rules activating substantive reasoning from a nation's private laws. In contrast, whereas our approach includes conflict reasoning in light of forum shopping, [16] identify two additional types of conflicts (e.g., when the legal systems of two involved nations yield conflicting outcomes for the same case). Last, [7] take a multi-modal approach. Their formalism does not deal with conflicts and represents the monotonic fragment of our CoL referral reasoning. In future work, we aim to extend CoLC to reason with public order constraints which prevent "outrages consequences" of CoL reasoning, that lead to an application of the private law of the lex fori instead (also see [16]).

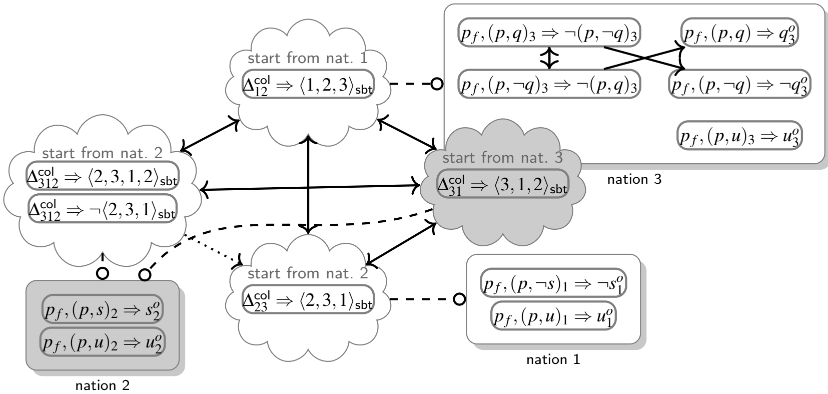

Figure 4. The AF based on Ex. 8. Boxes, clouds, dashed —∘ arrows and dotted arrows have the same meaning as in Fig. 3. Solid arrows represent substantive and forum shopping attacks. Arrows from clouds indicate that at least one argument in the cloud attacks all arguments in another cloud resp. supports every substantive argument in a rounded box. Highlighted is one out of three stable extensions.

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