

Particle Swarm Optimization Algorithm Based on Levy Flight

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Abstract: On account of the shortcomings of particle swarm optimization algorithm(PSO), such as poor global search capability and low convergence accuracy, levy flight ^[1] and Gaussian mutation ^[2] are used to propose particle swarm optimization algorithm based on levy flight. In iteration, the particle aggregation degree ^[3] is calculated, and the corresponding probability is selected to carry out levy flight ^[4] according to the particle aggregation degree, it strengthen the global optimization ability. Gaussian variation is carried out on particle positions of each iteration, and particles with better fitness are selected for iteration, which enhances the local search capability. At the same time, adaptive perturbation is carried out to the global optimal position of each iteration to increase the optimization precision of the optimal value. The test shows that the convergence rate of the improved algorithm is better than that of PSO, and the convergence accuracy is higher.

Keywords: levy flight; particle aggregation degree; Gaussian mutation

1. Introduction

PSO is a random search algorithm. It is a new intelligent optimization technology. It has fast calculation speed and better global optimization capability. However, PSO has the problems of the low traversal capability in the whole feasible solution regin and poor calculation accuracy. Aim to solve these defects, this paper proposes a particle swarm optimization algorithm based on levy flight, which increases the ability to traverse the entire solution space in the early stage of iteration. The late iteration can improve the convergence precision of the optimal value.

2. PSO

PSO is used to solve non-convex optimization problems. In this algorithm, each particle is affected by the search behavior of other particles in the group. Each particle will adjust its speed and position affected by the best value of each particle's own

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history and the best value of whole particles's history. The speed and position update formula of PSO is as follows :

$$V_{i+1} = w * V_i + c_1 * r_1 (P_{best} - X_i) + c_2 * r_2 (G_{best} - X_i) \quad (1)$$

$$X_{i+1} = X_i + V_{i+1} \quad (2)$$

In the above formula, V is the velocity of each iteration; X is the position of each iteration. P_{best} is the best adaptation position of each particle's own history; G_{best} is the best adaptation position of whole particles's history. w is the inertia weight. c_1, c_2 are individual learning factor and group learning factor. In PSO, the two random number generally taken in (0,1) uniform distribution.

3. Particle Swarm Optimization Algorithm Based on Levy Flight

In order to enhance the traversal ability over the whole feasible solution region, convergence speed and accuracy of the algorithm, the value of aggregation degree is calculated and normalized during the iterative optimization. The closer the value of aggregation degree is to 0, the smaller the distance between particles is, and levy flight is carried out with a larger probability. The closer the aggregation degree is to 1, the greater the distance between particles is. At this time, PSO is used to optimize. At the same time, as the iteration increases, the probability of levy flight is continuously reduced, the probability of Gaussian mutation of particles is increased, and the convergence precision in the later stage of iteration is improved. In the whole iteration process, Gaussian perturbation is performed on the global optimal position after each iteration to advance the accuracy of the global optimal value. The improved method of the algorithm in this paper is as follows :

3.1. Aggregation Calculation

The distance between particles is positively correlated with the traversal ability of the algorithm in the whole solution space. This paper proposes a calculation method of particle aggregation degree.

$$h(i) = \frac{\sqrt{\sum_{i=1}^N (x_i - P_{best})^2}}{N} \quad (3)$$

$$H = \frac{h_{max} - h(i)}{h_{max}} \quad (4)$$

$$P_m = e^{-\frac{H}{3}} \quad (5)$$

In the above formula, the aggregation degree after each iteration, H is the normalized value. The closer H is to 1, the greater the degree of aggregation is, the smaller the odds of particles falling into local minima is ; the closer H is to 0, the greater the aggregation degree is, the smaller the particle is, and the greater the possibility of the particle falling into the local minimum is.

3.2. Levy Flight

Levy flight is a random search path that follows the Levy distribution between short-distance and occasional long-distance walking^[5]. It has better global search ability than random walking. The formula of Levy flight is:

$$x_i^{t+1} = x_i^t + a \oplus t^{-\lambda}, \quad 1 < \lambda \leq 3 \tag{6}$$

where \oplus is point-to-point multiplication; a represents the step control quantity. The step size of Levy flight conforms to Levy distribution, which is often simulated by Mantegna algorithm. The step size s is calculated by :

$$s = \frac{\mu}{|v|^{\frac{1}{\beta}}} \tag{7}$$

$$\sigma_{\mu} = \left\{ \frac{\Gamma(1+\beta) \sin(\frac{\pi\beta}{2})}{\Gamma(\frac{1+\beta}{2}) \beta \cdot 2^{\frac{\beta-1}{2}}} \right\}^{\frac{1}{\beta}} \tag{8}$$

$$\sigma_v = 1 \tag{9}$$

Where $\mu \sim N(0, \sigma_{\mu}^2)$, $v \sim N(0, \sigma_v^2)$ and β usually takes 1.5.

3.3. Gaussian Variation

In the later iteration, the otherness of particles will gradually descend, and the phenomenon of iteration stagnation will appear. In order to find the exact solution to the global optimal value, the following Gaussian mutation formula is introduced^[7] :

$$x_{new} = x * (1 + 0.5\mu) \tag{10}$$

In the above formula, μ is a random number taken in (0,1) uniform distribution, x_{new} is a new population generated by Gaussian mutation of particle position x . The x_{new} and x populations are merged, and the first N particles with good fitness are selected by calculating the fitness value for the next iteration.

3.4. Global Optimal Position Gaussian Disturbance

Finding the global optimal position with better fitness can greatly improve the convergence speed. Therefore, this paper performs Gaussian perturbation on the global optimal position after each iteration, and selects the optimal fitness position to update. The perturbation formula is as follows :

$$G2_{best} = G_{best} * (1 + 0.5\mu) \tag{11}$$

4. Brief Introduction of Algorithm Flow

The flow chart of algorithm design is shown in Figure 1.

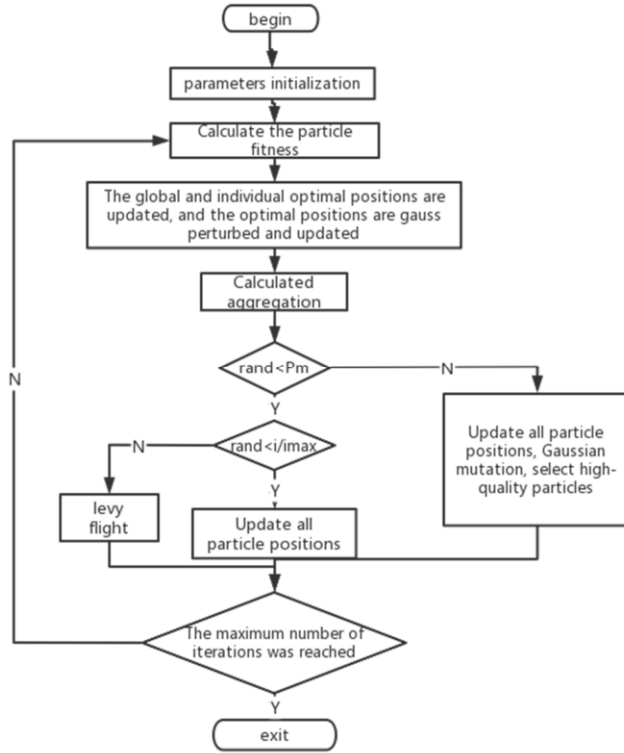


Figure 1. Algorithm flow chart

As shown in Figure 1, the specific operation steps of the algorithm are as follows:

(1) Data initialization such as population number, dimension, speed, position, inertia weight and learning factor.

(2) Plug each particle into the objective function and calculate the result ;

(3) Upgrade the best value of each particle’s history, the best fitness of group and the best position of each particle’s history. The best adaptive position is selected, and the Gaussian disturbance is applied to the optimal fitness position by Formula (11) to select a better global optimal position.

(4) According to the formula (3)~(5), the particle aggregation degree and the corresponding value of the aggregation degree value are calculated.

(5) Randomly generate a random number a in (0,1) and compare with it. If $a < P_m$, then turn step (6), otherwise turn step (8) ;

(6) Randomly generate a random number b in (0,1) and compare it with it. If $b < \frac{i}{i_{max}}$, the particle swarm is updated in (1) ~ (2) standard particle swarm update mode, otherwise turn to step (7) ;

(7) All particles carry out Levy flight according to the formula (6) ~ (9), turn to step (9) ;

(8) The particle swarm is updated in the (1) ~ (2) standard particle swarm update method. After the update is completed, the Gaussian mutation is performed according to the formula (10), and the fitness of the mutated particle swarm and the original particle swarm is calculated. Select the first N particles with good fitness to update ;

(9) If the number of iterations is reached, exit; otherwise, turn step (2) to continue the loop ;

5. Experimental Verification

For the sake of confirming the superiority of the improved algorithm, PSO and Particle swarm optimization algorithm based on levy flight are used to compare the Ackley function, Rastrigin function and Griewank function as the optimization objective function. By calculating the minimum value of these three functions, the global traversal ability and search precision of the two algorithms are compared.

5.1. Experiment on Global Optimization Ability

In the two algorithms, the value range of each dimension of the initial particles is [-5.12,5.12], the inertia weight is linearly decreasing between [0.4,0.9], and the learning factor is 2. F1 is used to represent the improved algorithm. F2 is used to represent PSO. The following figure shows the convergence curves of the two algorithms under 50 particles, 10 dimensions and 500 iterations :

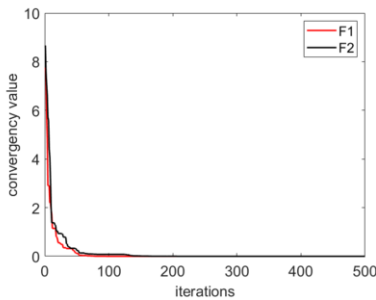


Figure 2. Fitness curve in Ackley test function

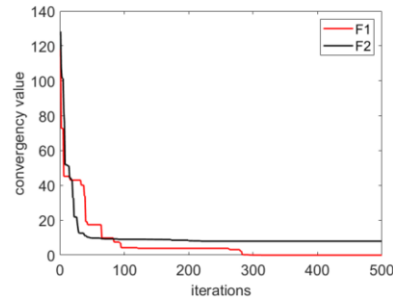


Figure 3. Fitness curve in Rastrigin test function

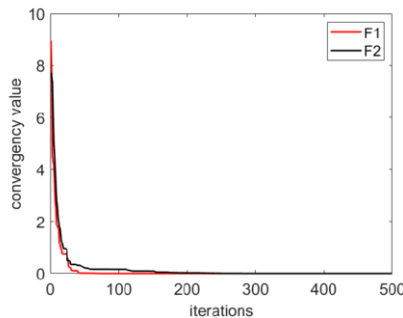


Figure 4. fitness curve in griewank test function

Ackley function, Griewank function has obvious global optimum advantage, and is often used to test the optimization accuracy of function. In Figure 2 and Figure 4, the fitness curve of improved algorithm is basically consistent with the optimization curve of PSO, indicating that the proposed algorithm has the same global optimization ability as the PSO in single-peak optimization. Rastrigin function is a typical multimodal function with a large number of local optimal values, it is often used to test the global optimization capability of algorithms in experiments. As shown in Figure 3, PSO falls into local optimal and cannot fly out of the local optimal area, it is unable to find the global optimal. However, the algorithm in this paper can jump out of the local optimal region by using Levy flight when it is failed in the local minimum^[6]. The global search capability is improved and the global optimal is found.

5.2. Search Accuracy Experiment

The following table is the average value of the maximum fitness of the two algorithms recorded in 20 consecutive experiments and the number of iterations when the maximum fitness is reached:

Table 1. Algorithm convergence times and convergence accuracy comparison table

test function	optimization method	optimum value	Average convergence value	Worst convergence value	Average iterations
Ackley	standard particle swarm optimization	0.0023	0.550	1.646223639394980	522
	proposed algorithm	0	1.85456632198742e-15	1.646223639394980e-12	451
Rastrigin	standard particle swarm optimization	3.9800	12.934483212133529	13.952950550003	496
	proposed algorithm	0	0.0556	3.3556	499
Griewank	standard particle swarm optimization	1.15001165990059e-06	3.04618209057139e-04	0.0074	474
	proposed algorithm	0	0	0	215

The above table shows that the convergence precision of Ackley function and Griewank improved algorithm is 12 and 13 orders of magnitude higher than that of standard particle swarm optimization algorithm respectively. In Rastrigin function, the algorithm not only overcomes the disadvantage of falling into the local optimum, but also converges to the global optimum. It can be concluded that the algorithm in this paper has very high optimization accuracy in the optimization problem of unimodal function. It can not only overcome the problem of poor global search ability of PSO, but also converge to the global optimum in the late iteration of the algorithm, which has a better application effect in the optimization of multi-local function. As shown in Figure 2, Figure 3, Figure 4 and Table 1, experiments show that the algorithm proposed in this paper has obvious advantages in global search ability, and the convergence accuracy is much higher than that of PSO.

6. Conclusion

In this paper, PSO is added to the aggregation degree based on the probability of levy flight method, so that the algorithm in the global search ability has been greatly enhanced; at the same time, Gaussian mutation is added to make it faster to find the global optimal point in the iterative process and improve the optimization accuracy. It has also been verified in experiments on basic test functions. Experiments show that the improved algorithm in this paper is obviously better than PSO in solving effect.

Acknowledgement

This paper is supported by the Postgraduate Innovation and Practice Ability Development Fund of Xi'an Shiyou University

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