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Constellation Configuration Analysis Based on Angle Condition Number for Tri-satellite TDOA Geolocation System

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Abstract. The geolocation performance of tri-satellite system based on time difference of arrival (TDOA) is significantly affected by the constellation configuration. In this paper, an angle condition number based method is proposed for constellation configuration analysis. First, the coefficient matrix of constellation configuration is derived by geolocation error analysis. Second, the angle condition number is adopted to measure the ill-condition of the coefficient matrix. Based on the measurement, the influence of constellation configuration on geolocation is deeply analyzed. The effectiveness of angle condition number on constellation configuration analysis is verified by simulations.

Keywords. Time difference of arrival, constellation configuration, angle condition number

1. Introduction

The geolocation of unknown radiation source is a hot research topic in satellite electromagnetic spectrum sensing. Time difference of arrival (TDOA) method is widely used to obtain the target location [1][2][3][4]. When the TDOA estimation accuracy is known, the geolocation accuracy is determined by the constellation configuration [5][6][7][8][9]. Therefore, many methods have been put forward to analyze the influence of constellation configuration on geolocation. The expressions of geolocation bias for TDOA geolocation system are derived in [10]. The geometry dilution of precision factor (GDOP) is derived for tri-satellite TDOA localization system in [11]. The optimum configuration, which minimize the geolocation error of single point, is derived in [12][13]. The relationship between GDOP and the triangle area formed by three satellites is derived in [14], which proved that the tri-satellite triangle area is inversely proportional to GDOP when the distances from radiation source to three satellites are equal. The ill-condition of coefficient matrix is preliminarily introduced to the geolocation error assessment in [15], but it is not discussed in detail.

In this paper, an angle condition number (ACN) based method for constellation configuration analysis is proposed. First, the coefficient matrix of constellation

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configuration is calculated by geolocation error analysis. In the following, the angle condition number is introduced to measure the ill-condition of the coefficient matrix. Based on the measurement, the influence of constellation configuration on geolocation is deeply analysed.

2. Geolocation error model

The tri-satellite TDOA geolocation solution equations are:

$$\begin{cases} r_2 - r_1 = c\tau_{12} \\ r_3 - r_1 = c\tau_{13} \\ x^2 + y^2 + z^2 = R_e^2 \end{cases} \tag{1}$$

where $r_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}, i = 1,2,3$ is the distance between the ith satellite and the radiation source. (x, y, z) denotes the coordinate of radiation source. (x_i, y_i, z_i) is the coordinate of the ith satellite. R_e denotes the radius of the earth. c denotes the light speed. τ_{12} is the TDOA measurement about satellite 1 and 2, τ_{13} is the TDOA measurement about satellite 1 and 3. The error equation can be obtained by differentiating the Eq. (1):

$$Cdu = d\tau + dR_e \tag{2}$$

where $du = [dx \quad dy \quad dz]^T$, $d\tau = [cd\tau_{12} \quad cd\tau_{13} \quad 0]^T$, $dR_e = [0 \quad 0 \quad dR_e]^T$ .The matrix C denotes the partial differential matrix of the radiation source, expressed as:

$$C = \begin{bmatrix} g_{x2} - g_{x1} & g_{y2} - g_{y1} & g_{z2} - g_{z1} \\ g_{x3} - g_{x1} & g_{y3} - g_{y1} & g_{z3} - g_{z1} \\ x / R_e & y / R_e & z / R_e \end{bmatrix} \tag{3}$$

where $g_{xi} = (x - x_i) / r_i, g_{yi} = (y - y_i) / r_i, g_{zi} = (z - z_i) / r_i, i = 1,2,3$. Therefore, the geolocation error can be measured by GDOP, which is calculated by:

$$GDOP(x, y, z) = \sqrt{tr(P)} \tag{4}$$

where matrix $P = E[du \cdot du^T]$ is the radiation source geolocation error covariance matrix. From the Eq. (2) and (4), it is found that the matrix C reflects the geometric constellation configuration relationship among three satellites and the radiation source, $d\tau$ is the TDOA estimation error, dR_e is the elevation error. For the definite signal, the $d\tau$ is determined when the TDOA estimation method is determined. For radiation source at sea level, the dR_e is determined meanwhile. Therefore, the geolocation error depends on the error propagation characteristics of the matrix C at this time [15], which can be calculated by condition number:

$$cond(C) = \| C^{-1} \| \cdot \| C \| \tag{5}$$

where $\| \cdot \|$ denotes the norm. The condition number reflects the influence of the perturbation. The condition number of matrix C reflects the disturbance degree of the constellation configuration on the geolocation error. When the condition number is too large, the system is ill-conditioned, which means the geolocation accuracy will decline.

3. Constellation configuration analysis based on angle condition number

Although the condition number can effectively analyze the geolocation accuracy, the interpretation is poor. For example, the condition number calculated by the 2-norm is related to the eigenvalue of matrix $C^T C$, which is not directly related to constellation configuration. To solve this problem, the concept of angle condition number is adopted, which can analyze the constellation configuration more easily. The detail is described in the following.

3.1. Concepts of angle condition number

For matrix $A \in R^{n \times n}$ with $\det(A) \neq 0$, $\alpha_i (i = 1, 2, ..., n)$ is the ith row vector, $A(i)$ is the Euclidean space composed of the row vectors $\alpha_1, \alpha_2, ..., \alpha_{i-1}, \alpha_{i+1}, ..., \alpha_n$, ξ is the orthogonal projection of α_i on $A(i)$, $w = \alpha_i - \xi$. So $\theta_i = \arcsin(\| w \|_2 / \| \alpha_i \|_2)$ can be called as the ith characteristic angle. The angle condition number of matrix A is [16]:

$$cond(A)_a = \min_{1 \le i \le n} \{\theta_i\} \tag{6}$$

For n-order nonsingular linear algebraic equations $AX = B$ and a given positive number $\varepsilon (0 < \varepsilon \le \pi / 2)$, if

$$cond(A)_a \le \varepsilon \tag{7}$$

then, $AX = B$ is an ill-conditioned system with respect to ε.

3.2. Geometry configuration analysis based on angle condition number

This part analyzes the error propagation characteristics of coefficient matrix C based on angle condition number. In order to simplify the expression, C can be rewritten as:

$$C = \begin{bmatrix} g_{x2} - g_{x1} & g_{y2} - g_{y1} & g_{z2} - g_{z1} \\ g_{x3} - g_{x1} & g_{y3} - g_{y1} & g_{z3} - g_{z1} \\ x / R_e & y / R_e & z / R_e \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \beta_2 - \beta_1 \\ \beta_3 - \beta_1 \\ \alpha_3 \end{bmatrix} \tag{8}$$

where $\beta_i = [(x-x_i)/r_i \quad (y-y_i)/r_i \quad (z-z_i)/r_i], i=1,2,3$ is the unit vector from the *i*th satellite pointing to the radiation source, and α_3 is the unit vector from the earth's core pointing to the radiation source. In the east-north-up (ENU) coordinate which is based on the radiation source, the three satellites can be marked as $s_i, i=1,2,3$, and the six vectors $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$ are shown in the Figure 1.(a).

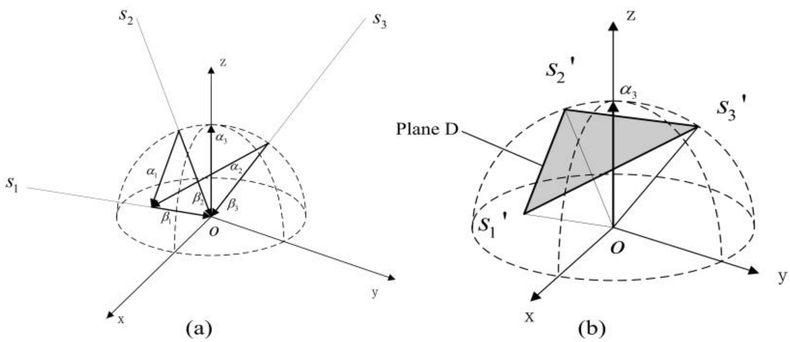

Figure 1. Geometry diagram: (a) the six vectors, (b) the Plane D

According to the concept of angle condition number, the angle between α_i and the plane composed of the remaining two vectors is used to analyze the geolocation error. Obviously, the vector α_3 is fixed, as the unit vector in the Z direction. The vectors α_1, α_2 change continuously with the motion of three satellites. Therefore, the plane composed by vectors α_1, α_2 (marked as plane D) should be discussed. Plane D is defined by three points $s_i', i=1,2,3$,which are the projections of three satellites on the unit sphere centered on the radiation source, just as showen in the Figure 1.(b). Thus, only the angle between Plane D and Z axis is required to be analyzed. When $cond(C)_a \le \varepsilon$, the system is ill-conditioned. It can be deduced that the plane D is almost perpendicular to plane XOY, while the constellation configuration is the worst at this time.

4. Numerical simulations

In this section, MATLAB and STK simulation are used to verify the proposed method. The simulation uses three satellites for radiation source geolocation. The orbital paraments at the initial time of three satellites are shown in the Table 1. The TDOA estimation error is 50 ns, the elevation error is 10 m. The radiation source is located at longitude 90 degrees west and latitude 10 degrees north. The condition number is calculated by the 2-norm.

Table 1. Simulation parameters of three satellites

	Satellite 1	**Satellite 2**	**Satellite 3**
Apogee Altitude (Km)	600	600	600
Perigee Altitude (Km)	600	600	600
Inclination (deg)	50	50	50
Argument of Perigee (deg)	0	0	0
Lon. Ascn. Node(deg)	-105	-100	-95
True Anomaly (deg)	5	3	5

As the satellites moving, the relative position between the radiation source and the tri-satellite constellation changes, as well as the geolocation error changes. The relationship of GDOP, condition number, angle condition number (ACN) and the reciprocal of ACN with time are shown in the Figure 2. It can be found that when $t = 256s$, GDOP is the largest, which means the constellation configuration has the greatest impact on the geolocation accuracy. At this time, the condition number reaches the maximum value, and the angle condition number is close to zero, which both reflect the system ill-condition.

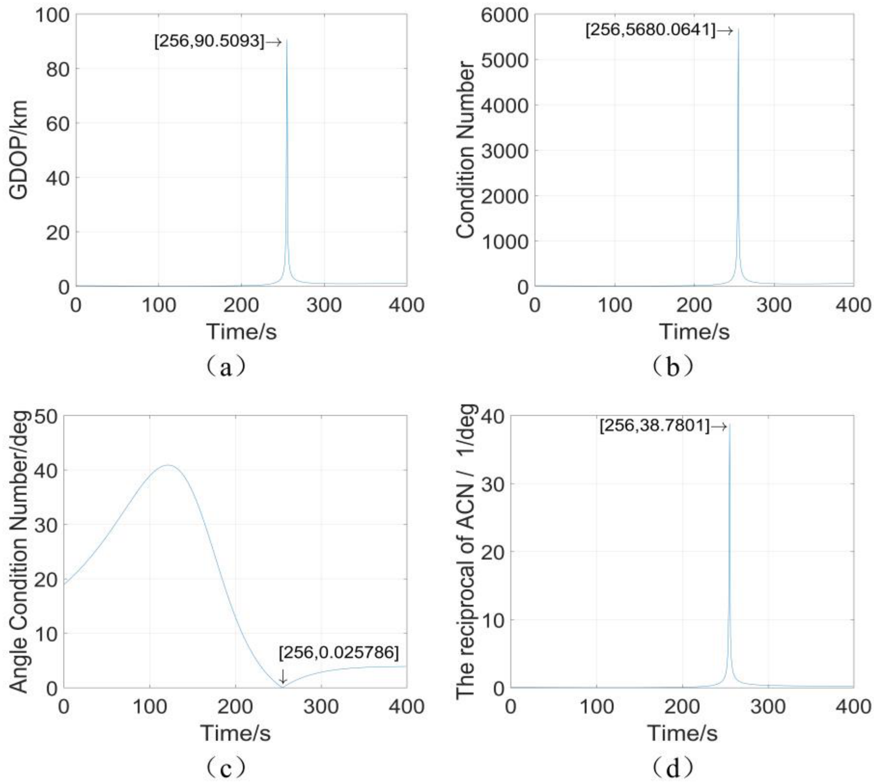

Figure 2. (a) GDOP with time, (b) condition number with time, (c) angle condition number with time, (d) the reciprocal of angle condition number with time

In order to further discuss the constellation geometric configuration when the geolocation accuracy is the worst, the three satellites coordinates can be obtained. The satellites' positions (in earth-centered earth-fixed coordinate system) and six vectors $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$ (in ENU coordinate system) are shown in the Table 2 and 3.

Table 2. Three satellites coordinate at $t = 256s$

Radiation source (Km)	[0 −6281.9 1100.2]
Satellite 1 (Km)	[−315.0 −6704.2 1910.3]
Satellite 1 (Km)	[126.2 −6757.7 1735.5]
Satellite 1 (Km)	[854.5 −6656.5 1912.0]

Table 3. Six vectors coordinate at $t = 256s$

Vector β_1	[0.3259 –0.7497 –0.5760]
Vector β_2	[–0.1571 –0.6756 –0.7203]
Vector β_3	[–0.6909 –0.5938 –0.4123]
Vector α_1	[–0.4830 0.0741 –0.1443]
Vector α_2	[–1.0169 0.1558 0.1637]
Vector α_3	[0 0 1]

After that, the six vectors geometric configuration can be got, as shown in the Figure 3. In this figure, the blue dotted line vector corresponds to $-\beta_1, -\beta_2, -\beta_3$, while the red solid line vector corresponds to $\alpha_1, \alpha_2, \alpha_3$. It can be seen that the plane D, where the vectors α_1, α_2 are located, is perpendicular to the XOY plane. In the physical sense, it corresponds that the plane composed of three points, which is projected by the three satellites on the unit sphere with the radiation source as the origin, is perpendicular to the XOY plane. That is consistent with the above analysis conclusion.

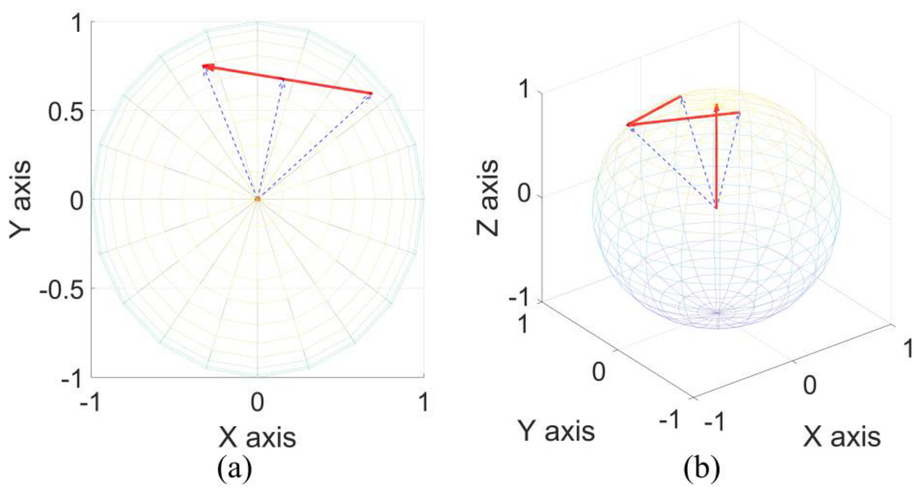

Figure 3. Six vectors geometric configuration: (a) view direct above, (b) view at an oblique 45°

5. Conclusion

This paper proposed a constellation configuration analysis method based on the angle condition number for tri-satellite TDOA geolocation system. This paper derived the constellation configuration coefficient matrix from geolocation error analysis model. Then the angle condition number is adopted to assess the ill-condition of the coefficient matrix. Finally, the influence of constellation configuration on geolocation is deeply analyzed. The simulation results showed that the angle condition number can effectively measure the influence of constellation configuration on geolocation accuracy.

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