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Precedential Constraint Derived from Inconsistent Case Bases

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Abstract. I explore a factor-based model of precedential constraint that, unlike existing models, does not rely on the assumption that the background set of precedent cases is consistent. The model I consider is a generalization of the reason model of precedential constraint that was suggested by Horty. I show that, within this framework, inconsistent case bases behave in a sensible and interesting way, both from a logical and a more practical perspective.

Keywords. precedential constraint, reason model, factors, inconsistent case-base

1. Introduction

According to the doctrine of precedent, the decisions of earlier courts constrain the decisions of later courts through the requirement that later decisions ought to be consistent with precedent decisions. Explaining how, exactly, precedent cases constrain future decisions—or what, exactly, "consistency" means—is a traditional problem in legal theory but has become, through the development of the reason model of constraint by Horty and Bench-Capon [1,2], a central concern in AI and Law as well.

The reason model, which builds on Lamond's theory of precedential constraint [3], supplements a factor-based representation of legal cases in the style of HYPO [4] and CATO [5] with priority orderings between sets of factors representing the strength of the reasons underlying the decisions of different courts. With respect to earlier proposals based on similar ideas [6,7], the key innovation is that these priority orderings are used to define a notion of consistency, and so a notion of constraint.

This has led to a number of developments in AI and Law that aim at refining the analysis of constraint by tackling, e.g., factors that can have multiple values [8,9,10], framework precedents [11], or issues [12]. A problem that has not been taken up in this literature, however, is that the reason model notion of consistency presupposes that the background case base is consistent to start with, which is unrealistic. Horty [1] sketches a generalization of the reason model notion of constraint that applies to inconsistent case bases as well. Yet, the idea is only presented and not verified. My aim is to take Horty's suggestion and study how, exactly, according to the generalized notion of constraint, inconsistent case bases constrain future decisions.[2]

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[2] A different approach to the problems presented by inconsistent case bases can be found in recent works by Peters and colleagues [13] and by van Woerkom and colleagues [14] where a version of the reason model is used to analyze how machine learning systems base their decisions on training data.

I proceed as follows. In Section 2, I review some basic definitions and present the generalized notion of constraint. In Section 3, I define what it means, in this framework, that it follows from a possibly inconsistent case base that a decision is obligatory or permitted and study the resulting logic of constraint. In Section 4, I address three more practical issues: first, whether it is feasible, in practice, to determine that a decision made in the context of an inconsistent case base is permitted; second, whether inconsistent case bases provide us with intuitive criteria to identify the fact situations that ought to be decided for a specific side; and, finally, whether inconsistent case bases provide us with intuitive criteria to compare different permissible decisions. Section 5 concludes.

2. Generalized reason model notion of precedential constraint

The reason model represents cases as consisting of three elements: a fact situation presented to the court; an outcome, which can be either a decision for the plaintiff or a decision for the defendant; and a rule that justifies the outcome on the basis of a reason that holds in the considered situation. I start by reviewing the definitions of these elements.

A *fact situation* is a set of facts that are legally relevant, called *factors*. Factors are assumed to have polarities: every factor favors either the plaintiff, denoted with π, or the defendant, denoted with δ. We take $\mathscr{F}^{\pi} = \{f_1^{\pi}, \ldots, f_n^{\pi}\}$ to be the set of factors favoring the plaintiff, $\mathscr{F}^{\delta} = \{f_1^{\delta}, \ldots, f_m^{\delta}\}$ to be the set of factors favoring the defendant, and $\mathscr{F} = \mathscr{F}^{\pi} \cup \mathscr{F}^{\delta}$ to be the set of all factors. Where s is one of the two sides, we will use $\bar{s}$ to represent the other, so $\bar{s} = \pi$ if $s = \delta$ and $\bar{s} = \delta$ if $s = \pi$. Where X is a fact situation, $X^s = X \cap \mathscr{F}^s$ is the set of factors from X that favor the side s.

Next, a *reason for the side s* is a set of factors uniformly favoring s; a *reason* is then a set of factors uniformly favoring *a* side. We say that a reason U *holds* in a fact situation X whenever $U \subseteq X$ and that U is *at least as strong as* another reason V favoring the same side as U whenever $V \subseteq U$. To illustrate, if $X_1 = \{f_1^{\pi}, f_2^{\pi}, f_1^{\delta}\}$, then $\{f_1^{\pi}\}$ and $\{f_1^{\pi}, f_2^{\pi}\}$ are reasons for π that hold in X_1 and such that $\{f_1^{\pi}, f_2^{\pi}\}$ is at least as strong as $\{f_1^{\pi}\}$.

We can now define a *rule* as a statement of the form $U \rightarrow s$, where U is a reason for the side s. Intuitively, $U \rightarrow s$ represents a defeasible rule that, roughly, says that, if U holds in a fact situation, then the court has a *pro tanto* reason to decide that situation for s. For any rule $r = U \rightarrow s$, we let $prem(r) = U$ and $conc(r) = s$. We say that r is *applicable in a fact situation* X whenever its premise holds in X, that is $prem(r) \subseteq X$.

Finally, a *case* is a triple of the form $\langle X, r, s \rangle$, where X is a fact situation, r is a rule applicable in X and whose conclusion is s, and s is either π or δ. For any case $c = \langle X, r, s \rangle$, we set $facts(c) = X$, $rule(c) = r$, and $out(c) = s$. A *case base* is simply a set of cases.

Turning to the notion of constraint, the reason model is based on two key ideas: first, that every case decided by a court induces a priority ordering among reasons and, second, that the decisions taken by later courts ought to be consistent with the priority ordering induced by precedent cases. We start by defining the priority ordering induced by a case:

Definition 1 (Priority ordering induced by a case)**.** Where $c = \langle X, r, s \rangle$ is a case, the priority ordering $<_c$ induced by c is defined by setting, for any pair of reasons $U \subseteq \mathscr{F}^{\bar{s}}$ and $V \subseteq \mathscr{F}^s$: $U <_c V$ if and only if $U \subseteq X$ and $prem(r) \subseteq V$.

To illustrate, let c_1 be the case $\langle X_1, r_1, \pi \rangle$, where X_1 is as above and $r_1 = \{f_1^{\pi}\} \rightarrow \pi$. The idea behind Definition 1 is that c_1 reveals that, according to the court, the reason

$\{f_1^\pi\}$ has higher priority than every reason for δ that holds in X_1—i.e., $\{f_1^\delta\}$—and that every reason for π that is at least as strong as $\{f_1^\pi\}$, for instance $\{f_1^\pi, f_2^\pi\}$, also has higher priority than every such reason. It is worth noting that Definition 1 ensures that the ordering $<_c$ is asymmetric: there are no reasons U and V such that $U <_c V$ and $V <_c U$.

We can now define the priority ordering induced by a case base as follows:

Definition 2 (Priority ordering induced by a case base)**.** Where Γ is a case base, the priority ordering $<_\Gamma$ induced by Γ is defined by setting, for any pair of reasons U and V: $U <_\Gamma V$ if and only if there is a case c in Γ such that $U <_c V$.

Definition 2 does not force $<_\Gamma$ to be asymmetric: there may be reasons U and V such that $U <_\Gamma V$ and $V <_\Gamma U$. This happens when some cases in Γ support conflicting information about the priority ordering among reasons. Such cases make Γ inconsistent. To make this precise, call a pair of reasons such that $U <_\Gamma V$ and $V <_\Gamma U$ (abbreviated as $U \perp_\Gamma V$) an *inconsistency in* Γ and let $inc(\Gamma)$ be the set of inconsistencies in Γ. We can then define:

Definition 3 (Inconsistent and consistent case base)**.** A case base Γ is *inconsistent* when $inc(\Gamma) \neq \emptyset$ and *consistent* when $inc(\Gamma) = \emptyset$.

So, if c_1 is as before and c_2 is the case $\langle X_2, r_2, \delta \rangle$, where $X_2 = \{f_1^\pi, f_1^\delta, f_2^\delta\}$ and $r_2 = \{f_1^\delta\} \rightarrow \delta$, then $\Gamma_1 = \{c_1, c_2\}$ is inconsistent, as $\{f_1^\delta\} \perp_{\Gamma_1} \{f_1^\pi\}$, and so $inc(\Gamma_1) \neq \emptyset$.

Now, the case base Γ_1 in our example is inconsistent in a way that is so obvious that it would be striking if any court actually had to work with a case base like it. But, in real life, case bases are much more complex than Γ_1 and it is not at all unusual that some precedents pull in different directions. The question *How do inconsistent case bases constrain?* thus becomes pressing. The reason model notion of constraint does not allow us to pose—let alone answer—this question. According to the reason model, decisions of later courts ought to preserve consistency of the underlying case base. Formally:

Definition 4 (Reason model notion of constraint)**.** Let Γ be a consistent case base. Then, against the background of Γ, the court is permitted to decide the fact situation X for the side s on the basis of the rule $r = U \rightarrow s$ just in case $inc(\Gamma \cup \{\langle X, r, s \rangle\}) = \emptyset$.

The problem is that Definition 4 explicitly requires that the underlying case base be consistent. Even worse, simply dropping this requirement would not give us a sensible account of how inconsistent case bases constrain—given an inconsistent case base, there would be no permitted way to decide any fact situation, which is absurd.

There is, however, another way to generalize the reason model notion of constraint so that it applies to inconsistent case bases as well. The idea, which was suggested in [1, p.15], is that, rather than being required to preserve consistency of a consistent case base, courts should be required to introduce no new inconsistencies into a possibly inconsistent case base. Let us take this suggestion and define:

Definition 5 (Generalized reason model notion of constraint)**.** Against the background of a case base Γ, the court is permitted to decide the fact situation X for the side s on the basis of the rule $r = U \rightarrow s$ just in case $inc(\Gamma \cup \{\langle X, r, s \rangle\}) \subseteq inc(\Gamma)$.

To make Definition 5 less abstract, suppose that a court has to decide the situation $X_3 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$ against the background of the inconsistent case base Γ_1. According to Definition 5, the court is *not* allowed to extend Γ_1 to the case base $\Gamma_2 = \Gamma_1 \cup \{c_3\}$, where $c_3 = \langle X_3, \{f_1^\delta\} \to \delta, \delta \rangle$. In fact, c_3 induces the priority $\{f_1^\pi, f_2^\pi\} <_{c_3} \{f_1^\delta\}$, which is inconsistent with the priority $\{f_1^\delta\} <_{c_1} \{f_1^\pi, f_2^\pi\}$. In addition, it is neither the case that $\{f_1^\pi, f_2^\pi\} <_{c_1} \{f_1^\delta\}$ (as $\{f_1^\delta\} <_{c_1} \{f_1^\pi, f_2^\pi\}$ and $<_{c_1}$ is asymmetric) nor that $\{f_1^\pi, f_2^\pi\} <_{c_2} \{f_1^\delta\}$ (as $\{f_1^\pi, f_2^\pi\}$ does not hold in X_2). So, we have $\{f_1^\pi, f_2^\pi\} \perp_{\Gamma_2} \{f_1^\delta\}$ but not $\{f_1^\pi, f_2^\pi\} \perp_{\Gamma_1} \{f_1^\delta\}$—that is, deciding for δ on the basis of $\{f_1^\delta\} \to \delta$ introduces a new inconsistency. Still, this does not mean that it is not permissible to decide X_3 for δ at all; for instance, deciding X_3 for δ on the basis of the rule $\{f_1^\delta, f_2^\delta\} \to \delta$ would not introduce any new inconsistencies and is thus allowed.[3]

To conclude this section, Observation 1 shows that Definition 5 is indeed a generalization of the reason model notion of constraint: it is permissible—in the sense of Definition 5—to extend a *consistent* case base just in case the extension is also consistent.

Observation 1. *Let Γ be a consistent case base and c any case. Then, $inc(\Gamma \cup \{c\}) \subseteq inc(\Gamma)$ if and only if $inc(\Gamma \cup \{c\}) = \emptyset$.*

3. Conflict-free deontic logic from inconsistent case bases

Observation 1 supports the idea that the notion of constraint set out in Definition 5 is a natural generalization of the reason model notion of constraint. But how exactly does the generalized notion work when the background case base is inconsistent? In this section, I begin to explore this question from a logical perspective: assuming the generalized notion, I aim to define, first, what it means that it follows from a possibly inconsistent case base that a decision for a side is obligatory or permitted and, second, to study the logic of constraint underlying the resulting notions of permission and obligation.

As to the first task, following [15, Sect. 1.2.4], I begin from the observation that Definition 5 characterizes the rules that a court is permitted to use in the context of a certain case base to justify its decisions. Given this notion, we can say that *it follows from a case base that a decision for a side is permitted* whenever there is a rule that is permitted in the context of that case base that supports that side and that *it follows from a case base that a decision is obligatory* whenever all the rules that are permitted in the context of that case base support that side. To state this formally, let $Perm(\Gamma, X)$ be the set of rules that are applicable to the fact situation X and are permitted in the context of the case base Γ. In addition, let $\Gamma \mathrel{|\!\sim} P_X(s)$ mean that it follows from Γ that deciding X for s is permitted and $\Gamma \mathrel{|\!\sim} O_X(s)$ mean that it follows from Γ that deciding X for s is obligatory. Then, $\Gamma \mathrel{|\!\sim} P_X(s)$ and $\Gamma \mathrel{|\!\sim} O_X(s)$ are defined as follows:

Definition 6 (Deontic operators)**.** Let Γ be a case base and X a fact situation. Then, $\Gamma \mathrel{|\!\sim} P_X(s)$ holds just in case there is an $r \in Perm(\Gamma, X)$ such that $conc(r) = s$, and $\Gamma \mathrel{|\!\sim} O_X(s)$ holds just in case, for all $r \in Perm(\Gamma, X)$, $conc(r) = s$.

Turning to the second task, it immediately follows from our definitions that a court ought to decide for a side if and only if it is not allowed to decide for the opposite side:

Observation 2. $\Gamma \mathrel{|\!\sim} O_X(s)$ *holds if and only if* $\Gamma \mathrel{|\!\sim} P_X(\overline{s})$ *does not hold.*

[3] Of course, as the case for δ and the case for π are symmetric, an analogous reasoning shows that the court is allowed to decide X_3 for π on the basis of the rule $\{f_1^\pi, f_2^\pi\} \to \pi$ but not on the basis of the rule $\{f_1^\pi\} \to \pi$.

Observation 2 tells us that the deontic operators introduced above are interdefinable in the usual way. A key question is whether they are standard also in the sense that they exclude the possibility of contradictory obligations: Can we exclude that, in the context of an inconsistent case base, a court is required to decide for s and also required to decide for $\bar{s}$? The question is not trivial because Definition 6 mirrors the semantics of standard deontic logics and, in standard deontic logics, inconsistent normative information does give rise to contradictory requirements. Now, in our case, the only situation in which both $\Gamma \mid\!\sim O_X(s)$ and $\Gamma \mid\!\sim O_X(\bar{s})$ could hold is when the set $Perm(\Gamma, X)$ is empty—that is, when no rule applicable in X is permitted in the context of Γ. It turns out that this situation can be excluded: no matter whether Γ is inconsistent or which factors are present in X, there is a permitted rule that can be used to decide X.

Observation 3. *Let Γ be a case base and X a fact situation. Then there exists some rule $r : U \to s$ applicable in X such that $inc(\Gamma \cup \{\langle X, r, s\rangle\}) \subseteq inc(\Gamma)$.*

Proof. If Γ is consistent, then, by Observation 1, Observation 3 is equivalent to the claim that, for any fact situation X, there is a rule $r : U \to s$ applicable in X such that $\Gamma \cup \{\langle X, r, s\rangle\}$ is consistent. A proof of the latter claim can be found in [15, App. A.2]. So, let Γ be inconsistent. Suppose, toward contradiction, that there is no rule $r : U \to s$ applicable in X such that $inc(\Gamma \cup \{\langle X, r, s\rangle\}) \subseteq inc(\Gamma)$. Then, letting $c_1 = \langle X, X^s \to s, s\rangle$ and $\Gamma_1 = \Gamma \cup \{c_1\}$, it follows that there is a pair of reasons $U \subseteq \mathscr{F}^{\bar{s}}$ and $V \subseteq \mathscr{F}^s$ such that $U \perp_{\Gamma_1} V$ but $not(U \perp_\Gamma V)$. By unfolding the definitions, it is not difficult to see that this can only happen when the following facts hold: (1) $U <_{c_1} V$, i.e., $U \subseteq X^{\bar{s}}$ and $X^s \subseteq V$; (2) $V <_\Gamma U$, i.e., there is a case $c_3 = \langle X_3, r_3, \bar{s}\rangle$ in Γ s.t. $V \subseteq X_3$ and $prem(r_3) \subseteq U$; and (3) $U \not<_\Gamma V$. Similarly, letting $c_2 = \langle X, X^{\bar{s}} \to \bar{s}, \bar{s}\rangle$ and $\Gamma_2 = \Gamma \cup \{c_2\}$, it follows from our hypothesis that there is a pair of reasons $U' \subseteq \mathscr{F}^{\bar{s}}$ and $V' \subseteq \mathscr{F}^s$ such that $U' \perp_{\Gamma_2} V'$ but $not(U' \perp_\Gamma V')$. Again, this can only happen when the following facts hold: (4) $V' <_{c_2} U'$, i.e., $V' \subseteq X^s$ and $X^{\bar{s}} \subseteq U'$; (5) $U' <_\Gamma V'$, i.e., there is a case $c_4 = \langle X_4, r_4, s\rangle$ in Γ s.t. $U' \subseteq X_4$ and $prem(r_4) \subseteq V'$; and (6) $V' \not<_\Gamma U'$. It is now easy to see that: (7) $U \subseteq X_4$, since $U \subseteq X^{\bar{s}}$ by 1, $X^{\bar{s}} \subseteq U'$ by 4, and $U' \subseteq X_4$ by 5; (8) $prem(r_4) \subseteq V$, since $prem(r_4) \subseteq V'$ by 5, $V' \subseteq X^s$ by 4, and $X^s \subseteq V$ by 1. But 7 and 8 entail that $U <_{c_4} V$, and so that $U <_\Gamma V$, which contradicts 3. □

An immediate consequence of Observation 3 is that, regardless of whether the background case base is inconsistent, the court will never be subject to contradictory requirements; in addition, in any situation, the court will be either required to decide for a side, or required to decide for the opposite side, or permitted to decide for either side:

Observation 4. *It is never the case that both $\Gamma \mid\!\sim O_X(s)$ and $\Gamma \mid\!\sim O_X(\bar{s})$ hold. In addition, it is always the case that exactly one of the following holds: either $\Gamma \mid\!\sim O_X(s)$, or $\Gamma \mid\!\sim O_X(\bar{s})$, or $\Gamma \mid\!\sim P_X(s)$ and $\Gamma \mid\!\sim P_X(\bar{s})$.*

We can thus conclude that possibly inconsistent case bases support a natural, *conflict-free* deontic logic. Let me now move on to three more practical issues.

4. Applying the generalized notion of constraint

4.1. Feasibility

Suppose that we have to decide a fact situation X against the background of a possibly inconsistent case base Γ and that we want to determine whether we are allowed to decide X for the side s on the basis of the rule $r = U \to s$. How should we proceed?

Simply applying Definition 5 will often be unfeasible. If we do that, we will have to consider all pairs of opposing reasons that can be built from the sets of factors $\mathscr{F}^s$ and $\mathscr{F}^{\overline{s}}$ and check, for each pair, that either it does not form an inconsistency in the extended case base $\Gamma' = \Gamma \cup \{\langle X, r, s\rangle\}$ or, if it does, that it also forms an inconsistency in Γ. The problem is that the number of pairs of opposing reasons grows exponentially with the number of factors: if $\mathscr{F}^s$ contains m factors and $\mathscr{F}^{\overline{s}}$ contains n factors, then there are 2^m reasons for s and 2^n reasons for $\overline{s}$, which results in $2^m \times 2^n = 2^{m+n}$ pairs of opposing reasons. Unless we work with only a few basic factors—which may not be possible if we want to model real cases—simply applying Definition 5 will thus not do.

Now, if our task were just to check that Γ' is consistent, a simplification would be available: As shown in [15, Sect. 2.2.1], it turns out that a case base is inconsistent just in case it includes two cases $c_i = \langle X_i, r_i, s\rangle$ and $c_j = \langle X_j, r_j, \overline{s}\rangle$ such that $prem(r_j) <_{c_i} prem(r_i)$ and $prem(r_i) <_{c_j} prem(r_j)$. This means that, if Γ' includes p cases decided for s and q cases decided for $\overline{s}$, we would have to check $p \times q$ pairs of case rule premises favoring opposite sides, which would make our problem more tractable—the search space would be polynomial in the number of cases rather than exponential in the number of factors. The central result of this section is that, fortunately, a similar simplification is available for our original task:

Observation 5. *Let Γ be a case base, c be the case $\langle X, r, s\rangle$, and $\Gamma' = \Gamma \cup \{c\}$. Then, $inc(\Gamma') \not\subseteq inc(\Gamma)$ if and only if the following two conditions obtain:*
1. there is a case $c_i = \langle X_i, r_i, \overline{s}\rangle$ in Γ s.t. $prem(r_i) <_c prem(r)$ and $prem(r) <_{c_i} prem(r_i)$;
2. there is no case $c_j = \langle X_j, r_j, s\rangle$ in Γ s.t. $X^{\overline{s}} \subseteq X_j^{\overline{s}}$ and $prem(r_j) \subseteq prem(r)$.

Proof. ***Left-to-right.*** Assume that there are reasons $U \subseteq \mathscr{F}^{\overline{s}}$ and $V \subseteq \mathscr{F}^s$ such that $U \perp_{\Gamma'} V$ but $not(U \perp_\Gamma V)$. By unfolding the definitions, it is easy to see that this can only happen when: (1) $U <_c V$, i.e., $U \subseteq X^{\overline{s}}$ and $prem(r) \subseteq V$, (2) $V <_\Gamma U$, i.e., there is $c_i = \langle X_i, r_i, \overline{s}\rangle$ in Γ s.t. $V \subseteq X_i$ and $prem(r_i) \subseteq U$; and (3) $U \not<_\Gamma V$. But then: (4) $prem(r_i) <_c prem(r)$, since $prem(r_i) \subseteq X$ by 2 and 1; and (5) $prem(r) <_{c_i} prem(r_i)$, since $prem(r) \subseteq X_i$ by 1 and 2. Facts 4 and 5 suffice to establish condition 1 in Observation 5. In order to establish condition 2, suppose, toward contradiction, that there is $c_j = \langle X_j, r_j, s\rangle$ in Γ such that (a) $X^{\overline{s}} \subseteq X_j^{\overline{s}}$ and (b) $prem(r_j) \subseteq prem(r)$. Then: (6) $U \subseteq X_j$, as $U \subseteq X^{\overline{s}}$ by 1, $X^{\overline{s}} \subseteq X_j^{\overline{s}}$ by (a), and $X_j^{\overline{s}} \subseteq X_j$ by definition; and (7) $prem(r_j) \subseteq V$, as $prem(r_j) \subseteq prem(r)$ by (b) and $prem(r) \subseteq V$ by 1. It follows from 6 and 7 that $U <_{c_j} V$, and so that $U <_\Gamma V$, which contradicts 3. ***Right-to-left.*** Suppose that Γ satisfies conditions 1 and 2 in Observation 5. So, let $c_i = \langle X_i, r_i, \overline{s}\rangle$ be a case in Γ satisfying condition 1. Then: (1) $prem(r) \subseteq X_i$, since $prem(r) <_{c_i} prem(r_i)$; and (2) $prem(r_i) \subseteq X^{\overline{s}}$, since $prem(r_i) <_c prem(r)$. Hence, $prem(r) <_{c_i} X^{\overline{s}}$. Since $X^{\overline{s}} <_c prem(r)$ by Definition 1, we have that $prem(r) \perp_{\Gamma'} X^{\overline{s}}$. It remains to be shown that $not(prem(r) \perp_\Gamma X^{\overline{s}})$, i.e., that there is no $c_j = \langle X_j, r_j, s\rangle$ in Γ such that $X^{\overline{s}} <_{c_j} prem(r)$. Suppose, toward contradiction, that there is such a c_j. Then, by Definition 1, $X^{\overline{s}} \subseteq X_j^{\overline{s}}$ and $prem(r_j) \subseteq prem(r)$. But this contradicts the hypothesis that Γ satisfies condition 2. □

According to Observation 5, we can determine whether it is permissible to extend Γ with the case $\langle X, r, s\rangle$ by applying the following procedure: first, take all pairs of opposing reasons consisting of $prem(r)$ and of a case rule premise from Γ that favors $\overline{s}$; if no pair is an inconsistency in $\Gamma \cup \{\langle X, r, s\rangle\}$, it is permissible to extend Γ with $\langle X, r, s\rangle$; otherwise, take, for each case c_j in Γ decided for s, two additional pairs of reasons, i.e., the pair consisting of $X^{\overline{s}}$ and $facts(c_j)^{\overline{s}}$ and the pair consisting of $prem(rule(c_j))$ and $prem(r)$; if there is a case for s for which the two pairs satisfy condition 2, then it is permissible to extend Γ with $\langle X, r, s\rangle$; otherwise, it is not. This means that, if Γ includes p cases decided for $\bar{s}$ and q cases decided for s, then, in the worst case scenario, we need to consider $p+(2\times q)$ pairs of opposing reasons. So, even when Γ is inconsistent, our search space is polynomial in the number of cases rather than exponential in the number of factors.

4.2. Identifying fact situations that ought to be decided for a side

Suppose now that we have to decide a fact situation X in the context of a possibly inconsistent case base Γ and that we want to determine whether we ought to decide X for a specific side. Are there intuitive relations between X and the fact situations already decided in Γ that would help us do that? Observation 5 is key to answer this question. To see this, I define two notions that will make its content more transparent, namely the notions of a defeater and of a supporter of a potential decision.

The idea is simple: when, given a case base Γ, we want to determine whether we can decide a fact situation X for the side s on the basis of the rule r, we consider the potential decision $c=\langle X,r,s\rangle$ and see if it is defeated or supported by some cases in Γ. A *defeater of c in* Γ is any case in Γ that is decided for $\bar{s}$ on the basis of a reason that is inconsistent with the reason presented in c. The set of deafeaters of c in Γ is thus:

$$\begin{aligned} \mathit{defeaters}_\Gamma(c) &= \{c_i=\langle X_i,r_i,\bar{s}\rangle\in\Gamma \mid \mathit{prem}(r) <_{c_i} \mathit{prem}(r_i) \text{ and } \mathit{prem}(r_i) <_c \mathit{prem}(r)\} \\ &= \{c_i=\langle X_i,r_i,\bar{s}\rangle\in\Gamma \mid \mathit{prem}(r)\subseteq X_i \text{ and } \mathit{prem}(r_i)\subseteq X\} \end{aligned}$$

A *supporter of c in* Γ is any case in Γ that is decided for s and such that c is at least as strong for s as that case, where c is at least as strong for s as a case $c_j=\langle X_j,r_j,s\rangle$ just in case, first, the reason for s presented in c is at least strong as the reason for s presented in c_j and, second, the strongest reason for $\bar{s}$ that holds in X is weaker than the strongest reason for $\bar{s}$ that holds in X_j.[4] The set of supporters of c in Γ is thus:

$$\mathit{supporters}_\Gamma(c) = \{c_j=\langle X_j,r_j,s\rangle\in\Gamma \mid \mathit{prem}(r_j)\subseteq \mathit{prem}(r) \text{ and } X^{\bar{s}}\subseteq X_j^{\bar{s}}\}$$

With these notions in place, Observation 5 simply says that, according to the generalized notion of constraint, a court is *not allowed*, in the context of Γ, to decide X for s on the basis of r just in case the potential decision $c=\langle X,r,s\rangle$ has at least one defeater and no supporter in Γ:

Observation 5′. Let Γ be a case base and c be the case $\langle X,r,s\rangle$. Then, $\mathit{inc}(\Gamma\cup\{c\})\not\subseteq \mathit{inc}(\Gamma)$ if and only if $\mathit{defeaters}_\Gamma(c)\neq\emptyset$ and $\mathit{supporters}_\Gamma(c)=\emptyset$.

Now, according to Observation 2, a court ought to decide for s just in case it is not allowed to decide for $\bar{s}$—i.e., no applicable rule supporting $\bar{s}$ is permitted. Observation 5′ then says that X ought to be decided for s just in case it has two features:

Feature 1. Every potential decision of the form $\langle X,r,\bar{s}\rangle$ has a defeater in Γ.

This happens when (and only when) every reason for $\bar{s}$ holding in X also holds in a situation already decided for s on the basis of a reason that itself holds in X. This, in turn, happens when (and only when) Γ includes a case whose reason contradicts the strongest reason for $\bar{s}$ holding in X—i.e., when $\langle X,X^{\bar{s}}\to\bar{s},\bar{s}\rangle$ has a defeater in Γ.

Feature 2. No potential decision of the form $\langle X,r,\bar{s}\rangle$ has a supporter in Γ.

[4] The notion of strength between cases proposed here is a generalization of the notion of an *a fortiori* case discussed in [16] and formalized in [1].

This happens when (and only when) every fact situation already decided for $\bar{s}$ *either* does not include X^s *or* is decided on the basis of a reason that does not hold in X.

To get a better sense of the two features and their intuitiveness, let us consider two cases in which an individual, who moved to the foreign country Z, files a request to change fiscal domicile with respect to income tax.[5] The defendant is the individual and the plaintiff her home country. The relevant factors are: δ spent only one month in country Z (f_1^π); δ owned a house in her home country (f_2^π); δ had a permanent job in country Z (f_1^δ); δ opened a bank account in country Z (f_2^δ); δ registered a car in country Z (f_3^δ).

Example 1. Suppose that Mr. C presents the situation $X_7 = \{f_1^\pi, f_2^\pi, f_1^\delta\}$ and that there are two mutually inconsistent precedents: the case of Mr. A is $c_5 = \langle X_5, r_5, \pi\rangle$, where $X_5 = \{f_1^\pi, f_1^\delta\}$ and $r_5 = \{f_1^\pi\} \to \pi$, and the case of Mrs. B is $c_6 = \langle X_6, r_6, \delta\rangle$, where $X_6 = \{f_1^\pi, f_1^\delta, f_2^\delta\}$ and $r_6 = \{f_1^\delta\} \to \delta$. Are we allowed to find for Mr. C? The answer is negative. We can see this formally as follows: The strongest reason for δ that holds in X_7 (i.e., $\{f_1^\delta\}$) also holds in X_5; in turn, the reason that decides X_5 (i.e., $\{f_1^\pi\}$) holds in X_7; so, c_5 defeats all potential decisions for δ (i.e., $\langle X_7, r_6, \delta\rangle$), as per Feature 1. What is more, f_2^π is a pro-π factor present in X_7 but not in X_6; so, no potential decision for δ is supported by c_6, as per Feature 2. Thus, we ought to decide for π. This formal argument has an intuitive informal counterpart: The only reason in favor of Mr. C is that he had a permanent job in country Z. But the case of Mr. A already established that this reason is not strong enough to find for Mr. C, given that he spent only one month in country Z. It is true that Mrs. B won because she, as Mr. C, had a permanent job in country Z and despite the fact that she, as Mr. C, spent only one month in country Z. Yet, unlike Mrs. B, Mr. C owned a house in his home country. Thus, the case for Mr. C is weaker than the case for Mrs. B, while the case for his home country is stronger than the case for Mr. A's home country. So, we ought to decide for Mr. C's home country.

Example 2. Instead of c_5 and c_6, suppose that the cases of Mr. A and Mrs. B are, respectively, $c_8 = \langle X_8, r_5, \pi\rangle$, where $X_8 = \{f_1^\pi, f_1^\delta, f_2^\delta, f_3^\delta\}$ and $r_5 = \{f_1^\pi\} \to \pi$, and $c_9 = \langle X_9, r_6, \delta\rangle$, where $X_9 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$ and $r_6 = \{f_1^\delta\} \to \delta$. The case base $\{c_8, c_9\}$ is inconsistent, but Mr. C presents the situation $X_{10} = \{f_1^\pi, f_2^\pi, f_2^\delta, f_3^\delta\}$. As before, we are not allowed to find for Mr. C. Formally: The strongest reason for δ that holds in X_{10} (i.e., $\{f_2^\delta, f_3^\delta\}$) also holds in X_8 and, in turn, the reason that decides X_8 (i.e. $\{f_1^\pi\}$), holds in X_{10}; so, c_8 defeats all potential decisions for δ, as per Feature 1. What is more, the reason that decides c_9 (i.e., $\{f_1^\delta\}$) does not hold in X_{10}; so, c_9 does not support any potential decision for δ, as per Feature 2. Thus, we ought to decide for π. Again, this formal argument has an intuitive informal counterpart: The case of Mr. A already established that no reason in favor of Mr. C is strong enough to find for him, given that he spent only one month in country Z. It is true that Mrs. B won even if she, as Mr. C, spent only one month in country Z; but, unlike Mr. C, Mrs. B had a permanent job in country Z—and this is why she won. Since this reason does not apply to Mr. C, his case cannot be compared to Mrs. B's case. In contrast, the case for Mr. C's home country is stronger than the case for Mr. A's home country. We then ought to decide for Mr. C's home country.

Interestingly, Example 1 and Example 2 suggest that there is nothing mysterious in the fact that we can derive consistent obligations from conflicting precedent cases; the

[5]The examples are inspired by some hypothetical cases introduced by Prakken and Sartor [7]

key is that two conflicting precedents can have different reach—one precedent supports all possible decisions for a side and the other no possible decision for the opposite side.

4.3. Comparing different permissible decisions

Example 1 and Example 2 concern situations that cannot be decided without contradicting a precedent. Yet, in the examples, the only potential decisions that, despite contradicting a precedent, are supported by some other precedents are decisions for π, which makes it clear that π should win. What about situations that cannot be decided without contradicting some precedents but that we are permitted to decide for either side? Are there situations of this sort? If so, are there criteria to guide our decisions in such cases?

The answer to the first question is that there are, indeed, situations of the kind just described. Suppose, for example, that the case base Γ_3 includes the case $c_{11} = \langle X_{11}, r_{11}, \pi \rangle$, where $X_{11} = \{f_1^\pi, f_1^\delta, f_2^\delta\}$ and $r_{11} = \{f_1^\pi\} \to \pi$, $c_{12} = \langle X_{12}, r_{12}, \pi \rangle$, where $X_{12} = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$ and $r_{12} = \{f_2^\pi\} \to \pi$, and $c_{13} = \langle X_{13}, r_{13}, \delta \rangle$, where $X_{13} = \{f_1^\pi, f_2^\pi, f_3^\pi, f_1^\delta, f_2^\delta, f_3^\delta\}$ and $r_{13} = \{f_2^\delta\} \to \delta$. Our task is to decide the situation $X_{14} = \{f_1^\pi, f_2^\pi, f_2^\delta\}$ in the context of the inconsistent Γ_3. It is easy to see that all the potential decisions available to us, i.e., $d_1 = \langle X_{14}, \{f_1^\pi\} \to \pi, \pi \rangle$, $d_2 = \langle X_{14}, \{f_2^\pi\} \to \pi, \pi \rangle$, $d_3 = \langle X_{14}, \{f_1^\pi, f_2^\pi\} \to \pi, \pi \rangle$, $d_4 = \langle X_{14}, \{f_2^\delta\} \to \delta, \delta \rangle$, contradict some precedent decisions but are supported by some other precedents. What should we do?

One answer might be that, since they all have some defeaters and some supporters, the four decisions are equally good; we can thus decide X_{14} however we like. But, if we look more carefully at their defeaters and supporters, it is not so obvious that d_1 to d_4 are really on a par: On the one hand, the pro-π cases d_1, d_2, and d_3 are defeated by only one case (i.e., c_{13}) and supported by at least one case (d_1 by c_{11}, d_2 by c_{12}, and d_3 by c_{11} and c_{12}). On the other hand, the pro-δ case d_4 is defeated by two cases (i.e., c_{11} and c_{12}), while it is supported by only one case (i.e., c_{13}). Since each of d_1, d_2, and d_3 has fewer defeaters than d_4 but at least as many supporters as d_4, it is reasonable to conclude that the former cases are better than the latter. This gives us an argument to decide X_{14} for π.

Generalizing from our example, the number of defeaters and supporters of competing potential decisions seems to provide us with an intuitive, and sensible, criterion to guide our decisions in fact situations like X_{14}. To make this precise, suppose that Γ is an inconsistent case base and X a situation that cannot be decided without contradicting some cases in Γ but that the court is allowed to decide for either side. Where $d = \langle X, r, s \rangle$ and $d' = \langle X, r', \bar{s} \rangle$ are two competing potential decisions, we can say that d *is better than* d' when d has fewer defeaters in Γ than d' and at least as many supporters in Γ as d', while d' is better than d when the opposite is true—in the other cases, the number of defeaters and supporters of d and d' does not support a preference for either case.

5. Conclusion

I investigated a generalization of the reason model notion of precedential constraint that can be used to address the question *How does an inconsistent set of precedents constrain?* The generalized notion provides an interesting, and sensible answer: inconsistent case bases support a natural, conflict-free deontic logic; they do not make verification that a decision is permissible substantially more complex than consistent case bases; finally, they provide us with intuitive criteria both to identify the fact situations that ought to be decided for a specific side and to compare different permissible decisions.

Concerning permissible decisions, I left a number of questions unanswered that are worth investigating. First, I suggested how to compare pairs of *potential decisions* for opposite sides, but I have not considered how to lift the comparison to a comparison between opposite *outcomes*. The example in Section 4.3 is simple in this respect because all potential decisions for π are better than the only potential decision for δ, which clearly makes π a better outcome than δ. Can we say something about cases in which some potential decisions for π are better than some potential decisions for δ and vice versa? Another issue concerns the criteria to evaluate potential decisions *for the same side*. Going back, once again, to the example in Section 4.3, once we have established that it is better to decide X_{14} for π, would it be best to extend Γ_3 with d_1, d_2, or d_3? Does the number of supporters and defeaters of the three cases help us answer this question? Finally, I said nothing about fact situations that may be permissibly decided by contradicting either some or no precedents. According to the criteria suggested in Section 4.3, contradicting no precedents is better than contradicting some. It would be interesting to consider arguments in favor and against this consequence.

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