

# Towards an Implementation of Merging Operators in Many-Valued Logics

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In this paper, we present work in progress on belief merging operators' implementation in many-valued logics. Belief merging is an active area of artificial intelligence and cognitive science [1,2]. Commonly, it studies the combination of consistent knowledge bases (possibly mutually inconsistent) to obtain a single consistent knowledge base, expressed in classical propositional logic. Recently, some authors restricted the theoretical study to some fragments of propositional logic with good computational properties such as the Horn fragment, which affords very efficient algorithms [3,4]. Likewise, some implementations have been proposed with practical applications using partial-satisfiability-based merging [5]. For instance, Kareem et al. [6] used belief merging for oral cancer diagnosis, and Pozos-Parra et al. [7] introduced Merginator, a belief merging tool for consensus decision making.

Traditionally, the logical approach has proposed a set of postulates that a merging operator has to meet to be considered rational. In previous works [8] we studied the logical postulates and theoretical properties of merging operators in the Horn fragment of many-valued logics. In particular, the formalism we used is signed logic. Signed logic was introduced as a generic treatment of many-valued logics [9], and its fundamental underlying idea is attaching a sign or label to an atomic formula to generalize the classical notion of a literal.

A *knowledge base* can be expressed by a signed Horn formula, defining a *profile* as a nonempty finite multiset of consistent but not necessarily mutually consistent knowledge bases. A profile can be viewed as a multiset of agents, represented by their sets of beliefs. Then, merging is finding a ranking that approximates the individual rankings as best as possible. These are the main results we will present:

1. A characterization of the class of models of a Horn signed formula in terms of the closure of this class under certain operations.
2. A sufficient condition for a signed Horn merging operator to satisfy the logical IC-postulates ([10,11]), showing that Horn merging can be seen as an aggregation problem on rankings of signed interpretations.

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3. An example of a signed Horn merging operator, defined by giving a notion of distance between signed interpretations (to induce preorders for each knowledge base) and using an aggregation function (to combine the individual rankings into a final preorder for the profile).
4. An example of an implementation of the belief merging process in the Horn fragment of signed logic. Two parts complete this task: the first one consists in implementing the transformation of signed regular Horn formulas into classical Horn formulas; the other one is defining a Horn merging operator meeting the rationality postulates of belief merging [10,11]. This operator is defined by giving a notion of distance between signed interpretations (to induce preorders for each knowledge base) and using an aggregation function (to combine the individual rankings into a final preorder for the profile). We consider a running example of a simple review conference process written in Python.

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