A Ranking Semantics for Abstract Argumentation Based on Serialisability

Lydia BLÜMEL and Matthias THIMM

Artificial Intelligence Group, University of Hagen, Germany

Abstract. We revisit the foundations of ranking semantics for abstract argumentation frameworks by observing that most existing approaches are incompatible with classical extension-based semantics. In particular, most ranking semantics violate the principle of admissibility, meaning that admissible arguments are not necessarily better ranked than inadmissible arguments. We propose new postulates for capturing said compatibility with classical extension-based semantics and present a new ranking semantics that complies with these postulates. This ranking semantics is based on the recently proposed notion of *serialisability* that allows to rank arguments according to the number of conflicts needed to be solved in order to include that argument in an admissible set.

Keywords. abstract argumentation, ranking semantics, serialisability

1. Introduction

Abstract argumentation frameworks [1] represent argumentative scenarios via directed graphs, where vertices represent arguments and a directed edge from an argument *a* to an argument *b* denotes an "attack" from *a* to *b*. This simple representation formalism is already powerful enough to analyse and discuss many facets of argumentative reasoning such as argumentation-based dialogue [2], strategic argumentation [3], and dynamics of belief [4], see also [5,6]. Abstract argumentation frameworks are interpreted through formal semantics that assess which arguments can be deemed "acceptable". The classical approach to formal semantics is by means of *extensions* [1,7], i.e., sets of arguments that form a plausible point of view on the outcome of the argumentation modelled by an abstract argumentation framework. Concrete extension-based semantics specify additional constraints that should be satisfied by their extensions, which capture, e.g., aspects such as conflict-freeness (no argument in an extension should attack another argument in the extension) or admissibility (arguments should be defended by the extension against attacks from outside). Another formal framework for the interpretation of abstract argumentation frameworks is given by ranking-based [8,9,10,11] or graded semantics [12,13,14,15,16,17]. There, *argument strength* is assessed by either *qualitative* (for ranking-based semantics) or *quantitative* (for graded semantics) rankings of arguments. For reasons of simplicity, we will use the term *ranking semantics* to capture both technical frameworks in the following.

Most ranking semantics such as [12,13,8,15] assess argument strength by weighing numbers of attackers and defenders and lengths of paths in the argumentation frame-

work, in some form or the other. As it has already been observed by Bonzon and colleagues [10], there are some fundamental differences in the way ranking semantics assess the acceptability (or better: strength) of arguments, compared to the way this is done by extension-based semantics. As a result, they proposed some hybrid approaches that combine both views by pairing a concrete extension-based semantics with a concrete ranking semantics. In this paper, we pursue another direction, namely the development of a family of ranking semantics that is *compatible* with extension-based semantics in the sense that they *refine* the acceptability assessment of extension-based semantics. As a motivation for this endeavour we start from the general principle of admissibility, a notion that is central to almost all extension-based semantics—with exceptions, of course [18,19]—and demands that acceptable arguments should be defended by acceptable arguments. This core principle is violated by most of the existing ranking semantics in the sense that admissible arguments are not necessarily ranked higher than inadmissible arguments (see Section 3 for details). We consequently propose novel postulates for ranking semantics that capture the intuition behind our aim of developing ranking semantics that are compatible with classical extension-based semantics. We then present and analyse a new ranking semantics that complies with this interpretation. This ranking semantics is based on the notion of *serialisability* [20], which is a principle satisfied by all semantics from [1] and allows the step-wise construction of extensions via iterative selection of non-empty minimal admissible sets—also called *initial sets* [21]—and consideration of the resulting reducts [19]. We will use the minimal number of steps required to include an argument in such a construction as an assessment of the acceptability of an argument. This basically amounts to the number of conflicts between arguments that have to be resolved in order to accept an argument.

To summarise, the contributions of this paper are as follows.

1. We revisit and re-assess the foundations of ranking semantics by introducing and analysing postulates aiming at compatibility with extension-based semantics (Section 3).
2. We discuss a novel ranking semantics based on serialisability (Section 4).

Section 2 presents the background on abstract argumentation and Section 5 concludes this paper. Proofs of technical results can be found in an online appendix.[1]

2. Preliminaries

We present basic background on abstract argumentation and extension-based semantics in Section 2.1 and ranking semantics in Section 2.2.

2.1. Abstract Argumentation

Let $\mathfrak{A}$ denote a universal set of arguments. An *abstract argumentation framework* AF is a tuple $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$ where $\mathsf{A} \subseteq \mathfrak{A}$ is a finite set of arguments and R is a relation $\mathsf{R} \subseteq \mathsf{A} \times \mathsf{A}$ [1]. Let $\mathfrak{AF}$ denote the set of all abstract argumentation frameworks. For two arguments $a, b \in \mathsf{A}$ the relation $a\mathsf{R}b$ means that argument a attacks argument b. For $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$ and

---

[1] http://mthimm.de/misc/lbmt_rankser_proofs.pdf

$AF' = (A', R')$ we write $AF' \sqsubseteq AF$ iff $A' \subseteq A$ and $R' = R \cap (A' \times A')$. For a set $X \subseteq A$, we denote by $AF|_X = (X, R \cap (X \times X))$ the projection of AF on X. For a set $S \subseteq A$ we define

$$S^+ = \{a \in A \mid \exists b \in S : bRa\} \qquad\qquad S^- = \{a \in A \mid \exists b \in S : aRb\}$$

If S is a singleton set, we omit brackets for readability, i. e., we write a^- (a^+) instead of $\{a\}^-$ ($\{a\}^+$). For two sets S and S' we write SRS' iff $S' \cap S^+ \neq \emptyset$.

Two abstract argumentation frameworks $AF = (A, R)$ and $AF' = (A', R')$ are *isomorphic*, written $AF \equiv AF'$, if there is a bijective function $\gamma : A \rightarrow A'$ such that aRb iff $\gamma(a)R'\gamma(b)$ for all $a, b \in A$ (γ is then called an *isomorphism*).

A set $S \subseteq A$ is *conflict-free* if $S \cap S^+ = \emptyset$. S is a *naive* (na) extension if it is maximal wrt. set inclusion among the conflict-free sets of AF. A set S *defends* an argument $b \in A$ if $b^- \subseteq S^+$. A conflict-free set S is called *admissible* if S defends all $a \in S$. Let $\mathsf{adm}(AF)$ denote the set of admissible sets of AF. Different *extension-based semantics* can be phrased by imposing constraints on admissible sets [7]. In particular, an admissible set E

- is a *complete* (co) extension iff for all $a \in A$, if E defends a then $a \in E$,
- is a *grounded* (gr) extension iff E is complete and minimal,
- is a *stable* (st) extension iff $E \cup E^+ = A$,
- is a *preferred* (pr) extension iff E is maximal.

All statements on minimality/maximality are meant to be with respect to set inclusion. For $\sigma \in \{\mathsf{co}, \mathsf{gr}, \mathsf{st}, \mathsf{pr}\}$ let $\sigma(AF)$ denote the set of σ-extensions of AF. The acceptance of an argument a wrt. a given semantics σ distinguishes three levels:

- a is *skeptically accepted* wrt. σ iff $a \in E$ for all $E \in \sigma(AF)$,
- a is *credulously accepted* wrt. σ iff there is $E \in \sigma(AF)$ with $a \in E$,
- a is *rejected* wrt. σ iff $a \notin E$ for all $E \in \sigma(AF)$.

2.2. Ranking Semantics

Directly comparing individual arguments with each other yields another class of argumentation semantics. Ranking semantics evaluate the acceptability (or better: strength) of single arguments instead of sets of arguments, their output is a (partial) preorder on the arguments of a given AF.

Definition 1. A ranking semantics is a mapping $\tau : \mathfrak{AF} \rightarrow 2^{\mathfrak{A} \times \mathfrak{A}}$ which assigns to each $AF = (A, R) \in \mathfrak{AF}$ a partial preorder $\succeq_{\tau(AF)}$ on A, i. e., $\succeq_{\tau(AF)}$ is transitive and reflexive.

If the AF we refer to is clear from the context, the shorthand $\succeq_\tau$ is used instead. The stronger an argument, the greater its rank among the other arguments, i. e., a is at least as strong as b is represented by $a \succeq_\tau b$. We use the standard shorthands $a \succ_\tau b$ to say that a is strictly stronger than b ($a \succeq_\tau b \wedge b \not\succeq_\tau a$) and $a \simeq_\tau b$ when both arguments are equally strong ($a \succeq_\tau b \wedge b \succeq_\tau a$). An example of a ranking semantics is the categoriser [22,15].

Definition 2. Let $AF = (A, R)$ be an AF. The *categoriser semantics* cat assigns to AF the ranking $\succeq_{\mathsf{cat}}$ defined by $a \succeq_{\mathsf{cat}(AF)} b$ iff $cat(AF)(a) \geq cat(AF)(b)$ where

$$cat(AF)(a) = \begin{cases} 1 & \text{if } a^- = \emptyset \\ \frac{1}{1 + \sum\limits_{b \in a^-} cat(AF)(b)} & \text{otherwise} \end{cases}$$
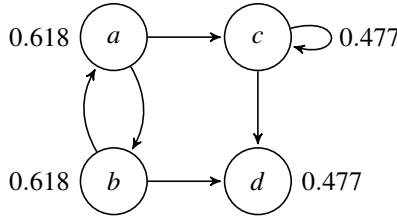
Figure 1. Example with Categoriser values

The above definition yields a system of equations, which can be uniquely solved [15] to obtain the ranks of the individual arguments.

Example 1. The arguments in the AF from Figure 1 are ranked $a \simeq_{\text{cat}} b \succ_{\text{cat}} c \simeq_{\text{cat}} d$ according to the resp. values of the categoriser function (depicted next to the arguments). Since a and b only attack each other, their rank is the same i.e. higher than c and d which are both additionally attacked by c (resulting in the same values for this pair, too).

Following the tradition of the principle-based analysis for extension-based semantics, desirable properties for ranking semantics have been formulated to compare these different approaches. The principles considered in this paper are a selection from [9,17]. Before stating them, we need further notation. A path P of length $l_P = n$ between two arguments a,b is a sequence of arguments $P(a,b) = (a, a_1, ..., a_{n-1}, b)$ with $a_i R a_{i+1} \forall i$ (with $a = a_0, b = a_n$). $cc(\mathsf{AF})$ is the set of all connected components of AF, i. e. all maximal subgraphs $\mathsf{AF'} = (\mathsf{A'}, \mathsf{R'})$ such that for every two arguments $a, b \in \mathsf{A'}$ an undirected path $P_u(a,b) = (a = a_0, a_1, ..., a_n = b) \subseteq \mathsf{AF'}$ with $a_i R a_{i+1}$ or $a_{i+1} R a_i \forall i$ exists.

Definition 3. A ranking semantics τ satisfies the respective principle iff for any $\mathsf{AF} = (\mathsf{A}, \mathsf{R}) \in \mathfrak{AF}$ and any $a, b \in \mathsf{A}$:

Abstraction If for any $\mathsf{AF'} = (\mathsf{A'}, \mathsf{R'})$ with $\mathsf{AF} \equiv \mathsf{AF'}$ and for every isomorphism $\gamma : \mathsf{A} \to \mathsf{A'} : a \succeq_{\tau(\mathsf{AF})} b$ iff $\gamma(a) \succeq_{\tau(\mathsf{AF'})} \gamma(b)$. (*The ranking on arguments should be defined only on the basis of the attacks between them.*)

Independence If for every $\mathsf{AF'} = (\mathsf{A'}, \mathsf{R'}) \in cc(\mathsf{AF})$ and for all $a, b \in \mathsf{A'} : a \succeq_{\tau(\mathsf{AF'})} b$ iff $a \succeq_{\tau(\mathsf{AF})} b$. (*The ranking between two arguments a and b should be independent of any argument that is neither connected to a nor to b.*)

Void precedence If $a^- = \emptyset$ and $b^- \neq \emptyset$ then $a \succ_{\tau(\mathsf{AF})} b$. (*A non-attacked argument should be ranked strictly higher than any attacked argument.*)

Self-contradiction If not aRa but bRb then $a \succ_{\tau(\mathsf{AF})} b$. (*A self-attacking argument should be ranked strictly lower than any non self-attacking argument.*)

Cardinality precedence If $|a^-| < |b^-|$ then $a \succ_{\tau(\mathsf{AF})} b$. (*The greater the number of direct attackers for an argument, the weaker the level of acceptability of this argument.*)

Quality precedence If there is $c \in b^-$ such that for all $d \in a^-$, $c \succeq_{\tau(\mathsf{AF})} d$ but not $d \succeq_{\tau(\mathsf{AF})} c$, then $a \succ_{\tau(\mathsf{AF})} b$. (*The greater the acceptability of one direct attacker for an argument, the weaker the level of acceptability of this argument.*)

Counter-Transitivity If some injective $f : a^- \to b^-$ exists such that $f(x) \succeq_\tau x \forall x \in a^-$ then $a \succeq_{\tau(\mathsf{AF})} b$. (*If the direct attackers of b are at least as numerous and acceptable as those of a, then a is at least as acceptable as b.*)
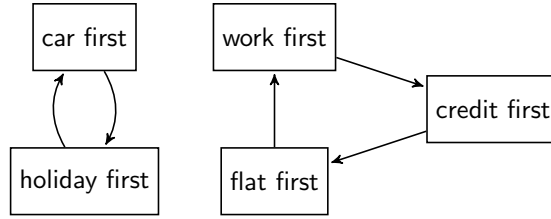
Figure 2. Two simple choice problems

Strict Counter-Transitivity If some injective $f : a^- \to b^-$ exists such that $f(x) \succeq_\tau x \; \forall x \in a^-$ and either $|a^-| < |b^-|$ or there exists some $x \in a^-$ with $f(x) \succ_\tau (x)$ then $a \succ_\tau b$. (*If the direct attackers of b are strictly more numerous or acceptable than those of a, then a is strictly more acceptable than b.*)

Defense precedence If $|a^-| = |b^-|$ and $(a^-)^- \neq \emptyset$ but $(b^-)^- = \emptyset$ then $a \succ_{\tau(\mathsf{AF})} b$. (*For two arguments with the same number of direct attackers, a defended argument is ranked higher than a non-defended argument.*)

Distributed Defense precedence If $|a^-| = |b^-|$ and $|(a^-)^-| = |(b^-)^-|$, and if the defense of a is simple—every direct defender of a directly attacks exactly one direct attacker of a—and distributed—every direct attacker of a is attacked by at most one argument—and the defense of b is simple but not distributed, then $a \succ_{\tau(\mathsf{AF})} b$. (*The best defense is when each defender attacks a distinct attacker (*distributed defense*).*)

Total If $a \succeq_{\tau(\mathsf{AF})} b$ or $b \succeq_{\tau(\mathsf{AF})} a$. (*All pairs of arguments can be compared.*)

Non-attacked Equivalence If $a^- = \emptyset$ and $b^- = \emptyset$ then $a \succeq_{\tau(\mathsf{AF})} b$ and $b \succeq_{\tau(\mathsf{AF})} a$. (*All the non-attacked arguments have the same rank.*)

Attack vs Full Defense If AF is acyclic and every path $P(u,a)$ in AF from an unattacked u to a has $l_p = 0 \bmod 2$ and there exists $u \in b^-$ unattacked then $a \succ_\tau b$. (*An argument without any unattacked indirect attackers should be ranked higher than an argument only attacked by one unattacked argument.*)

3. Rankings, Admissibility, and Reinstatement

Let us start with the motivation for our work in form of a practical example.

Example 2. The argumentation framework depicted in Figure 2 illustrates two everyday choice problems. While the average employee might have to choose between a new car or an overseas holiday, those who have hit rock bottom often find themselves in unsolvable dilemmas. Suppose a homeless person with no money decided to change her life and look for work. Any company would need her address to handle taxes. So she first has to find herself a place to live. But she cannot pay the deposit for renting a flat. So she needs a credit. However any credit institute would require her to have a job in the first place.

When we apply the categoriser semantics to the argumentation framework from the example, all five arguments receive the same value of approximately 0.618. But ranking the impossible choices of the second case just as high as the two options of the first, which can actually materialize, seems inaccurate. The same issue is present in Ex-

ample 1, where the self-attacker c and the credulously acceptable d end up having the same value. In practice we would have to execute caution when interpreting this ranking. For example, one known strategy of conspiracy theories is to convince with the sheer amount of arguments in favor of their hypothesis. Under existing ranking semantics, a classically acceptable argument with lots of attackers could end up with a lower rank than, e. g., one of its attackers which is in turn attacked by an unattacked argument like a fact (objective measurement etc.). This potentially leads to a nonsensical argument being ranked higher than a scientifically supported position. We therefore suggest the acceptability of an argument should be represented in its rank somehow. Strictly speaking, if an argument has no chance of being classically accepted, we should expect it to rank lower than any argument which is actually acceptable. Since existing ranking semantics do not conform to this, a need to investigate new options for ranking semantics emerges. That is not to say acceptance has been completely ignored in ranking semantics so far. A few existing principles already incorporate some aspects of classic defense, e. g., *defense precedence* demands that a ranking-semantics prefers an argument with defenders over one with only attackers provided they have the same number of attackers. The principle we introduce here is a more general approach to integrate extension-based acceptability into ranking semantics. In this paper we limit our investigations to its implications under classic admissible semantics, though.

Definition 4. Let σ be an extension-based semantics. A ranking semantics τ satisfies σ-*compatibility* iff for any $AF = (A, R)$ and any $a, b \in A$, if a is credulously accepted under σ and b is rejected then $a \succ_{\tau(AF)} b$.

σ-*compatibility* ensures no non-acceptable argument can rank as high or higher than any of the acceptable arguments under σ. A special case of σ-compatibility is *na-compatibility* which results in self-attackers ranking strictly lower than the rest. This is actually equivalent to the existing principle of *self-contradiction*.

Proposition 1. *A ranking semantics τ satisfies self-contradiction iff it satisfies* na-compatibility.

Let us turn our attention to *adm-compatibility*. The classic admissible, complete and preferred semantics credulously accept the same arguments, so *adm-compatibility* covers all three. *adm-compatibility* is incompatible with a number of known principles for ranking semantics, notably with *strict counter-transitivity*. This also generalizes some observations from [11].

Proposition 2. *Let τ be a ranking semantics satisfying* adm-compatibility. *Then τ does not satisfy any of the following four principles:* strict counter-transitivity, counter-transitivity, cardinality precedence*, and* quality precedence.

We briefly demonstrate the contradiction with *strict counter-transitivity* using Example 2. The credulously accepted arguments under classic admissibility are car first and holiday first. Suppose some ranking semantics τ satisfies *adm-compatibility*, then both car first $\succ_{\tau(AF)}$ credit first and holiday first $\succ_{\tau(AF)}$ flat first hold. But under *strict counter-transitivity* car first $\succ_{\tau(AF)}$ credit first implies flat first $\succ_{\tau(AF)}$ holiday first, so τ can at most satisfy one of the two principles. Many existing ranking semantics such as the categoriser semantics from above, but also the burden- and discussion-based semantics
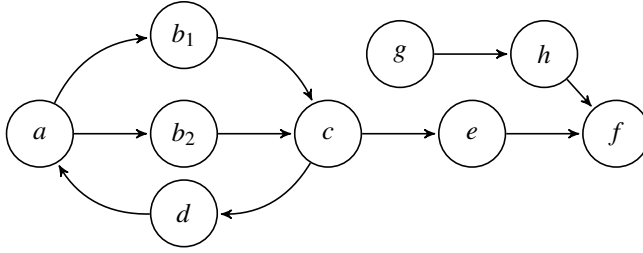
Figure 3. Weak and strong σ-support

[8] and the social abstract argumentation semantics [14] satisfy *strict counter-transitivity* [9]. Therefore none of them satisfies *adm-compatibility*.

For our next new properties we investigate the innate relational structure of extensions or, more precisely, the relations between acceptable arguments under a given extension-based semantics. A longstanding defense-related principle for extension-based semantics is *reinstatement*, the inclusion of defended arguments in an extension [23]. Now, an argument depending on other arguments to become defended should not be ranked higher than its defenders and strictly lower, if its defenders are acceptable independently from it. A way to express this is as follows.

Definition 5. Let $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$, $a, b \in \mathsf{A}$, and σ some extension-based semantics.

- *a weakly σ-supports b* if b is credulously accepted wrt. σ and for all $E \in \sigma(\mathsf{AF})$, if $b \in E$ then $a \in E$.
- *a strongly σ-supports b* if b is credulously accepted wrt. σ and for all $E \in \sigma(\mathsf{AF})$, if $b \in E$ then there is $E' \in \sigma(\mathsf{AF})$ with $E' \subseteq E$, $a \in E'$, and $b \notin E'$.

Informally, an argument a weakly σ-supports an argument b, if a is a part of any σ-extension containing b which intuitively amounts to a being an unavoidable side-effect of accepting b. Moreover, a strongly σ-supports b, if the presence of b in an extension (which necessarily also includes a) implies the existence of a smaller extension with a but without b. In that case a becomes a prerequisite for accepting b while b can be said to be irrelevant for accepting a. It is clear that strong σ-support implies weak σ-support.

Example 3. Let $\sigma = \mathsf{adm}$, then in the AF depicted in Figure 3 the arguments a and c weakly σ-support each other, since they take care of each others attackers in the 4-cycle. Neither of them can be accepted on its own, so a does not strongly σ-support c nor vice versa. Both of them strongly σ-support f because c is the only attacker of e and cannot be accepted without a. Note that although a and c strongly σ-support f, this does not imply they are sufficient for accepting f, g strongly σ-supports f as well. Take also note of d weakly σ-supporting b_1 but not the other way around. Since no admissible subset of b_1, d containing only d exists, d does not strongly σ-support b_1, though.

Using these two notions of σ-support, we can define the according principles for ranking semantics as follows.

Definition 6. Let τ be a ranking semantics.

- τ satisfies *weak σ-support* iff for every $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$, $a, b \in \mathsf{A}$, if a weakly σ-supports b then $a \succeq_{\tau(AF)} b$.

- τ satisfies *strong σ-support* iff for every $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$, $a, b \in \mathsf{A}$, if a strongly σ-supports b then $a \succ_{\tau(AF)} b$.

The two principles are independent from each other, unlike the argument relations they are based upon.

Proposition 3. Strong σ-support *does not imply* weak σ-support *and* weak σ-support *does not imply* strong σ-support.

Reflecting on these two principles for ranking semantics leads again to some interesting observations. Weak σ-support firmly links the rank of an argument with the situations (extensions) in which it is accepted. If two arguments are always accepted together they are of equal rank.

Remark 1. Let τ be a ranking semantics satisfying *weak σ-support*, $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$ an AF and $a, b \in \mathsf{A}$. If a weakly σ-supports b and b weakly σ-supports a then $a \simeq_{\tau(AF)} b$.

In case of a single-status semantics (or semantics providing a single extension for some framework) the above observation results in all accepted arguments sharing the same rank.

Corollary 1. *Let τ be a ranking semantics satisfying* weak σ-support, $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$ *an AF. If* $|\sigma(\mathsf{AF})| = 1$ *then for all credulously accepted* $a, b \in \mathsf{A}$, $a \simeq_{\tau(AF)} b$.

For the same reason *weak σ-support* enforces the same rank on all skeptically accepted arguments wrt. any semantics.

Corollary 2. *Let τ be a ranking semantics satisfying* weak σ-support, $\mathsf{AF} = (\mathsf{A}, \mathsf{R})$ *an AF. For all skeptically accepted* $a, b \in \mathsf{A}$, $a \simeq_{\tau(AF)} b$.

While *weak σ-support* only blocks arguments from ranking better than those they depend on, *strong σ-support* discriminates between arguments in order to express one-sided dependencies as asymmetric rank differences. It has a kind of chain effect, an argument is not only ranked lower than the arguments it depends on but also lower than the arguments those arguments depend on in turn. When applied to classic admissibility, this property leads to a preference of short defense routes. Let us now investigate the relationships of *weak/strong adm-support*.

Proposition 4. *Let τ be a ranking semantics satisfying* strong adm-support. *Then τ does not satisfy any of the following four principles:* strict counter-transitivity, counter-transitivity, cardinality precedence *and* quality precedence.

Proposition 5. *Let τ be a ranking semantics satisfying* weak adm-support. *Then τ does not satisfy* strict counter-transitivity *or* cardinality precedence.

As expected, the two forms of *adm-support* do not go well with attacker-focused properties like *strict counter-transitivity* for which we already demonstrated their contradiction with *adm-compatibility*. Since, e.g., the categoriser semantics, burden- and discussion-based semantics, and social abstract argumentation semantics all satisfy *strict counter-transitivity* [9], none of them satisfies *weak/strong adm-support*.

4. A Ranking Semantics Based on Serialisability

In this section we will introduce a ranking semantics capable of expressing both the acceptability differences and the dependencies between arguments under the classic admissible semantics formalized in the previous section as *adm-compatibility* and *weak/strong adm-support*. In order to do this, we take the admissible extensions apart and analyze them for the relevant dependencies. The smallest units within an admissible set, which still maintain admissibility are the so-called *initial sets* introduced in [21].

Definition 7. For $AF = (A, R)$, a set $S \subseteq A$ with $S \neq \emptyset$ is called an *initial set* if S is admissible and there is no admissible $S' \subsetneq S$ with $S' \neq \emptyset$. Let $IS(AF)$ denote the set of initial sets of AF.

Example 4. The initial sets of the AF depicted in Figure 3 are $\{g\}, \{a,c\}, \{b_1,d\}$ and $\{b_2,d\}$.

Note that not all credulously accepted arguments wrt. admissibility are members of initial sets, e. g., f in Figure 3 is part of the admissible set $\{a,c,f,g\}$ but not contained in any initial set. Arguments like f are exactly those which depend on others for their defense while not being necessary for the defense of their defenders. In [20], a construction method for admissible sets is presented, which implements a form of step-by-step addition for including such arguments. This approach relies on the reduct [19] of an argument set in an argumentation framework.

Definition 8. For $AF = (A, R)$ and $S \subseteq A$, the *reduct* of S wrt. AF is $AF^S = AF|_{A \setminus (S \cup S^+)}$.

Using the reduct, the central idea of [20] can be formalised with the following notion of a serialisation sequence.

Definition 9. A *serialisation sequence* for $AF = (A, R)$ is a sequence $\mathscr{S} = (S_1, ..., S_n)$ with $S_1 \in IS(AF)$ and for each $2 \leq i \leq n$ we have $S_i \in IS(AF^{S_1 \cup ... \cup S_{i-1}})$.

It has been shown that admissible sets can be characterized by serialisation [20]:

Proposition 6. *Let* $AF = (A, R)$ *be an AF and* $E \subseteq A$. $E \in adm(AF)$ *if and only if there is a serialisation sequence* $(S_1, ..., S_n)$ *with* $E = S_1 \cup ... \cup S_n$.

Let us demonstrate this for some of the admissible sets of our previous example.

Example 5. Consider the admissible sets $S_1 = \{b_1, d\}$, $S_2 = \{b_1, b_2, d, g\}$, and $S_3 = \{a, c, g, f\}$ and corresponding serialisation sequences:

$$\mathscr{S}_1 = (\{b_1, d\}) \qquad\qquad\qquad (\text{for } S_1)$$

$$\mathscr{S}_2 = (\{b_1, d\}, \{b_2\}, \{g\}) \qquad\qquad (\text{for } S_2)$$

$$\mathscr{S}_3 = (\{a, c\}, \{g\}, \{f\}) \qquad\qquad (\text{for } S_3)$$

Serialisation sequences are not necessarily unique, but certain arguments can only be selected after they appear in some initial set. For example, $\{f\}$ only becomes an initial set after g (and $\{a,c\}$) are already part of the sequence. This dependency between sets in a serialisation sequence is similar to the *strong adm-support* introduced in Section 3.

Now that we have a tool for representing the structure of admissible sets, we can use it for defining a new argument ranking that is based on the length of shortest serialisation sequences.

Definition 10. For $AF = (A, R)$ and $a \in A$ define the *serialisation index* $\mathsf{ser}_{AF}(a)$ via

$$\mathsf{ser}_{AF}(a) = \min\{n \mid (S_1, \dots, S_n) \text{ is a serialisation sequence and } a \in S_n\}$$

with $\min \emptyset = \infty$.

Intuitively, the value of $\mathsf{ser}_{AF}(a)$ represents the minimal number of conflicts, which have to be solved before an argument a can be accepted. In this context, $\mathsf{ser}_{AF}(a) = 1$ means a can solve all relevant conflicts by "itself" or—to be correct—by being a member of an initial set itself. The serialisation-index $\mathsf{ser}_{AF}(a) = \infty$ for non-acceptable arguments can be read as no serialisation sequence of any length will be sufficient for this argument. From the choice of a trivial value for all non-acceptable arguments, it already becomes clear that our ranking will only represent differences between acceptable arguments. To foster our understanding of these values let us compute the serialisation indices for our running example.

Example 6. For the arguments of the AF from Figure 3 we get $\mathsf{ser}_{AF}(x) = 1$ for $x \in \{a, c, b_1, b_2, d, g\}$ a member of an initial set, $\mathsf{ser}_{AF}(e) = 2$, since the two smallest admissible sets containing e are $\{b_1, d, e\}$ and $\{b_2, d, e\}$ which both can be serialised in $k = 2$ steps, $\mathsf{ser}_{AF}(f) = 3$ because two initial sets, $\{g\}$ and $\{a, c\}$ are needed for the defense of f and have to be included first before $\{f\}$ becomes an initial set in $AF^{\{a,c\} \cup \{g\}}$ and $\mathsf{ser}_{AF}(h) = \infty$ for the non-acceptable argument h.

The ranking semantics naturally arising from the serialisation index is as follows.

Definition 11. For $AF = (A, R)$ and $a, b \in A$, we say that a is *at least as preferred as b* (wrt. serialisability), written $a \succeq_{\mathsf{ser}} b$ iff $\mathsf{ser}_{AF}(a) \leq \mathsf{ser}_{AF}(b)$.

The lower the serialisation index, the higher the rank of an argument with the members of initial sets all being ranked equally at the top. Applying this ranking semantics to our running example yields $a \simeq_{\mathsf{ser}} b_1 \simeq_{\mathsf{ser}} b_2 \simeq_{\mathsf{ser}} c \simeq_{\mathsf{ser}} d \simeq_{\mathsf{ser}} g \succ_{\mathsf{ser}} e \succ_{\mathsf{ser}} f \succ_{\mathsf{ser}} h$. We will now prove that this ranking semantics indeed has the desired properties defined in Section 3 and begin by demonstrating that $\succeq_{\mathsf{ser}}$ produces the intended results for our motivating example.

Example 7. The options for the decision problems represented in Figure 2 are assigned the serialisation indices $\mathsf{ser}_{AF}(\mathsf{holiday\ first}) = \mathsf{ser}_{AF}(\mathsf{car\ first}) = 1$ and $\mathsf{ser}_{AF}(\mathsf{flat\ first}) = \mathsf{ser}_{AF}(\mathsf{work\ first}) = \mathsf{ser}_{AF}(\mathsf{credit\ first}) = \infty$ respectively, resulting in a higher ranking for the viable options of the average employee.

Indeed, the serialisation ranking satisfies *adm-compatbility* per definition, since the serialisation index for non-acceptable arguments of ∞ cannot be reached by acceptable arguments. The conformity to *weak* and *strong adm-support* is not that trivial, but can also be shown.

Theorem 1. $\succeq_{ser}$ *satisfies* adm-compatibility *and both* strong *and* weak adm-support.

The similarities of our new ranking semantics to classical extension-based semantics do not stop with the above result. Another important property of admissibility semantics is *directionality*, i.e., the admissible sets of an unattacked subset of an AF are also admissible in the AF as a whole [23]. The intuition behind this principle is that an argument *a* which has no directed path to an argument *b* should not have any impact on the acceptability of *b*. This idea makes sense for ranking semantics as well and an according principle for graded semantics was formulated in [24]. Here we generalize this directionality principle for ranking semantics.

Definition 12. A ranking-based semantics τ satisfies *directionality* iff for any AF = (A, R) and any $a, b, x, y \in$ A such that $(a, b) \notin$ R and no directed path from *b* to neither *x* nor *y* exists, then $x \succ_{\tau(\mathsf{AF})} y$ if and only if $x \succ_{\tau((\mathsf{A}, \mathsf{R} \cup \{(a,b)\}))} y$.

Proposition 7. $\succeq_{\mathsf{ser}}$ *satisfies* directionality.

Regarding the principles from Definition 3, $\succeq_{\mathsf{ser}}$ satisfies the general ones such as *abstraction* and *independence*. Most of the other principles are not satisfied, in particular because of the incompatibilities we already showed in Propositions 2, 4, and 5. Further principles are not satisfied because they demand rank differences under certain structural conditions, like *distributed defense precedence*. Since all non-acceptable arguments have the same rank under $\succeq_{\mathsf{ser}}$, those principles are violated if their conditions can apply to pairs of non-acceptable arguments. For example, defense precedence is only uphold in case the stronger argument is acceptable. The following proposition summarizes our findings.

Proposition 8. $\succeq_{\mathsf{ser}}$ *satisfies* abstraction, independence, totality, non-attacked equivalence, *and* attack vs full defense. *All other principles from Def. 3 are not satisfied.*

5. Summary and Conclusion

We revisited the foundation of ranking semantics for abstract argumentation and proposed a new interpretation of ranking semantics as *refinements* of classical extension-based semantics. For that aim, we presented the new postulates σ-*compatibility* as well as *weak* and *strong* σ-*support* and showed that these are generally incompatible with existing postulates for ranking semantics. We proposed a new ranking semantics based on the concept of serialisibility and showed that this new semantics behaves well wrt. these postulates.

Our contributions should be regarded as an additional aspect of interpreting ranking semantics and not as disregarding previous approaches. The central aspect of existing ranking semantics is that they aim at assessing *strength* of arguments, which is—as we have seen in this paper — not necessarily the same as acceptability. Here, we aimed at comparing acceptability (wrt. admissibility) of arguments. An interesting avenue for future work is also to investigate more general foundations for acceptability such as *weak admissibility* [19,23] or to exploit different notions of *defense* [25] for our formalisation of weak and strong σ-support.

References

[1] Dung PM. On the Acceptability of Arguments and its Fundamental Role in Nonmonotonic Reasoning, Logic Programming and n-Person Games. Artificial Intelligence. 1995;77(2):321-58.

[2] Black E, Maudet N, Parsons S. Argumentation-based Dialogue. In: Handbook of Formal Argumentation. vol. 2. College Publications; 2021. p. 511-76.

[3] Governatori G, Maher MJ, Olivieri F. Strategic Argumentation. In: Handbook of Formal Argumentation. vol. 2. College Publications; 2021. p. 577-662.

[4] Baumann R, Doutre S, Mailly JG, Wallner JP. Enforcement in Formal Argumentation. In: Handbook of Formal Argumentation. vol. 2. College Publications; 2021. p. 445-510.

[5] Baroni P, Gabbay D, Giacomin M, van der Torre L, editors. Handbook of Formal Argumentation. College Publications; 2018.

[6] Gabbay D, Giacomin M, Simari GR, Thimm M, editors. Handbook of Formal Argumentation. vol. 2. College Publications; 2021.

[7] Baroni P, Caminada M, Giacomin M. Abstract Argumentation Frameworks and Their Semantics. In: Handbook of Formal Argumentation. College Publications; 2018. p. 159-236.

[8] Amgoud L, Ben-Naim J. Ranking-based Semantics for Argumentation Frameworks. In: Proceedings of the 7th International Conference on Scalable Uncertainty Management (SUM'13); 2013. p. 134-47.

[9] Bonzon E, Delobelle J, Konieczny S, Maudet N. A Comparative Study of Ranking-based Semantics for Abstract Argumentation. In: Proceedings of the 30th AAAI Conference on Artificial Intelligence (AAAI'16); 2016. p. 914-20.

[10] Bonzon E, Delobelle J, Konieczny S, Maudet N. Combining extension-based semantics and ranking-based semantics for abstract argumentation. In: Sixteenth International Conference on Principles of Knowledge Representation and Reasoning; 2018. p. 118-27.

[11] Amgoud L, Ben-Naim J. Axiomatic foundations of acceptability semantics. In: International Conference on Principles of Knowledge Representation and Reasoning (KR 2016); 2016. p. 2-11.

[12] Cayrol C, Lagasquie-Schiex MC. Graduality in Argumentation. Journal of Artificial Intelligence Research. 2005;23:245-97.

[13] Matt PA, Toni F. A Game-Theoretic Perspective on the Notion of Argument Strength in Abstract Argumentation. In: Proceedings of the 11th European Conference on Logics in Artificial Intelligence (JELIA'2008); 2008. p. 285-97.

[14] Leite J, Martins J. Social Abstract Argumentation. In: Proceedings of the 22nd International Joint Conference on Artificial Intelligence (IJCAI'11); 2011. .

[15] Pu F, Luo J, Zhang Y, Luo G. Argument Ranking with Categoriser Function. In: Proceedings of the 7th International Conference on Knowledge Science, Engineering and Management (KSEM'14). Springer; 2014. p. 290-301.

[16] Grossi D, Modgil S. On the graded acceptability of arguments in abstract and instantiated argumentation. Artificial Intelligence. 2019;275:138-73.

[17] Thimm M, Cerutti F, Rienstra T. Probabilistic Graded Semantics. In: Proceedings of the Seventh International Conference on Computational Models of Argumentation (COMMA'18); 2018. p. 369-80.

[18] Baroni P, Giacomin M, Guida G. SCC-recursiveness: a general schema for argumentation semantics. Artificial Intelligence. 2005;168(1–2):162-210.

[19] Baumann R, Brewka G, Ulbricht M. Revisiting the Foundations of Abstract Argumentation - Semantics Based on Weak Admissibility and Weak Defense. In: Proceedings of the 34th AAAI Conference on Artificial Intelligence (AAAI'20); 2020. p. 2742-9.

[20] Thimm M. Revisiting initial sets in abstract argumentation. Argument & Computation. 2022 July.

[21] Xu Y, Cayrol C. Initial Sets in Abstract Argumentation Frameworks. In: Proceedings of the 1st Chinese Conference on Logic and Argumentation (CLAR'16); 2016. p. 1-20.

[22] Besnard P, Hunter A. A logic-based theory of deductive arguments. Artificial Intelligence. 2001;128(1-2):203-35.

[23] Baroni P, Giacomin M. On principle-based evaluation of extension-based argumentation semantics. Artificial Intelligence. 2007;171(10-15):675-700.

[24] Amgoud L, Ben-Naim J, Doder D, Vesic S. Acceptability Semantics for Weighted Argumentation Frameworks. In: Twenty-Sixth International Joint Conference on Artificial Intelligence; 2017. p. 56-62.

[25] Blümel L, Ulbricht M. Defining Defense and Defeat in Abstract Argumentation From Scratch - A Generalizing Approach. In: Proceedings of the 19th International Conference on Principles of Knowledge Representation and Reasoning (KR'22); 2022. .