Information Modelling and Knowledge Bases XXXIII
M. Tropmann-Frick et al. (Eds.)

doi:10.3233/FAIA210479

Reasoning Through Models

Model-Based Reasoning

Bernhard THALHEIM [1]
Christian-Albrechts-University Kiel, Christian-Albrechts-Platz 4, 24118 Kiel, Germany

Abstract. Models are used everywhere, in daily life, sciences, engineering, and thoughts. They represent, support, and enable our thinking, acting, reflecting, communication, and understanding. They are universal instruments. Reasoning through and by models is, however, different from those that we use in 'exact' sciences and is far less understood. The notion of model is becoming nowadays well-accepted. Reasoning through models is far less understood and a long lacuna. This keynote aims at closing this gap.

Keywords. reasoning through models, model-based reasoning, plausible reasoning, approximative reasoning, abduction, induction, explanation, hypotheses, empiric

1. Introduction

Humans intensively use models everywhere, at any time, for any reason, by everybody, for everybody, and at any sphere of human activity. They transform complex, abstract, or partial ideas, systems, and theories into more easily to understand and simpler to use things, i.e. humanise them in dependence on human abilities. Pupils learn natural sciences through models. They are already used to deploy models with their first thoughts. The very first intellectual instrument we use is a model. It is not surprising that babies quickly develop their own models or at least concepts of the 'mother' and 'father'. They cannot yet use a natural language but they know already models of their surroundings [12]. Later they realise that their models are completely different from those of their contemporaries.

We are using the word 'model' widely in our daily life as well as in sciences and engineering. Models are also widely used in the social sphere, in religion, in communication, in interaction, and collaboration. They must not be correct but should be useful as an instrument ('*model_for*'). Models can be understood as a collection of competing interpretations, perception, prehension, ideas, comprehension, imaginations, or conceptions about the world a human observes and understands, each with a utility core, which nevertheless must prove to be progressive over time. This wide usage of models direct us to consider models as the fourth sphere of our life beside sensing and reflecting the *world of the be-*

[1]bernhard.thalheim@email.uni-kiel.de

ing, acting and mastering the *world we create*, and the *intelligible world* of science, knowledge, and concept(ion)s. Models can be considered as our 'third reflection eye' ('*model_of*') we use for comprehension, acceptance, understanding, finding our way around, socialising, communication, planing, and actuation.

Models are developed for being used as an instrument, for instance, as a reasoning instrument. We concentrate here on three kinds of rational reasoning through and by models. The simplest kind of this reasoning is model-backed reasoning; a more advanced is model-driven reasoning. The rational and non-rational reasoning through and by models is far from being understood so far.

The theory and practice of models and modelling is already fairly rich (see, for instance, [32]) and resulted in a large body of knowledge for almost all disciplines of science and engineering. So, we should ask ourselves first: What is a model? Next, we have to ask: what is model-based reasoning? How do we perform reasoning through and by models? The first question got already hundreds of answers. The second and third ones almost none. Therefore, the paper aims at answering the third question after gaining an understanding of the first and second answer.

2. Modellkunde – Towards a Study of Models and Modelling

2.1. The Notion of Model

The notion we use since [28] generalises almost all notions or pre-notions used and known so far in general model theory [13,20,21,27,32][2]:

"A **model** *is a well-formed, adequate, and dependable instrument that represents origins*[3] *and that functions in utilisation scenarios.*
Its criteria of well-formedness[4], adequacy[5], and dependability[6] must be commonly accepted by its community of practice (CoP) within some context and correspond to the functions that a model fulfills in utilisation scenarios." [29]

This notion also allows consideration of the *model-being of any instrument*[7]. Anything – any thought and any thing – can be a model as long as it is used as such. The model-being is, therefore, an assignment for an instrument that is used in scenarios.

[2] Its advantage is that all notion we have seen so far can be understood as a parametric specialisation. More specific notions can be declined by parameter refinement and hardening from this notion.

[3] The 'origin' is different from 'original'. 'Origin' means the source of something's existence or from which it derives or is derived. It points to the place, event, the point of origination, the initial stage of a developmental process, etc. where something begins, where it springs into being.

[4] *Well-formedness* is often considered as a specific modelling language requirement.

[5] The criteria for *adequacy* are analogy (as a generalisation of the mapping property that forms a rather tight kind of analogy), being focused (as a generalisation of truncation or abstraction), and satisfying the purpose (as a generalisation of classical pragmatics properties).

[6] The model has another constituents that are often taken for granted. The model is based on a background, represents origins, is accepted by a community of practice, and follows the accepted context. The model, thus, becomes *dependable*, i.e. it is justified or viable and has a sufficient quality. Most notions assume dependability either as a-priori given or neglect it completely.

[7] We note that the instrument-being is based on the function that a model plays in some scenario.

2.2. Functions of Models in Scenarios

Models function in application scenarios, i.e. they have in those scenarios a function[8,9]. Typical functions in science and engineering scenarios are reflection, illustration, visualisation, being a theory surrogate, guiding thoughts and activities, aiding for theory construction, mediating, and substituting theories.

Models are used instruments. The instrument-being is, thus, a pre-requisite for the model-being. The means that models have to be optimised on the function that the model has in the given application scenario. Instead of considering holistic models, model suites with a sophisticated and explicit association schema among models in the model suite are far better accommodated to reasoning through and by models and deployment in scenarios. A scenario consists of a task space and an envisioned delivery space. Instruments may functions in a variety of ways. Therefore, a model may serve in several functions. Also, a scenario may consist of a collection of scenarios. The upper part in Figure 1 depicts this 'landscape' of the model-being.

2.3. The Model-Being of Things and Thoughts

The model-being is determined by the function of an instrument in an application scenario. Nothing is a-priori a model. Things and thought have not to be models forever. Models have their journey in the model-being. They can be used in one function, remain to be useful or pass away as model. They can be used in a different function at a later stage. Criteria for the model-being seem to be necessary for some demarcation, i.e. a discrimination between things and thoughts as different and distinct on the basis of their characteristics or attributes. The demarcation can be derived from the model-being of an instrument and from the instrument-being of something:

1. A model functions in scenarios. It may functioning well, optimally, flawlessly, properly, satisfactorily, or primarily. Or barely and poorly.
2. A model may have several functions. The function might change during model's existence.
3. Functions can be characterised. This characterisation is an essential element of the mission, determination, meaning and identity of something[10].
4. Functioning may be matured. The maturity level depends on the model objectives.

[8]The word 'function' has seven word fields for the noun and three for the verb. We use here the meaning of a function that is associated with purpose, role, use, utility, usefulness, i.e. what something is used for.

[9]The word 'function' is often considered a synonym of 'goal' or 'purpose'. We distinguish the three word and use a layered approach: *Goal* is definable as a ternary relation between initial state, desired states and community of practice who may assess the states and follow their beliefs, desires, and intentions. *Purposes* extend goals by means, e.g. methods, techniques, and operations. *Functions* embed the model into practices in applications and, thus, relate the purpose to the application, i.e. as a role and play of the model in an application scenario.

[10]B. Mahr [19,20] introduced the notion of *cargo* as a carrier of main properties and objectives of origins to important issues for the result. It describes the instrument, the main functions, the forbidden usages, the specific values of the instrument, and the context for the usage model.

5. Functioning can only be defined for specified scenarios. There is no universal function of a model.
6. Model functions determine the adequacy and dependability.

Typical engineering functions are blueprint for realisation, starting point, prescription, mould, guide, companion, modernisation, integration, replacement, deploy, informative, recording, and assess. These functions, the usefulness, the utility, and the quality in and of use determines whether an instrument is a model. The instrument-being is based on the actual, practised, skilled, ideal, and desired play of a role in an application scenario. There are two main roles: the reflection of those origins the model have to be represented and the achievable result through use of the instrument. The instrument-being depends on the temporal, spatial, and disciplinary context of the community of practice.

The model-being is based on three viewpoints that determine the model utility as a mediator (see Figure 1): (1) the model-being as a 'model_of'[11] (2) the ends for the model-being as a 'model_for' [12] (3) the model-being based on the mediator function of the instrument[13]. Mediation includes transfer of main properties of origins that are essential in the given scenario to the result by means of the model, i.e. the model 'transports' those properties to the results of model application as *invariants*. The explication of the mediation can be directly given as an informative model of the model (as an instrument). Informative models [31] are, thus, essentially the 'product insert' of the model. This model of the model is used as some kind of a leaflet or model suite insert that represents the essentials of the model suite. That means, we use already a model suite consisting of at least two models.

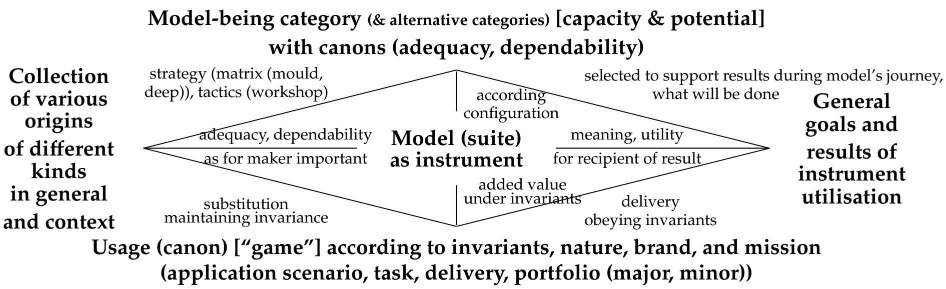

Figure 1. Characterising the model-being of instruments according to reflected origins and results to be accomplished by means of the model

The large variety of models in life, science, engineering, and thought seems to prevent development of a general 'Modellkunde' as a systematic study of models and modelling. There are (a) perception or mental, (b) representation

[11] Representing a collection of origins that are really of interest and relevant, i.e. the whereof as the source for the models existence or from which it derives or is derived.

[12] Essentially, a plan that is intended to achieve and that (when achieved) terminates behavior intended to achieve it, i.e. the for what as the cause or intention underlying model usage.

[13] Determined by a function in a given context and scenario, i.e. whereby as the helper that offers benefits and supporting means for achieving a result.

or reflection, (c) communication or collaboration, (d) actuation and activity, (e) guidance or steering, (f) thought or reasoning, (g) substitution or sense-making, (h) socialisation or interaction, and (i) orientation models. These nine kinds of model follow however their specific adequacy and dependability canon and provide means for their usage. We, thus, realise that we have essentially nine different model categories of models.

As any instrument, a model has its own additional properties that are neither of importance for origins nor for results, its own authority, its obstinacy[14], its profile (goals, purposes, functions) and anti-profile, its modus agendi and mode action, and its materiality. A model may, thus, also be misleading, disorienting, and of lower quality than other models.

3. Model-Based Reasoning Beyond Classical Logics

Sciences are oriented on true statements and consistency of theories, at least to certain extent. This explains the omnipresence of deductive systems as the main reasoning mechanism. Models must not be true. They can be contradictious or even paraconsistent. Models should be useful at least in some application scenario, for some time, for some community of practice, in some context, for some origins, for some results, within some background, on the basis of some supporting and enabling mechanisms, and within human restrictions. Model-based reasoning does not have to be entirely based on classical logics. A similar observation can be made for engineering[15]. The study of models and reasoning through and by models has also to be based on other kinds of reasoning.

The study of models is also concerned with obstacles, mismatches, limitations, and restrictions of reasoning through and by models. Despite the common belief in most books and research on a theory of models (e.g. [27]), model-based reasoning is, however, rather seldom based on deduction and deductive-nomological reasoning.

3.1. *The Obstinacy of Classical Logics*

Classical mathematical logics mainly considers deductive systems and various mechanisms of deduction. Already C.S. Peirce [22] distinguished three reasoning mechanisms: deduction, induction, and abduction. Their difference is illustrated in Table 1 for a set of premises, supporting means, and results: These three reasoning styles are well-known. Deduction is considered to be the main mechanism. It is the basis for Mathematical Logics. Deductive reasoning is based on three postulates that are too restrictive: (A) completeness of the specification, (B) agreement on the background and the matrix, and (C) context-independence. The Peirce

[14]M.W. Wartofsky already states in [35]: "*There is an additional trivial truth, which may strike some people as shocking: anything can be a model! ... And although it is the case that anything can be a model of anything else, it is taken as a model which makes an actual out of a potential model; and every case of being taken as a model involves a restriction with respect to relevant properties.*"

[15]See, for instance, [30] on the problematic side of first-order predicate logics for database engineering.

	Deduction	Induction	Abduction
Reasoning style	Rule-Data-Result	Data-Result-Rule	Rule-Result-Data
First	general rules	observed primary phenomena	general rules
Second	specific observations	dependent secondary phenomena	dependent secondary phenomena
Finally	conclusion for observations and new rules	rule supposition and questions	potential (causal) explanations

Table 1. Subduction: deduction, induction, abduction

triangle is more general, however: From its *abductive* suggestion, *deduction* can draw a prediction which can be tested by *induction*. Abduction is well-known. Induction is far less accepted since rule suppositions are only hypothetical results and have to be revised whenever primary and secondary phenomena are not matching anymore. Many researchers, e.g. K.R. Popper[16], however, strictly deny usefulness and utility of induction and avoid usage of induction.

3.2. Inductive Model-Based Reasoning

Induction is the most prominent and important reasoning mechanism in daily life and for model building based on evidences or observations. Inductive conclusions are uncertain due to the incompleteness of observations. Worlds that are potentially infinite are and will be, however, never completely observable. The inherent incompleteness of the world of phenomena shows that induction is the best logical mechanism for human reasoning.

We distinguish between:

- Induction in broad sense as explanatory inferences, as well as analogical and 'more-of-the-same" inferences in the style: *'All observed Xs have property P'* to *'The next X observed will have property P'*.
  It includes explanatory inferences, as well as analogical and 'more-of-the-same" inferences.
- Induction in narrow sense is based on a random sample (with test/validation set) and results in simple enumerative induction (or the straight rule).

Induction degrees are either strong inductive argument based on authority, on evidence, or stronger inductive argument based on better evidence.

The inductive reasoning schema is based on given knowledge $\mathcal{K}$, beliefs $\mathcal{B}$, models $\mathcal{M}$ known so far, and data $\mathcal{D}$ observed. Within the setting of some reasoning systems Γ, we assume $\mathcal{K} \cup \mathcal{M} \not\models_{\Gamma} \mathcal{D}$.

The induction task aims at discovery of a formula α (not uniquely defined) such that

- it is coherent with $\mathcal{K}$, $\mathcal{M}$, and $\mathcal{D}$ and

[16] [23]: *"Induction simply does not exist, and the opposite view is a straight-forward mistake. ... I hold that neither animals nor men use any procedure like induction, or any argument based on repetition of instances. The belief that we use induction is simply a mistake."*

- that allows to explain $\mathcal{D}$, i.e.
  - $\mathcal{K} \cup \mathcal{M} \cup \mathcal{D} \not\models_{\Gamma} \neg\alpha$ and
  - $\mathcal{K} \cup \mathcal{M} \cup \{\alpha\} \models_{\Gamma} \mathcal{D}$

Inductive reasoning schemata can be extended to Solomonoff induction bound by Kolmogorov complexity. In this case, sophisticated inductive reasoning generates most relevant, most simple, and preferred generalisations from facts and/or observations. The conclusions can be revised whenever the fact or observation set is extended. Inductive reasoning inherits the obstinacy of the representation language of facts and observations.

Induction is a kind of compilation-so-far reasoning with uncertainty, preferences for conclusions, and complexity reduction for result presentation.

Induction is transfer of likely truth from a number of observations to a general principle. It is based on conjecture spaces and specific approaches, experience, (tacit) knowledge, parsimony, economy, clever sampling, and wise experimentation. Induction is a very strong modelling principle. We appreciate statistical, probabilistic, possibilistic, eliminative, and mathematical induction. Induction can be treated as 'blind' search (depth-first, breath-first). It can also be clever search for suppositions as humans like to do.

3.3. Abductive Model-Based Reasoning

Abductive reasoning (e.g. [2,18,24]) is a kind of concise reasoning that infers particular cases from general observations and rules. It is a weak kind of inference because we cannot say that the explanation is true, but that it can be true.

Premises are given in the form:

- D is a collection of data, facts, observations
- M explains D within a given reasoning mechanism.
- No other model can explain D as well as M does.

Conclusion: Therefore, the model M is probably acceptable.

Abductive model-based reasoning is a process that tries to form plausible models for some situations. It covers also abnormal situations. The inference result is a model, which is somehow acceptable within the given reasoning mechanism and, thus, could explain the occurrence of the given facts. This approach can be used for detection of good explanations and especially good causal explanations.

A typical abductive hypothetical reasoning schema is the following:

1. Searching somehow anomalous, surprising, or disturbing phenomena and observations.
2. Observing details, little clues, and tones.
3. Continuous search for hypotheses and noting their hypothetical status.
4. Aiming at finding what kind or type of explanations or hypotheses might be viable to constraint the search in a preliminary way.
5. Aiming at finding explanations (or ideas) which themselves can be explained (or be shown to be possible).
6. Searching for 'patterns" or connections that fit together to make a reasonable unity.

7. Paying attention to the process of discovery and its different elements and phases.

Abductive reasoning also allows to consider negative information by modus tollens

$$\frac{H \rightarrow I\,,\ \neg I}{\neg H}$$

.

The Mathematical Model of Meaning [15] is a third kind of abductive reasoning schema that is used for categorisation of observations:

- Empirical observations can be represented by data representing the importance of some feature for the observation.
- Importance data should be normalised, e.g. 0, .., 10.
- Data can be represented as a table (or matrix) with some features/indicators as attributes.
- Attributes (in the universal world approach) can be related to categories. The database is then a universal relation with tuples where those values that are $\neq 0$ show belongness to a category.
- Multiplication of tuples from the observations with the feature-category matrix results in a tuple that characterises the belongness of an observation to a category.

Feature $F_j\,(1 \leq j \leq f)$ are relevant $d_{i,j}\,(1 \leq i \leq m,\ 1 \leq j \leq f)$ for observations. $o_i\,(1 \leq i \leq m)$. These features belong to categories $C_k\,(1 \leq k \leq R)$ by a knowledge or abduction matrix $c_{j,k}\,(1 \leq j \leq f,\ 1 \leq k \leq R)$.

$$\begin{pmatrix} & F_1 & \dots & F_f \\ o_1 & d_{1,1} & \dots & d_{1,f} \\ \dots & \dots & \dots & \dots \\ o_m & d_{m,1} & \dots & d_{m,f} \end{pmatrix} \times \begin{pmatrix} & C_1 & \dots & C_R \\ F_1 & c_{1,1} & \dots & c_{1,R} \\ \dots & \dots & \dots & \dots \\ F_f & c_{f,1} & \dots & c_{f,R} \end{pmatrix} = \begin{pmatrix} & C_1 & \dots & C_R \\ o_1 & r_{1,1} & \dots & r_{1,R} \\ \dots & \dots & \dots & \dots \\ o_m & r_{m,1} & \dots & r_{m,R} \end{pmatrix}$$

The result of multiplication of the observation matrix with the abduction matrix is a matrix of relevance of a category $C_1, ..., C_R$ for an observation o_i.

3.4. Principles and Assumptions of Reasoning by Models

Models are instruments that properly function in utilisation scenarios. The utility is given by the quality of appropriateness in use. Therefore, we have to understand which objects, artifacts, and thoughts can really be used as models whenever we base reasoning on models. Appropriateness of models is a specific variant of the design principle 'form-follows-function'.

Goals, purposes, and function must be well-defined, well-thought and achievable: In most cases, utilisation scenarios are neither an ad-hoc, nor chaotic, or nor trial-and-error flows of work. They must not be fully defined. We have to understand to a greater or lesser extent what should be done and, especially, which instruments might be useful in which way on which grounds. The profile of an instrument is given by the goal we follow, by the means we could use for our goal (i.e. purposes of the instrument), and by the way how the instrument is going to be used according to the purpose (i.e. function of the instrument). Instruments shall be effective. From the other side, appropriateness of an instrument is also determined whether the goal is accomplishable.

Models are mental compilations of observed worlds: Models are a product of our thoughts. As a referent, we observe some situation in our world. Following the consideration by [16] on the three analogies by Platon (analogy of the cave, of the sun, and of the divided line), the referent recognises shadows in the observable world, builds some comprehension based on the thoughts and his/her intellect, and uses some language (not necessarily natural one; potentially some visual one) for reflection by terms, e.g. signs and images.

The background of models strikes through or is limiting reasoning: As already noted, models are often only given as the normal model while the deep model is implicit and the matrix of model application is commonsense in a discipline. As long as the deep model and the matrix are unconditionally acceptable and have not to be changed, the results of model functioning are reliable. There are, however, reasons to reconsider this background and these application frames. The potential and capacity of a model is restricted to these assumptions. Models are, however, not really context-free. They have their anti-profile also due to restrictions and their focus.

Evidential reasoning as initial point for model development: Evidential reasoning starts with evidences or glues and compiles guesses or conclusions, thus, providing hints about possible or likely conclusions with an explicit representation of uncertainty. Evidences, thus, support or refute hypotheses about the current status of the existing and observable situation. Unobservable propositions can be then determined on the basis of observable evidence, e.g. the observable data are used to reason on the almost unobservable real and micro-data. Evidential reasoning should be distinguished from causal reasoning which orients on explaining observable evidences by a hypothesised cause. It has its limits which should be integrated into this reasoning style.

Living in a world without necessity for a universal world formula — Almost plausibility and inherent incompleteness: Models have to be incomplete whenever they are based on the principles of reduction, decontextualisation, vagueness, and ignorance. Models used in some domain have not to be consistent. A property that is acceptable in this case is coherence what means that sub-models of models which express the same set of properties are compatible and partially homogeneous to certain degree (so-called non-adjunctive model suites) [14]. Inconsistency is handled in a controlled way by many-facetted coherence without integration. A classical example is Bohr's theory of atom and the system of Maxwell's equations. Paraconsistency treats a collection of models as consistently as possible without requiring full consistency. Model suites represent then some kind of 'knowledge islands' with partial bridge axioms.

It is surprising that neither form-follows-function, form-restricts-function, function-and-form-determine-techniques, nor inherent incompleteness and almost plausibility have been explicitly discussed in model theory and practice. A model property that is commonly accepted is well-formedness (some times called 'beauty', stronger well-defined) of models. They allow proper application of methods that support functioning of models.

4. Model-Backeded Reasoning Mechanisms

Model-backed reasoning is completely different from classical logical reasoning techniques. It might use deduction. Models don't have to be true, consistent, fully integrateable with other models, based on a homogen understanding, at the most recent state-of-art, or acceptable by everybody. They can be certain to some limited extent, somehow coherent or even paraconsistent, heterogenous, representing islands without homogeneity, combine various generations of knowledge, or personal opinions. The two lists are not complete but demonstrate the difficulty to develop a sophisticated theory of model-backed reasoning. Instead, let us consider some of the most essential reasoning procedures for models.

4.1. Plausible Reasoning

Models must not be complete and are considered within the given but changeable context. Models don't have to be true. They have to be useful and functioning as instruments in the given scenario. Models focus and scope on certain parts while neglecting others, i.e. they are using approaches of ignorance. Therefore, they are typically incomplete. The premise set does not strictly allow for every conclusion with certainty. Model development is often based on inductive and evidential reasoning which is another source for incompleteness. Model-backed reasoning has to additionally use more appropriate reasoning mechanisms. We especially use approaches based on plausibility and approximation.

Plausible reasoning stand for reasoning with uncertain conclusions for both certain or uncertain premisses. Typical forms are abductive, analogical, autoepistemic, counterfactual, default, defeasible, endorsement-backed, presumptive, and non-monotonic reasoning techniques.

The classical approach to plausible reasoning is given by the following schema:
The lack of soundness makes the conclusion plausible with a certainty below 1.0 based on evidence $CertF(\alpha|e)$ and reasonable (called believable) $ReasonF(\alpha|e)$ with reasonability below or equally certainty.
Certainty factors and reasonability factors may follow empirical rules to aggregate pieces of evidence, e.g.

$$CertF(\alpha|e_1 \wedge e_2) = min(CertF(\alpha|e_1), CertF(\alpha|e_2)) \text{ and}$$
$$CertF(\alpha|e_1 \vee e_2) = max(CertF(\alpha|e_1), CertF(\alpha|e_2))$$

in the Dempster/Shafer reasoning style or in the possibility theory style.

These rules shall be applied in dependence on the context since they may lead to unpredictable, problematic, and counterintuitive results. Negation can be handled as negation-by-failure or in a multi-valued or paraconsistent form [25].

4.2. Approximative Reasoning

Models must not be precise although precision is necessary whenever models are used for automatic generation of solution from a given model [4,7,10]. Instead, model can be reduced, abstracted, truncated, imprecise, and raw. We follow the principle of parsimony and economy. Models must support efficient and effective thinking and actuation. The final and optimal solution might not exist at all or is infeasible both in time for its generation and in space for its presentation. Although, tools might not exist.

Approximation supports aggregative, generative, imprecise and robust reasoning. Approximate reasoning based on models is a common form to avoid complexity throwback due to over-detailing. Typical kinds are reasoning systems supporting aggregation and cumulation, generalisation and categorisation, imprecision, heuristics, robust thinking, and shallow consideration. These reasoning styles are used in daily life and especially for models, e.g. best characterised by the Austrian saying 'paßt schon' (fits somehow, fits already, close enough, suits, somehow convenient). In Computer Science, approximate algorithms provide a reasonable solution to problems at polynomial time instead of optimal solutions computable at (hyper-)exponential time in dependence on the problem complexity measured by Kolmogorov complexity [17]. The principle of Occam's razor orients on models as 'simple" as possible. We, thus, do not miss simple models. We may also use approximative rules with preference and simplicity in the Solomonoff style [8], e.g. for model-based explanation.

4.3. Hypothetical Model-Based Reasoning

Hypothetical model-based reasoning is based on the following schema:

1. Given a hypothesis model M that implies a statement E which describes observable phenomena.
2. The statement E has been observed as true.
3. The conclusion is that M is true.

The method of the hypothesis is not deductively valid because wrong hypotheses can also have real consequences.

Different assumptions are considered in order to see what follows from them, i.e. reasoning about alternative possible models, regardless of their resemblance to the actual world. Potential assumptions with their possible world conclusions assertions are supported by a number of hypotheses (allowing to derive them). Inductive model-based reasoning can be combined with abductive reasoning.

Hypothetical model-based reasoning restricts inductive reasoning by specific forms of inductive conclusions:

1. *statistical inductive generalizations*, in which the premise that x percent of observed A's have also been B's, so that the conclusion is, x percent of all A's are B's;
2. *predictive conclusions*, in which the premises are that x percent of the observed A's have also been B's, and a is an A and where the conclusion is that a is a B;

3. *direct conclusions*, in to which the premises are that x percent of all A's are also B's, and that a is an A and where the conclusion is that a is a B, and
4. *conclusion by analogy* in which the premises are that certain individual objects have the properties $F_1, ..., F_n$ and a have the properties $F_1, ..., F_{n-1}$, and where the conclusion is that a also has the property F_n.

4.4. Model-Based Explanation

The reasoning schema used for inductive reasoning can be extended to model-based explanation that describes, explains, illustrates, clarifies and characterises in a guiding way in a mediation scenario essential, central and in the given scenario important elements of complex origins in a comprehensible, concrete and coherent form for the recipient.

The reasoning schema for model-based explanation, elaboration, and comprehension can be defined as follows:
Given

some theoretical background $\mathcal{T}$ based on the context, background, knowledge, concepts, etc.,

a model class $\mathfrak{M}$ with orders for preference $\leqq$ and simplicity $\lesssim$,

a deducability operator $\Vdash$ as an advanced operator for deductive, abductive, inductive, non-monotonic, approximative, and plausible derivation of conclusions, and

data under consideration O (observations) from the data space derived from input model suite data and prepared for analysis while being $\mathcal{T} \nVdash O$ (non-trivial for $\mathcal{T}$) and $\mathcal{T} \nVdash \neg O$ (not in conflict with $\mathcal{T}$).

The model $\mathcal{M}_E$ from $\mathfrak{M}$ is an **explanation model** for O within $\mathcal{T}$ if

it explains O within $\mathcal{T}$, i.e. $\mathcal{T} \sqcup \mathcal{M}_E \Vdash O$ while being
non-trivial (or parsimonious) for O, i.e. $\mathcal{M}_E \nVdash O$ and
coherent with O and $\mathcal{T}$, i.e. $\mathcal{M}_E \nVdash \neg O$ and $\mathcal{T} \sqcup \mathcal{M}_E \nVdash \perp$.

Based on the orders in $\mathfrak{M}$ we may be interested in the weakest (best) explanation model that is additionally parsimonious, i.e. $\mathcal{M}_E \nVdash O$.

Model-based explanation is not consolidative modelling that uses the model as a surrogate for the system, for instance, by consolidating known facts about the system for purposes of analysis whether the model adequately represents the system. Model-based explanation explores how the world would behave if various models were correct. Many details and mechanisms of a system are uncertain. The model has not to be a reliable image of the world. Relevant 'ground truth' data for evaluating model may not be obtainable. We, thus, identify an ensemble of plausible models and modelling assumptions, identify the range of outputs predicted by plausible models under plausible assumptions, and identify the relationship between modelling assumptions and model outputs. A trick is to find assumptions that have a large impact on model outputs. Another trick is to identify predictions that are robust across different sets of modelling assumptions.

There is no methodological approach for derivation of a good explanation model. It seems that the embracement method is the strongest one. This method considers at the same time the generation of models from one side and the generation of partially explaining models from the other side. These partially explaining models can be seen as hypotheses which would form a good explanation model together with the generated model. A simple embracement method are Mill's methods of agreement, difference, joint methods of agreement and difference, residues, and concomitant variations.

4.5. The Model as Mediator in Empiric Model-Based Reasoning

Empiric investigation and reasoning is based on data spaces. Data are structured according to properties of parameters. It is often not possible to observe all parameters. We may distinguish outer parameters that can be observed and inner parameters that cannot be accessed or are not yet observed or are not yet observable. This separation is similar to genotype and phenotype observations. The problematic accessibility for inner parameters has already been discussed by Platon in his analogy of the cave (see, for instance, [16]). Development of an understanding based on outer parameters is a real challenge that is difficult to overcome. The data space should give an insight into potential quantitative observation concepts or conceptions. We need an insight into the data space for the inner parameters in order to reason on the reality situation.

Empirical reasoning starts with an investigation of data sources for the outer parameters and develops some quantitative observation concepts that might be embedded into a theory offer. A *theory offer* is a scientific, explicit and systematic discussion of foundations and methods, with critical reflection, and a system of assured conceptions providing a holistic understanding. A theory offer is understood as the underpinning of technology and science similar to architecture theory [26] and approaches by Vitruvius [34] and L.B. Alberti [1]. Theory offers do not constitute a theory on their own, rather are some kind of collection consisting of pieces from different and partially incompatible theories, e.g. sociology theories such as the reference group theory, network theories, economic theories such as the agent theories, Darwinian evolution theories, subjective rationality theories, and ideology theories.

The main target is however, to form a theory that is based on the data, that is based on concept or conceptions, and that allows to draw conclusions on this theory. Concepts or conceptions to be developed should be qualitative and theory-forming. If qualitative concepts cannot be drawn then we need quantitative concepts that allow reasoning. Figure 2 displays this challenge. The challenge is solved if a number of functions exist.

The best solution for this challenge would be if we can map the inner parameters to the outer ones by a function f_o^i and use some kind of abstraction function g_c^o for association of data to concept(ion)s. In this case, we might succeed in constructing functions g_c^i resp. F_i^o from the reality situation resp. outer concept(ion)s to quantitative inner concept(ion)s. Then we could use the theory embedding of outer quantitative concept(ion)s h_t^o for construction of such inner functions h_t^i. This would also result in a coherence condition and a commuting

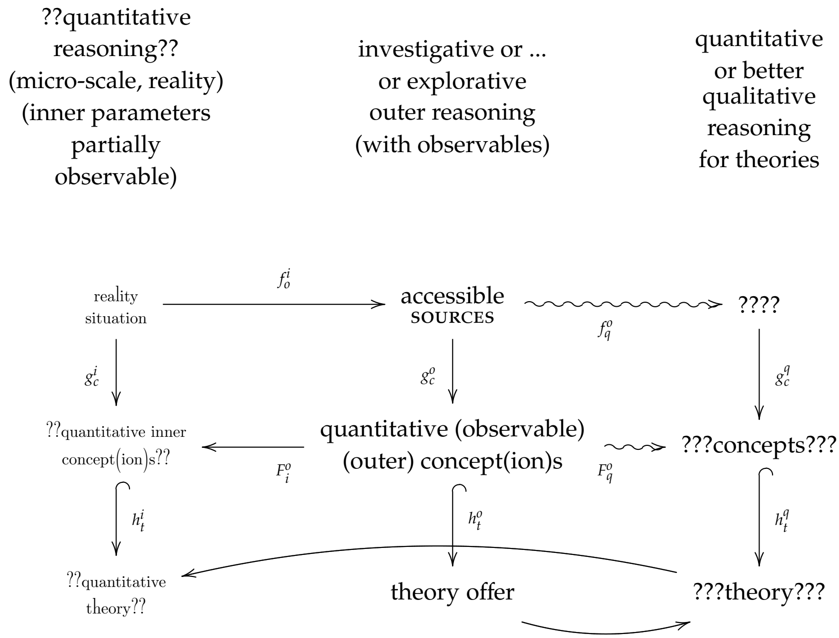

Figure 2. The challenge of empirical reasoning targetting on qualitative concepts and theory development with only partially known data for the inner parameters

diagram $F_i^o(g_c^o(f_o^i(situation))) = g_c^i(situation)$. We will be able to use the embedding function h_t^c for construction of a corresponding supposition h_t^i for inner parameter theories.

The next step is a constrcution of a reasoning system. We use some aggregation function f_q^o for compiling sources in support of concepts by a function g_c^q and for embedding these concepts into a theory by h_t^q. If we succeed then we can use the theory offer for the construction of a theory for reasoning and as the next step for mapping this theory back the to inner quantitative theory.

We arrive therefore with the big challenge of empiric research: *How we can close the gap between quantitative theory offers and qualitative theories?*

This program for empirical reasoning is not really feasible. The construction of the functions is a higher-order challenge. Instead we can use model-based reasoning as displayed in Figure 3. The model is then used as a mediating means between qualitative and quantitative reasoning. The model is at the same time (1) a means, (2) a mediator, and (3) a facilitator [3], i.e. (1) an instrumentality for accomplishing some end, (2) a negotiator who acts as a link between quantitative and qualitative issues, and (3) an instrument that makes reasoning easier. Since models are more focused, we do not have to have fully-fledged functions. Instead, we can concentrate on the main issues. At the same time, we properly support qualitative reasoning based on our data spaces. This would also allow to formulate proper hypotheses from the model world to the quantitative world. The validity power of the model would then support qualitative reasoning.

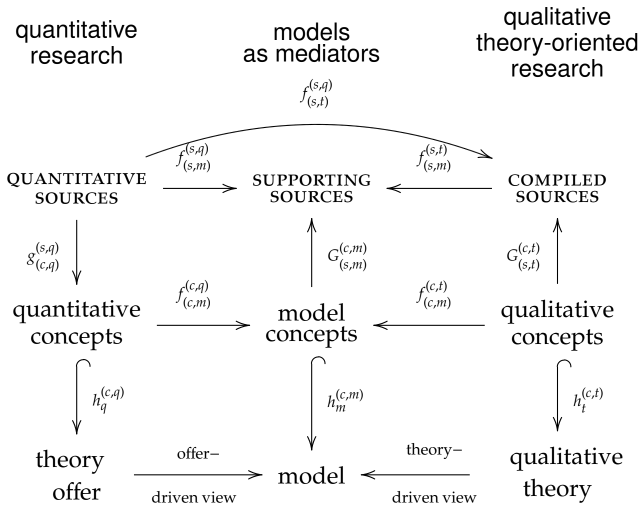

Figure 3. Models as integrating and mediating instrument in empiric research

In this case, we may succeed in constructing insights that go far beyond data-backed reasoning, e.g. in data science. We could then also construct massives of supporting sources for our models. The model accommodates the quantitative theory, the theory offers, and the qualitative theory.

Theories can be built on the basis of theoretical concepts which are supported by sources. Quantitative concepts should be associated with qualitative concepts. The association can only be developed in the case when the association among the data has been clarified. So far, the explanations that can be generated are mainly developed for explaining the observations made on the basis of outer data.

4.6. Meta-Model Reasoning Used for Model-Backed Steering

Thinking in models should be supported by a systematic methodology. Model-backed steering use meta-models (i.e. *steering models*) as a guiding or motivating model that directs the direction of reasoning through and by models. This kind of meta-reasoning enables us to explore potential opportunities in the opportunity and possibility spaces. We arrive at some proposal as a result of reasoning through and by models, i.e. putting forward or stating something for consideration by making or offering a formal plan or suggestion for acceptance, adoption, or performance. The *proposal model* is used in a second step as an additional origin. The trick we use is then based on second-order modelling. We know already *governing models*. Such meta-models make and administer the selection of opportunities based on possibilities, regulate, and control model-based reasoning while keeping exploration under control. They exercise a deciding or determining influence on selection and thoughts.

As a result of steering and governing, we obtain an *advice model* as a recommendation regarding a decision or course of conduct. It could be considered to state an opinion about what could or should be done about a situation or problem, what is going to be recommended offered as worthy to be followed. Advice models are used as a counsel and denote an opinion as to a decision or course of action.

We may use meta-techniques such as specific question-answer forms (or, more specifically, query-answer or input-output forms [9]). These question-answer forms have their inner meta-structure and inner meta-flow that could be used in investigative research, e.g. what-if analysis, what-would-be-if, 5-why-drill-down, rolling-up distancing, context-enhancement, assumption-slicing with attention restriction, why-it-must-be, why-this-question-and-not-other, why-not-rephrasing, observation-in-context dependencies, immersion-into-context, why-finish, how-we-can-know, why-this-question, question-reformulation by opening or closing the parameter space, and parameter-space-reduction by dicing with tolerance of errors, e.g. by principal component analysis. Essentially, these meta-techniques are steering models.

We use the steering model for driving into a problem space and detecting opportunities and possibilities. Sciences, engineering and daily life are full of such 'wisdom' techniques.

This approach can be generalised to meta-models for research, i.e. moulds[17]. Methodologies are simple moulds. They provide a guidance for a flow of work. Frameworks are complex moulds that can be adapted to the given situation[18]. Civil engineering uses moulds as frame on which something can be constructed.

Steering models are used to control or to direct or to guide the course of actuation. They set, follow, pursue, and hold to a course of action and reasoning and especially a hint as to procedure. Steering models are used as a piece of advice or information concerning the development of a situation. They allow to control a situation so that it goes in the direction that you want. They enable to take a particular line of action. Such meta-models are models of the models, of the modelling activities, and of the model association within a model suite. Their goal is to improve the quality of model outcomes by spending some effort to decide what and how much reasoning to do as opposed to what activities to do. It balances resources between the data-level actions and the reasoning actions. A typical case is design of activities in data mining or analysis [11] where agents are preparation agents, exploration agents, descriptive agents, and predictive agents. Meta-models for a model suite contain decisions points that require macro-model control according to performance and resource considerations. This understanding supports introspective monitoring about performance for the data mining

[17] A *mould* is a distinctive form in which a model is made, constructed, shaped, and designed for a specific function a model has in a scenario. It is similar to mechanical engineering where a mould is a container into which liquid is poured to create a given shape when it hardens. Im Mathematics, it is the general and well-defined, experienced framework how a problem is going to be solved and faithfully mapped back to the problem area.

[18] For instance, analytical solution of differential equations use a set of solution methods formulated as ansatz. Database development can be guided by specific frameworks. Artists and investigative researchers are guided by moulds, i.e. by steering or governing models. Software engineering is overfull of meta-models for design, development, and quality management.

process, coordinated control of the entire mining process, and coordinated refinement of the models. Meta-level control is already necessary due to the problem space, the limitations of resources, and the amount of uncertainty in knowledge, concepts, data, and the environment.

Steering models extend the origin' collection (see Figure 1) by meta-reasoning origins. These origins enable us to use second-order cybernetics [33], i.e. to continuously reason on insight we got in previous steps and to change our mind whenever new insight has been obtained. The methodology follows a mould of continuous changing method application.

This approach is based on control by meta-reasoning [6] as displayed in Figure 4. We distinguish the activity layel that is based on methods ground level,

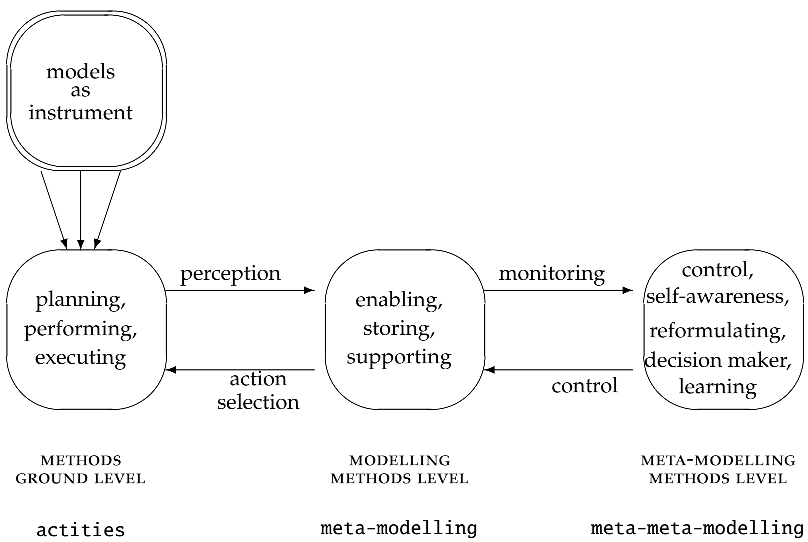

Figure 4. The meta-control mould for meta-reasoning

the meta-modelling level that contains the modelling methods level and rules the selection of actions, and the meta-meta-modelling level that contains the abstraction to meta-modelling methods and controls the middle level section. This approach is similar to government and binding [5] where the utterance playout is based on a second layer playout selection for the utterance that is again ruled by controller for settling the kind of utterance and its general form.

5. Conclusions for 'Modellkunde' – The Study of Models and Modelling

Models must not be true or consistent. They should be useful as instruments in application scenarios. Usefulness presupposes the existence of techniques for model utilisation. Reasoning is one kind of technique. Reasoning is a daily life practice that is rarely based on deductive systems. Instead, induction and abduction are used for model-based reasoning. These reasoning techniques are especially useful for models since we do not require consistency and truth maintenance.

We demonstrated the power of such techniques by the generalisation to plausible, approximative, hypothetical, and explanatory reasoning. These mechanisms are really sophisticated. For instance, pattern recognition and Modellkunde for pattern can be based on explanatory model-backed reasoning in combination with mediator approaches. One of the best achievements is mediator-based reasoning that allows to overcome pitfalls of middle-range theories and badly associated theory offers in empiric research. Instead we use models as a mediating device between empiric and qualitative reasoning. The utilisation of models may be governed by other models. Steering meta-models guide application and usage of models.

This paper can be extended by application of other model-backed reasoning techniques such as separation and concentration of concern, playing with ignorance and de-contextualisation, qualitative techniques used for data analysis, and probabilistic calculi.

This paper has been centred around reasoning techniques and reasoning through and by models. There are many other techniques beside reasoning that

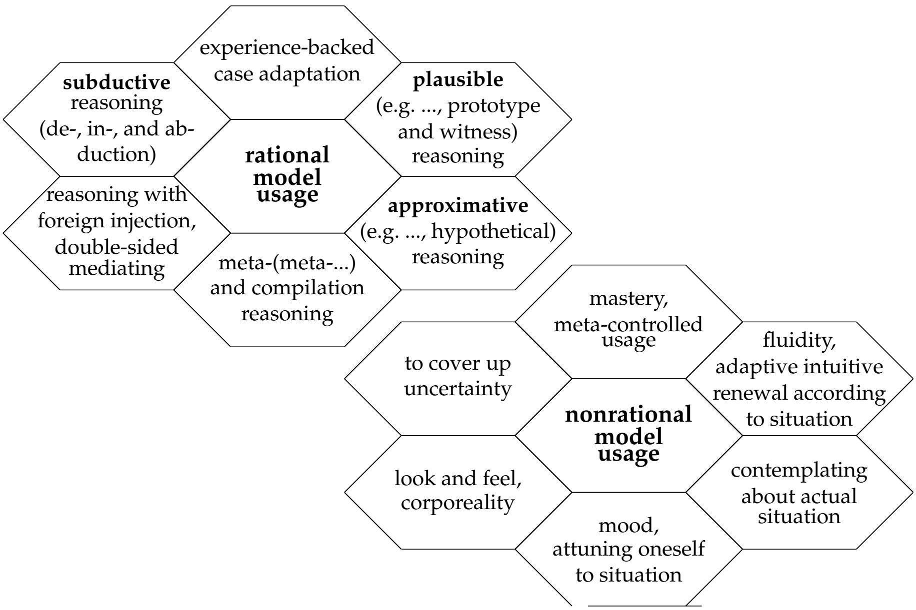

Figure 5. Rational and non-rational reasoning through and by models

can be applied to models as shown in Figure 5 (boldface shows the techniques we considered). Typical techniques are enhancements similar to conceptualisation, model inheritance from generic or reference models, parameter hardening used for inverse modelling in physics, model-based checking and control for systems, and simulation of behaviour for some of the parameters. Cognitive modelling is another technique that has been left out for this paper. Shallow and deep reasoning techniques are another lacuna for the study of models.

The study of models has to considers also other techniques for model utilisation. Models form a landscape. Some models are partially isolated. These isolated models should be supported by bridging techniques. Models are focused and have, thus, their abstraction level. Model-based problem solving use, therefore, also techniques for generalisation and governed specialisation.

Models can also be composed in vertical or horizontal layering. Models can be also origins for other models. The composition should supported by techniques similar to nested data warehousing, i.e. roll-up, drill-down, dice, slice, rotate, algebraic construction, peaceful renovation and updating, unnesting, and nesting. Models may consists of a well-associated collection of models, i.e. of a model suite. Association techniques allow also management of coherence of models in a model suite. Some models in a model suite may play the role of a master (or order) model while others are slaved. The model suite should, thus, be enhanced by control models for adaptation of the master models to a given situation, e.g. in inverse modelling.

References

[1] L.B. Alberti. *On the art of building in ten books*. MIT Press, Cambridge. Promulgated in 1475, published in 1485, 1988.

[2] A. Aliseda-Llera. *Seeking explanations: Abduction in logic, philosophy of science and artificial intelligence*. PhD thesis, Universiteit van Amsterdam, 1997.

[3] C. Blättler. *Wissenschaft und Kunst der Modellierung: Modelle, Modellieren, Modellierung*, chapter Das Modell als Medium. Wissenschaftsphilosophische Überlegungen, pages 107–137. De Gruyter, Boston, 2015.

[4] E. Börger and A. Raschke. *Modeling Companion for Software Practitioners*. Springer, 2018.

[5] N. Chomsky. *Some concepts and consequences of the theory of government and binding*. MIT Press, 1982.

[6] M.T. Cox and A. Raja, editors. *Metareasoning - Thinking about Thinking*. MIT Press, Cambridge, 2011.

[7] A. Dahanayake, O. Pastor, and B. Thalheim. *Modelling to Program: Second International Workshop, M2P 2020, Lappeenranta, Finland, March 10–12, 2020, Revised Selected Papers*, volume 1401 of *CCIS*. Springer Nature, 2021.

[8] David L. Dowe, editor. *Algorithmic Probability and Friends. Bayesian Prediction and Artificial Intelligence*, volume 7070 of *LNCS*. Springer, 2013.

[9] H. Jaakkola and B. Thalheim. Supporting culture-aware information search. In *Information Modelling and Knowledge Bases XXVIII*, Frontiers in Artificial Intelligence and Applications, 280, pages 161–181. IOS Press, 2017.

[10] H. Jaakkola and B. Thalheim. Models as programs: The envisioned and principal key to true fifth generation programming. In *Proc. 29'th EJC*, pages 170–189, Lappeenranta, Finland, 2019. LUT, Finland.

[11] K. Jannaschk. *Infrastruktur für ein Data Mining Design Framework*. PhD thesis, Christian-Albrechts University, Kiel, 2017.

[12] M. Kangassalo and E. Tuominen. Inquiry based learning environment for children. In *Information Modelling and Knowledge Bases XIX*, volume 166 of *Frontiers in Artificial Intelligence and Applications*, pages 237–256. IOS Press, 2007.

[13] R. Kaschek. *Konzeptionelle Modellierung*. PhD thesis, University Klagenfurt, 2003. Habilitationsschrift.

[14] R. Kauppi. Einführung in die Theorie der Begriffssysteme. *Acta Universitatis Tamperensis, Ser. A, Vol. 15, Tampereen yliopisto, Tampere*, 1967.

[15] Y. Kiyoki, T. Kitagawa, and T. Hayama. A metadatabase system for semantic image search by a mathematical model of meaning. *SIGMOD Rec.*, 23(4):34–41, 1994.

[16] C. Lattmann. *Vom Dreieck zu Pyramiden - Mathematische Modellierung bei Platon zwischen Thales und Euklid*. Habilitation thesis, Kiel University, Kiel, 2017.

[17] M. Li and P.M.B. Vitanyi. *An introduction to Kolmorov complexity*. Springer, New York, 2008.

[18] L. Magnani, W. Carnielli, and C. Pizzi, editors. *Model-Based Reasoning in Science and Technology: Abduction, Logic, and Computational Discovery*. Springer, 2010.

[19] B. Mahr. *Visuelle Modelle*, chapter Cargo. Zum Verhältnis von Bild und Modell, pages 17–40. Wilhelm Fink Verlag, München, 2008.

[20] B. Mahr. Information science and the logic of models. *Software and System Modeling*, 8(3):365–383, 2009.

[21] B. Mahr. Modelle und ihre Befragbarkeit - Grundlagen einer allgemeinen Modelltheorie. *Erwägen-Wissen-Ethik (EWE)*, Vol. 26, Issue 3:329–342, 2015.

[22] C.S. Peirce. *The Collected Papers of Charles S. Peirce,* (C. Hartshorne, P. Weiss, and A. W. Burks (eds.)). Cambridge: Harvard University Press, 1931-1966.

[23] K. R. Popper. *Logik der Forschung*. J.C.B. Mohr (Paul Siebeck), Tübingen, 10th. edition, 1994.

[24] M. V. Rodrigues and C. Emmeche. Abduction and styles of scientific thinking. *Synth.*, 198(2):1397–1425, 2021.

[25] K.-D. Schewe and B. Thalheim. NULL value algebras and logics. In *Information Modelling and Knowledge Bases*, volume XXII, pages 354–367. IOS Press, 2011.

[26] G. Semper. *Die vier Elemente der Baukunst*. Braunschweig, 1851.

[27] H. Stachowiak. *Allgemeine Modelltheorie*. Springer, 1973.

[28] B. Thalheim. The conceptual model ≡ an adequate and dependable artifact enhanced by concepts. In *Information Modelling and Knowledge Bases*, volume XXV of *Frontiers in Artificial Intelligence and Applications, 260*, pages 241–254. IOS Press, 2014.

[29] B. Thalheim. Conceptual models and their foundations. In *Proc. MEDI2019, LNCS 11815*, pages 123–139. Springer, 2019.

[30] B. Thalheim. Semiotics in databases, keynote paper. In *Proc. MEDI2019, LNCS 11815*, pages 3–19. Springer, 2019.

[31] B. Thalheim and A. Dahanayake. Comprehending a service by informative models. *T. Large-Scale Data- and Knowledge-Centered Systems*, 30:87–108, 2016.

[32] B. Thalheim and I. Nissen, editors. *Wissenschaft und Kunst der Modellierung: Modelle, Modellieren, Modellierung*. De Gruyter, Boston, 2015.

[33] S. A. Umpleby. Second-order cybernetics as a fundamental revolution in science. *Constructivist Foundations*, 11(3):455–465, 2016.

[34] Vitruvius. *The ten books on architecture (De re aedificatoria)*. Oxford University Press, London, 1914.

[35] M.W. Wartofsky. *Conceptual foundations of scientific thought*. MacMillian, New York, 1968.

Remark: More detailed information on our research papers can be found on research gate in collections at https://www.researchgate.net/profile/Bernhard_Thalheim .

See also the youtube channel "Bernhard Thalheim", videos at https://vk.com/id349869409 or https://vk.com/id463894395 (in German).