Proceedings of CECNet 2021
A.J. Tallón-Ballesteros (Ed.)

doi:10.3233/FAIA210430

A Novel Dual Mode Decision Directed Multimodulus Algorithm (DM-DD-MMA) for Blind Adaptive Equalization

Sagi TADMOR, Sapir CARMI and Monika PINCHAS [1]
Department of Electrical and Electronic Engineering, Ariel University, Ariel 40700, Israel

Abstract. In this paper, we propose for the 16 quadrature amplitude modulation (QAM) input case, a dual-mode (DM), decision directed (DD) multimodulus algorithm (MMA) algorithm for blind adaptive equalization which we name as DM-DD-MMA. In this new proposed algorithm, the MMA method is switched to the DD algorithm, based on a previously obtained expression for the step-size parameter valid at the convergence state of the blind adaptive equalizer, that depends on the channel power, input signal statistics and on the properties of the chosen algorithm. Simulation results show that improved equalization performance is obtained for the 16 QAM input case compared with the DM-CMA (where CMA is the constant modulus algorithm), DM-MCMA (where MCMA is the modified CMA) and MCMA-MDDMA (where MDDMA is the modified decision directed modulus algorithm).

Keywords. Blind equalization; CMA; Adaptive equalization; Dual mode algorithm; Decision directed algorithm

1. Introduction

Data transmission over band-limited communication channels requires the application of equalizers to remove intersymbol interference (lSI) caused by the channel properties [1]. We deal in this paper with the blind adaptive equalization issue where the CMA [2] and the MMA [3], [4] are involved in updating the equalizer's coefficients. According to [1] and [5]-[11], the CMA algorithm is one of the most widely used blind equalization algorithm and according to [12], it has become the workhorse for blind equalization. The CMA [2] is a computationally simple algorithm with outstanding equalization performance for source signals such as M-Ary Phase Shift Keying (MPSK). But, for input signals belonging to the 16QAM constellation, it reaches at the steady state a non negligible residual ISI which may not be sufficient for the system to obtain adequate performance. Thus, the idea proposed by [9] was to switch this computationally simple algorithm to a

[1]Corresponding Author: Department of Electrical and Electronic Engineering, Ariel University, Ariel 40700, Israel; E-mail: monikap@ariel.ac.il.

DD algorithm based on the output decision error calculated from the equalizer's output. In other words, the CMA algorithm [2] was used for the initial acquisition phase for removing the heavy ISI and then switched to a tracking algorithm, namely to the DD algorithm in order to obtain a faster convergence speed with a lower residual ISI compared to the algorithm that does not involve the DD algorithm. Indeed, according to [9], the equalization performance was seen improved with the DM-CMA algorithm compared with the CMA method [2] for the 16QAM source constellation input. According to [5], the fact that the CMA is carrier phase independent, is actually considered as an advantage of the CMA since no carrier synchronization is required before blind equalization. According to [7], the constellation seen at the equalized output sequence suffers from an arbitrary phase rotation. Thus, a phase rotator is required at the convergence state of the equalizer in order to rotate the constellation back in the right position. In order to eliminate the need for a rotator to perform separate constellation-phase recovery in steady-state operation, [3], [4] proposed the MMA algorithm that indeed eliminates the need for a phase rotator. Moreover, according to [1], the MMA compensates also minor frequency errors. The advantage of the MMA is due to a separate error-calculation, that is, for real and imaginary part of the received signal, individually [1].

When the equalizer has converged to a relative low residual ISI level, the convolutional noise seen at the equalizer's output is also relative low and considered as Gaussian [13]. But, this is not the case at the beginning of the equalization process. At the beginning of the equalization process, the ISI is usually high and the convolutional noise sequence is more uniform than Gaussian [13], [14]. Please note that the convolutional noise exists at the equalized output in addition to the input signal. The convolutional noise exists since the values for the equalizer's coefficients are not the optimal ones leading to zero residual ISI. The author in [15], considered the case of input signals where the real and imaginary parts of the input signal are independent and where the error involved in the update mechanism of the equalizer is a polynomial function of order three. Please note that the 16QAM source input belongs to this case and that the error involved in the MMA algorithm can be modeled as a polynomial function of order three. According to [15], if the equalizer converges to a very low residual ISI, the convolutional noise probability density function (pdf) can be considered approximately as Gaussian if the step-size parameter complies on some constraints depending on the input constellation statistics, channel power, chosen equalization method and equalizer's tap-length. Based on this outcome, [16] obtained a novel algorithm for a blind adaptive equalizer that supplies the time it takes the equalizer entering the steady state operation without the knowledge of the initial ISI, depending on the input signal statistics and properties of the chosen equalizer. The main idea used in [16] was looking at the squared error obtained by the difference between the step-size parameter obtained from [15] that does not contain any convolutional noise, to the step-size parameter obtained from [15] that involves the convolutional noise by using in the step-size parameter calculation, the equalized output signal statistics instead of the input signal statistics. Please note that for the noiseless case, the equalized output signal statistics are equal to the input signal statistics when the convolutional noise (namely the ISI) tends to zero.

In this paper, we propose a dual-mode, decision directed multimodulus algorithm (DM-DD-MMA) for blind adaptive equalization, where the MMA algorithm is switched to the DD algorithm. The switching mechanism is based on [16] and [15] where we use the normalized squared error obtained by the difference between the step-size parameter ob-

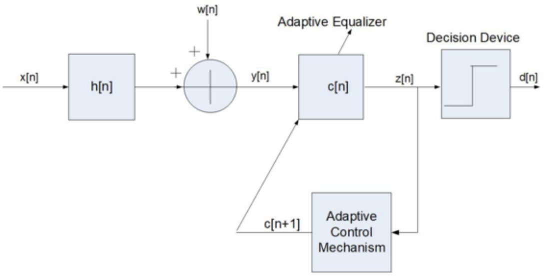

Figure 1. The considered system.

tained from [15] that does not contain any convolutional noise to the step-size parameter obtained from [15] that involves the convolutional noise by using in the step-size parameter calculation, the equalized output signal statistics instead of the input signal statistics. Simulation results will show that for the high SNR case, the new algorithm (DM-DD-MMA) achieves better equalization performance from the convergence speed point of view compared with the DM-CMA,DM-MCMA and MCMA-MDDMA methods while leaving the system with the lowest residual ISI that is approximately obtained also by the DM-MCMA algorithm which has almost the slowest convergence speed compared to all the other methods.

2. System Description

The system under consideration is a linear system, given in Figure 1, where we want to recover its input using an adjustable linear filter (equalizer). In addition, we apply the following assumptions:

- A 16QAM source is used for $x[n]$ where the real and imaginary parts of $x[n]$ are denoted as $x_1[n]$ and $x_2[n]$ respectively.
- The channel $h[n]$ is modeled as a finite impulse response (FIR) filter with channel tap length of $\tilde{R}$.
- The equalizer $c[n]$ is a FIR filter with equalizer's tap length of N.
- $w[n]$ is modeled as a Gaussian white noise process.
- The real part of the equalized output sequence is denoted as $z_1[n]$.
- The output sequence of the decision device is denoted as $d[n]$.

The equalizer's output is given by:

$$z[n] = y[n] * c[n] = x[n] * s[n] + w[n] * c[n], \tag{1}$$

where "$*$" stands for the convolutional operation, $y[n] = x[n] * h[n] + w[n]$ and,

$$s[n] = c[n] * h[n] = \delta[n] + \xi[n] \tag{2}$$

where $\delta[n]$ is the Kronecker delta function and $\xi[n]$ stands for the error not having perfect equalization. By perfect equalization we mean that $s[n] = \delta[n]$ which leads for the noiseless case to $z[n] = x[n]$. Now, based on (1) and (2) we may write:

$$z[n] = y[n] * c[n] = x[n] * (\delta[n] + \xi[n]) + w[n] * c[n] = x[n] + p[n] + w[n] * c[n], \tag{3}$$

where $p[n]$ is named as the convolutional noise and is given by $p[n] = x[n] * \xi[n]$. The equalizer's performance is measured via the ISI, given by:

$$ISI = \frac{\sum_t |s_t[n]|^2 - |s|^2_{max}}{|s|^2_{max}}, \tag{4}$$

where $t = N + \tilde{R} - 1$ and $|s|_{max}$ is the maximal absolute value of $s[n]$, given in (2). Based on [9], the equalizer's coefficients are updated for the DM-CMA algorithm by:

$$\begin{array}{l}
e[n] = sign\{z[n]\}\left[|z[n]| - R\right]; \quad e_D[n] = z[n] - d[n]; \quad R^q = \frac{E\left[|x[n]|^{2q}\right]}{E\left[|x[n]|^q\right]} \\
\text{if} \quad sign\{Re[e[n]]\} \neq sign\{Re[e_D[n]]\} \quad \text{or} \quad sign\{Im[e[n]]\} \neq sign\{Im[e_D[n]]\} \quad then \\
\qquad \underline{c}[n+1] = \underline{c}[n] - \mu_{DM-CMA_1}\left[z[n]\left(|z[n]|^2 - R^2\right)\right]\underline{y}^*[n] \\
\text{if} \quad sign\{Re[e[n]]\} = sign\{Re[e_D[n]]\} \quad \text{or} \quad sign\{Im[e[n]]\} = sign\{Im[e_D[n]]\} \quad then \\
\qquad \underline{c}[n+1] = \underline{c}[n] - \mu_{DM-CMA_2}\left[z[n]\left(|z[n]|^2 - |d[n]|^2\right)\right]\underline{y}^*[n]
\end{array} \tag{5}$$

where $|(\cdot)|$, $sign\{(\cdot)\}$ and $(\cdot)^*$ are the absolute operator, signum function and conjugate operator respectively on $(\cdot)$. μ_{DM-CMA_1} and μ_{DM-CMA_2} are step size parameters, $d[n]$ is the output sequence of the decision device, $Re[\cdot]$ and $Im[\cdot]$ are the real and imaginary parts of $[\cdot]$ respectively and the vector $\underline{c}[n]$ holds the equalizer's coefficients. The input vector $\underline{y}[n]$ is of length N which is the equalizer's tap length. Please note that R^q is a constant modulus depending on the input signal statistics where for $q = 1$ and $q = 2$ we have R and R^2 respectively. Based on (5), when $sign\{Re[e[n]]\} \neq sign\{Re[e_D[n]]\}$ and $sign\{Im[e[n]]\} \neq sign\{Im[e_D[n]]\}$, the CMA algorithm [2] is used to remove the ISI which means that for that case, the probability of correct decisions from the decision device is not very high. Otherwise, the equalizer's coefficients are updated based on the output of the decision device. It should be pointed out that it is assumed in [9], that when $sign\{Re[e[n]]\} = sign\{Re[e_D[n]]\}$ or $sign\{Im[e[n]]\} = sign\{Im[e_D[n]]\}$, the update of the equalizer's coefficients has the right direction and contributes to convergence. According to [1], the equalizer's coefficients are updated for the MCMA-MDDMA algorithm by:

$$\begin{array}{l}
e_R[n] = Re[z[n]]\left[|Re[z[n]]|^2 - \frac{E\left[|x_1[n]|^4\right]}{E\left[|x_1[n]|^2\right]}\right] + Re[z[n]]\left[|Re[z[n]]|^2 - |sign\{Re[z[n]]\}|^2\right] \\
e_I[n] = Im[z[n]]\left[|Im[z[n]]|^2 - \frac{E\left[|x_2[n]|^4\right]}{E\left[|x_2[n]|^2\right]}\right] + Im[z[n]]\left[|Im[z[n]]|^2 - |sign\{Im[z[n]]\}|^2\right] \\
\quad Re[\underline{c}[n+1]] = Re[\underline{c}[n]] - \mu_{MCMA-MDDMA}\left[e_R[n]Re\left[\underline{y}[n]\right] + e_I[n]Im\left[\underline{y}[n]\right]\right] \\
\quad Im[\underline{c}[n+1]] = Im[\underline{c}[n]] + \mu_{MCMA-MDDMA}\left[e_R[n]Im\left[\underline{y}[n]\right] - e_I[n]Re\left[\underline{y}[n]\right]\right]
\end{array} \tag{6}$$

where $\mu_{MCMA-MDDMA}$ is the step size parameter. Please note that according to [1], the MCMA-MDDMA algorithm belongs to a group of algorithms named as joint algorithms. This group additively combines two single algorithms which in our case are the MCMA and MDDMA algorithms. Based on [1], we denote DM-MCMA as the algorithm that updates the equalizer's coefficients according to:

$$\begin{array}{l}
\text{if} \quad sign\{e_R[n]\} \neq sign\{Re[e_D[n]]\} \quad \text{or} \quad sign\{e_I[n]\} \neq sign\{Im[e_D[n]]\} \quad then \\
\quad Re[\underline{c}[n+1]] = Re[\underline{c}[n]] - \mu_{DM-MCMA_1}\left[e_R[n]Re\left[\underline{y}[n]\right] + e_I[n]Im\left[\underline{y}[n]\right]\right] \\
\quad Im[\underline{c}[n+1]] = Im[\underline{c}[n]] + \mu_{DM-MCMA_1}\left[e_R[n]Im\left[\underline{y}[n]\right] - e_I[n]Re\left[\underline{y}[n]\right]\right] \\
\text{if} \quad sign\{e_R[n]\} = sign\{Re[e_D[n]]\} \quad \text{or} \quad sign\{e_I[n]\} = sign\{Im[e_D[n]]\} \quad then \\
\qquad \underline{c}[n+1] = \underline{c}[n] - \mu_{DM-MCMA_2}\left[z[n]\left(|z[n]|^2 - |d[n]|^2\right)\right]\underline{y}^*[n]
\end{array} \tag{7}$$

where $\mu_{DM-MCMA_1}$ and $\mu_{DM-MCMA_2}$ are the step-size parameters. The expression for $e_D[n]$ is given in (5) and those for $e_R[n]$ and $e_I[n]$ are given in (6). Please note that based on (7), when $sign\{e_R[n]\} \neq sign\{Re[e_D[n]]\}$ and $sign\{e_I[n]\} \neq sign\{Im[e_D[n]]\}$, the MCMA-MDDMA algorithm is used to remove the ISI which means that for that case, the probability of correct decisions from the decision device is not very high. Otherwise, the equalizer's coefficients are updated based on the output of the decision device. According to [1], [3], [4], [6], the equalizer's coefficients for the MMA algorithm are updated by:

$$\underline{c}[n+1] = \underline{c}[n] - \mu_{MMA}\left[Re[z[n]]\left[|Re[z[n]]|^2 - \frac{E\left[|x_1[n]|^4\right]}{E\left[|x_1[n]|^2\right]}\right] + j[Im[z[n]]]\left[|Im[z[n]]|^2 - \frac{E\left[|x_2[n]|^4\right]}{E\left[|x_2[n]|^2\right]}\right]\right]\underline{y}^*[n] \quad (8)$$

where μ_{MMA} is the step size parameter. As was already mentioned earlier in this paper, the MMA algorithm eliminates the need for a phase rotator due to the fact that it uses a separate error-calculation, that is, for real and imaginary part of the received signal, individually, which is not the case in the CMA algorithm [2].

3. The New Proposed Algorithm

In this section we propose a dual-mode, decision directed multimodulus algorithm (DM-DD-MMA) for blind adaptive equalization, where the MMA algorithm [3], [4] is switched to the DD algorithm [9], based on a previously obtained expression for the step-size parameter [15] valid at the convergence state of a blind adaptive equalizer where the error involved in the update mechanism of the equalizer is a polynomial function of order three. According to [15], the Gaussian model holds for the convolutional noise pdf at the steady state operation if the step-size parameter complies to:

$$\mu \ll \frac{2|a_1+3a_3n_2|}{3\left(\sigma_x^2 N \sum_{k=0}^{\tilde{R}-1}|h_k[n]|^2\right)\left|a_1^2+12a_3a_1n_2+15a_3^2n_4\right|} \quad (9)$$

where $n_a = E[x_1^a[n]]$ ($a = 2,4$ and $E[(\cdot)]$ is the expectation operator) and the values for a_1 and a_3 are given by $a_1 = -\frac{E\left[|x_1[n]|^4\right]}{E\left[|x_1[n]|^2\right]}$ and $a_3 = 1$ respectively for the MMA algorithm [3], [4]. Now, for the noiseless case we have that the equalized output ($z[n]$) tends to the input signal ($x[n]$) when $p[n] \to 0$. Thus, if we consider the following expression:

$$\underline{nor}_{err} = \frac{\underline{err}}{\max(\underline{err})}; \quad err[n] = (\mu_p[n] - \mu_x)^2$$
$$\mu_p[n] = \frac{2|a_1+3a_3\tilde{n}_2|}{3\left(\sigma_x^2 N \sum_{k=0}^{\tilde{R}-1}|h_k[n]|^2\right)\left|a_1^2+12a_3a_1\tilde{n}_2+15a_3^2\tilde{n}_4\right|}; \quad \mu_x = \frac{2|a_1+3a_3n_2|}{3\left(\sigma_x^2 N \sum_{k=0}^{\tilde{R}-1}|h_k[n]|^2\right)\left|a_1^2+12a_3a_1n_2+15a_3^2n_4\right|} \quad (10)$$

where $\tilde{n}_a = \frac{1}{L}\sum_{b=0}^{b=L-1} z_1^a[n-b]$, $\underline{err}$ is a vector (an error vector) containing the elements defined by $err[n]$, $\max(\underline{err})$ contains the maximal value of $\underline{err}$ and $\underline{nor}_{err}$ is the normalized error vector. Please note that for the noiseless case, when $p[n] \to 0$ we have that $\mu_p[n] \to \mu_x$ which leads to $err[n] \to 0$. Thus, based on (10) we have some indication on how far away we are from the convergence state of the equalizer. Now, based on (10), the equalizer's taps of our new algorithm, denoted as the DM-DD-MMA method are updated according to:

$$
\begin{aligned}
&\text{if} \quad nor_{err}[n] \quad < \quad 0.01 \quad then \\
&\underline{c}[n+1] = \underline{c}[n] - \mu_2 \left[z[n] \left(|z[n]|^2 - |d[n]|^2 \right) \right] \underline{y}^*[n] \\
&\text{if} \quad nor_{err}[n] \quad \geq \quad 0.01 \quad then \\
&\underline{c}[n+1] = \underline{c}[n] - \mu_1 \left[Re[z[n]] \left[|Re[z[n]]|^2 - \frac{E\left[|x_1[n]|^4\right]}{E\left[|x_1[n]|^2\right]} \right] + j\left[Im[z[n]] \right] \left[|Im[z[n]]|^2 - \frac{E\left[|x_2[n]|^4\right]}{E\left[|x_2[n]|^2\right]} \right] \right] \underline{y}^*[n]
\end{aligned}
\tag{11}
$$

and μ_1 and μ_2 are the step size parameters. Please note that according to (11), the MMA algorithm [3], [4] is used to remove the ISI when $nor_{err}[n] \geq 0.01$ which means that for that case, the probability of correct decisions from the decision device is not very high. Otherwise, the equalizer's coefficients are updated based on the output of the decision device.

4. Simulation Results

In this section we compare our new proposed equalization method (11) for the 16QAM input case with three different equalization methods (with the MCMA-MDDMA (6), DM-MCMA (7) and DM-CMA (5) methods) for three different channel cases and different values for the SNR (30 dB down to 10 dB). In this work we considered three channels:

Channel1 [17]: $h_n = \left\{0 \quad \text{for} \quad n<0; -0.4 \text{ for} \quad n=0; \quad 0.8(0.4^{n-1}) \quad \text{for} \quad n>0\right\}$.

Channel2 [18]: $h_n = (0.4851, -0.72765, -0.4851)$.

Channel3 based on [19] where we down decimated the channel parameters by 32 and normalized them so that $h^T h = 1$: $h_n = (0.6069, -0.2023, -0.6069, -0.2529, -0.1517, 0.0506, 0.1011, 0.1517, 0.2023, 0.1517, 0.1517, 0.1011, 0.0506)$

In this work, we use the tap-centering initialization strategy where the equalizers' taps were initialized by setting the center tap equal to one and all others to zero. For Channel1, Channel2 and Channel3 we used an equalizer with 13, 15 and 57 taps, respectively. The step-size parameters μ_1, μ_2, μ_{DM-CMA_1}, μ_{DM-CMA_2}, $\mu_{DM-MCMA_1}$, $\mu_{DM-MCMA_2}$, $\mu_{MCMA-MDDMA}$, were chosen for fast convergence with low steady state residual ISI (please refer to Figures 2-9). Figures 2-4 describe the ISI as a function of the iteration number obtained by our new proposed method (DM-DD-MMA) compared to the DM-CMA, DM-MCMA and MCMA-MDDMA methods for the 16QAM constellation input sent via Channel1 and with SNR values of 30dB, 20dB and 10dB. Please note that the iteration number describes the number of updating the equalizer's coefficients during the equalization process. According to Figures 2-3, ($SNR = 30dB$ and $SNR = 20dB$ cases), our new proposed method (DM-DD-MMA) achieves the fastest convergence speed while leaving the system with the lowest residual ISI that is approximately obtained also by the DM-MCMA method which has almost the slowest convergence compared to all the other methods. According to Figure 4, (the $SNR = 10dB$ case), our new proposed method (DM-DD-MMA) has similar equalization performance with the MCMA-MDDMA method and both algorithms lead to a faster convergence speed and to a lower residual ISI compared to the DM-CMA and DM-MCMA methods. Figures 5-7 describe the ISI as a function of the iteration number obtained by our new derived method (DM-DD-MMA) compared to the DM-CMA, DM-MCMA and MCMA-MDDMA methods for the 16QAM constellation input sent via Channel2 and with SNR values of 30dB, 20dB and 10dB. As was the case with Channel1, for SNR values down to 20dB, our new proposed method (DM-DD-MMA) achieves the fastest convergence speed while leaving the system with the lowest residual ISI that is approximately obtained also by the DM-MCMA method which has almost the slowest convergence speed compared to all the other methods (please refer to Figures 5-6). For the $SNR = 10dB$ case (Figure 7), our new derived method (DM-DD-MMA) has similar equalization performance with the MCMA-MDDMA method and both algorithms lead to a faster convergence speed and to a lower residual ISI compared to the DM-CMA and DM-MCMA methods. Figures 8-9 describe the ISI as a function of the iteration number obtained by our new derived method (DM-DD-MMA) compared to the DM-CMA, DM-MCMA and MCMA-MDDMA methods for the 16QAM constellation input sent via Channel3 and with SNR values of 30dB and 20dB. As was already seen with Channel1 and Channel2 for the $SNR = 30dB$ case, our new derived method (DM-DD-MMA) achieves also for Channel3 the fastest equalization convergence speed while leaving the system with the lowest residual ISI that is approximately achieved also by the DM-MCMA algorithm which has almost the slowest convergence speed compared to all the other methods (please refer to Figure 8). For the $SNR = 20dB$ case (Figure 9), our new derived method (DM-DD-MMA) has similar equalization performance with the MCMA-MDDMA algorithm and both algorithms lead to a faster convergence speed and to a lower residual ISI compared to the DM-CMA and DM-MCMA methods. Figures 2-9 clearly show the effectiveness of the switching mechanism involved in our new proposed algorithm (DM-DD-MMA) that is responsible to switch to the update mechanism of the equalizer's taps based on the decisions made by the decision device, when relative reliable decisions can be done by the decision device. As already was noted earlier in this paper, $\mu_p[n]$ tends to μ_x (for the noiseless case) when the convolutional noise (namely the residual ISI) tends to zero. Thus, for the noiseless case, where $\mu_p[n]$ is very close to μ_x, reliable decisions can be obtained by the decision device. In this paper, we switched to the decision directed algorithm when $nor_{err}[n] < 0.01$ (where $\mu_p[n]$ is not equal to μ_x but is also not too far away from it) in order to get improved equalization performance improvement from the residual ISI as well as from the convergence rate point of view. Since μ_x was obtained for the noiseless case, it is not a surprise that our proposed algorithm does not show the same equalization performance improvement for the very low SNR case as it does for the high SNR situation. It is possible that better equalization performance can be obtained for the very low SNR case if we set $nor_{err}[n] < t_r$ where $t_r \neq 0.01$ or if a new expression for μ_x is obtained that is applicable also for the noisy case.

5. Conclusions

We proposed in this work a dual-mode, decision directed multimodulus algorithm (DM-DD-MMA) for blind adaptive equalization, where the MMA algorithm is switched to the DD algorithm, based on a previously obtained expression for the step-size parameter valid at the convergence state of the equalizer for which the convolutional noise pdf can be considered approximately as Gaussian. The new proposed algorithm (DM-DD-MMA) was compared with the DM-CMA,DM-MCMA and MCMA-MDDMA methods with three different channel cases and with SNR values down to 10dB. Simulation results

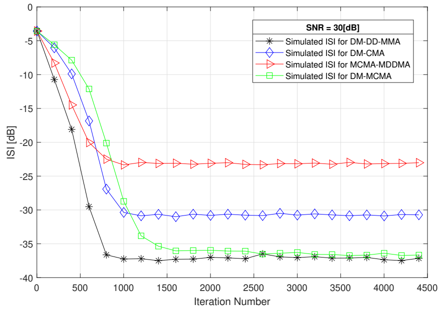

Figure 2. Simulation results (averaged results with 100 trials) with Channel1 for the DM-D-D-MMA, DM-CMA, DM-MCMA and MCMA-MDDMA methods. $SNR = 30dB$, $\mu_1 = 0.0002$, $\mu_2 = 0.0001$, $\mu_{DM-CMA_1} = 0.000025$, $\mu_{DM-CMA_2} = 0.0002$, $\mu_{DM-MCMA_1} = 0.000005$, $\mu_{DM-MCMA_2} = 0.0001$, $\mu_{MCMA-MDDMA} = 0.00005$

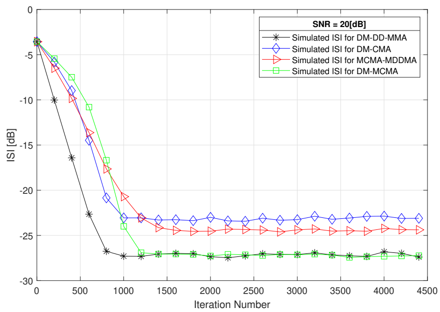

Figure 3. Simulation results (averaged results with 100 trials) with Channel1 for the DM-D-D-MMA, DM-CMA, DM-MCMA and MCMA-MDDMA methods. $SNR = 20dB$, $\mu_1 = 0.0002$, $\mu_2 = 0.0001$, $\mu_{DM-CMA_1} = 0.000025$, $\mu_{DM-CMA_2} = 0.0002$, $\mu_{DM-MCMA_1} = 0.000002$, $\mu_{DM-MCMA_2} = 0.0001$, $\mu_{MCMA-MDDMA} = 0.00003$

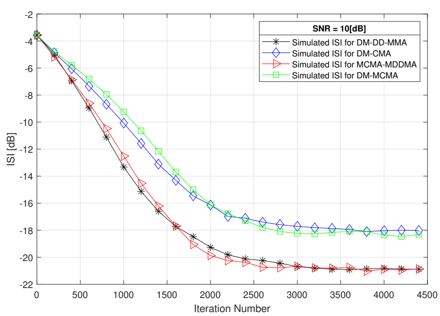

Figure 4. Simulation results (averaged results with 100 trials) with Channel1 for the DM-D-D-MMA, DM-CMA, DM-MCMA and MCMA-MDDMA methods. $SNR = 10dB$, $\mu_1 = 0.00004$, $\mu_2 = 0.00002$, $\mu_{DM-CMA_1} = 0.00005$, $\mu_{DM-CMA_2} = 0.00005$, $\mu_{DM-MCMA_1} = 0.00002$, $\mu_{DM-MCMA_2} = 0.00005$, $\mu_{MCMA-MDDMA} = 0.00002$

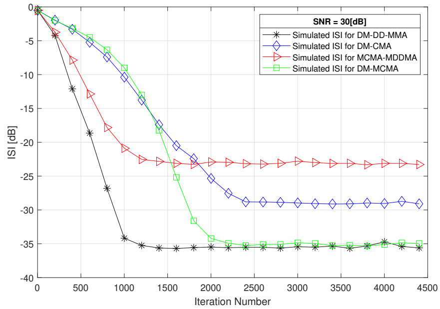

Figure 5. Simulation results (averaged results with 100 trials) with Channel2 for the DM-D-D-MMA, DM-CMA, DM-MCMA and MCMA-MDDMA methods. $SNR = 30dB$, $\mu_1 = 0.0002$, $\mu_2 = 0.0001$, $\mu_{DM-CMA_1} = 0.000027$, $\mu_{DM-CMA_2} = 0.0002$, $\mu_{DM-MCMA_1} = 0.000005$, $\mu_{DM-MCMA_2} = 0.0001$, $\mu_{MCMA-MDDMA} = 0.00004$

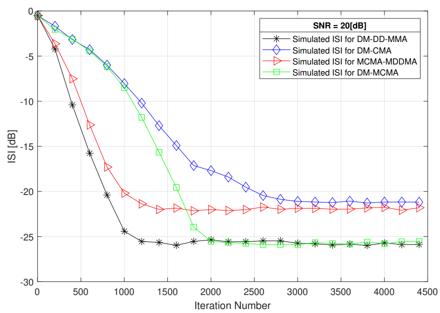

Figure 6. Simulation results (averaged results with 100 trials) with Channel2 for the DM-DD-MMA, DM-CMA, DM-MCMA and MCMA-MDDMA methods. $SNR = 20dB$, $\mu_1 = 0.0002$, $\mu_2 = 0.0001$, $\mu_{DM-CMA_1} = 0.000027$, $\mu_{DM-CMA_2} = 0.0002$, $\mu_{DM-MCMA_1} = 0.0000027$, $\mu_{DM-MCMA_2} = 0.0001$, $\mu_{MCMA-MDDMA} = 0.00004$

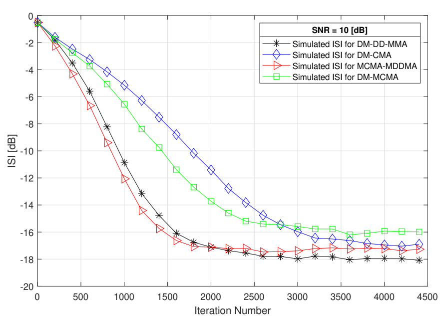

Figure 7. Simulation results (averaged results with 100 trials) with Channel2 for the DM-D-D-MMA, DM-CMA, DM-MCMA and MCMA-MDDMA methods. $SNR = 10dB$, $\mu_1 = 0.00005$, $\mu_2 = 0.00001$, $\mu_{DM-CMA_1} = 0.00005$, $\mu_{DM-CMA_2} = 0.00004$, $\mu_{DM-MCMA_1} = 0.00005$, $\mu_{DM-MCMA_2} = 0.00005$, $\mu_{MCMA-MDDMA} = 0.00003$

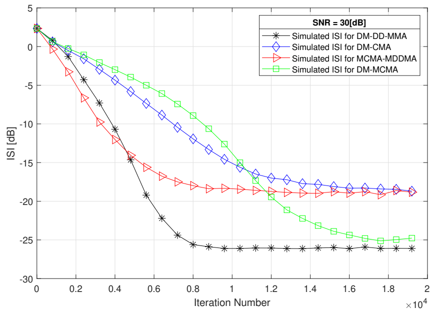

Figure 8. Simulation results (averaged results with 100 trials) with Channel3 for the DM-DD-MMA, DM-CMA, DM-MCMA and MCMA-MDDMA methods. $SNR = 30dB$, $\mu_1 = 0.00005$, $\mu_2 = 0.00005$, $\mu_{DM-CMA_1} = 0.00003$, $\mu_{DM-CMA_2} = 0.00003$, $\mu_{DM-MCMA_1} = 0.000007$, $\mu_{DM-MCMA_2} = 0.00003$, $\mu_{MCMA-MDDMA} = 0.00002$

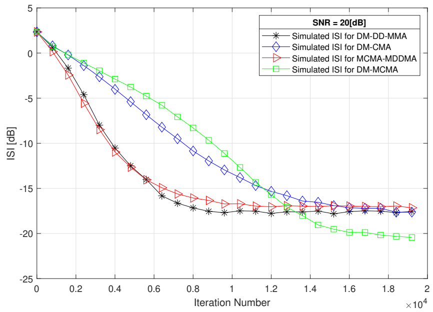

Figure 9. Simulation results (averaged results with 100 trials) with Channel3 for the DM-DD-MMA, DM-CMA, DM-MCMA and MCMA-MDDMA methods. $SNR = 20dB$, $\mu_1 = 0.00005$, $\mu_2 = 0.00005$, $\mu_{DM-CMA_1} = 0.00003$, $\mu_{DM-CMA_2} = 0.00003$, $\mu_{DM-MCMA_1} = 0.000007$, $\mu_{DM-MCMA_2} = 0.00003$, $\mu_{MCMA-MDDMA} = 0.00002$

has confirmed that for the three channel cases with $SNR = 30dB$ as well as for the first two channel cases (Channel1 and Channel2) with $SNR = 20dB$, a faster convergence speed is obtained with our new derived algorithm (DM-DD-MMA) compared with the DM-CMA,DM-MCMA and MCMA-MDDMA methods while leaving the system with the lowest residual ISI that is approximately obtained also by the DM-MCMA algorithm which has almost the slowest convergence speed compared to all the other methods. For the $SNR = 10dB$ case, simulation results have shown that for the first two channel cases (Channel1 and Channel2), our new proposed method (DM-DD-MMA) has similar equalization performance with the MCMA-MDDMA algorithm and both algorithms lead to a faster convergence speed and to a lower residual ISI compared to the DM-CMA and DM-MCMA algorithms. As already mentioned earlier, the new proposed algorithm (DM-DD-MMA) is based on a previously derived expression for the step-size parameter valid at the convergence state of the equalizer. This expression for the step-size was obtained for the noiseless case. Thus, this may be the reason that our new proposed method (DM-DD-MMA) achieved the best equalization performance for SNR values above 20dB compared to the DM-CMA,DM-MCMA and MCMA-MDDMA methods. Therefore, our future direction will be deriving a new expression for the step-size parameter valid at the convergence state of the equalizer that is suitable also for the very noisy case.

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