Modern Management based on Big Data II and Machine Learning and Intelligent Systems III
A.J. Tallón-Ballesteros (Ed.)

doi:10.3233/FAIA210235

The SAPEVO-M-NC Method

Sérgio Mitihiro do Nascimento MAÊDA[a,d,1], Marcio Pereira BASÍLIO[b], Igor Pinheiro de Araújo COSTA[a,d], Miguel Ângelo Lellis MOREIRA[a,d], Marcos dos SANTOS[c,d], and Carlos Francisco Simões GOMES[d]
[a] *Federal Fluminense University, Brazil*
[b] *Military Police of the State of Rio de Janeiro, Brazil*
[c] *Military Institute of Engineering, Brazil*
[d] *Naval Systems Analysis Center, Brazil*

Abstract. This article aims to present with more details, the multicriteria decision aid SAPEVO-M-NC (Simple Aggregation of Preferences Expressed by Ordinal Vectors - Non-Compensatory - Multi Decision Makers). It is a new version of the SAPEVO-M method, of an ordinal, non-compensatory nature and with the possibility of acting by multiple decision makers. As a result, the method provides information on the partial weights, indicating the relative importance of the criteria for each of the decision makers, the relative dominance values and two evaluations on the performance of the alternatives: a partial one, which considers the absolute dominance indices, being used to assess existing dominance relationships; and a global one, which provides the performance rates of the alternatives, making it possible to order them as well as to carry out a sensitivity analysis on the observed performances, reflecting in greater transparency in the decision-making process.

Keywords. Ordinal ranking methods; Multicriteria decision aid; Non-compensatory.

1. Introduction

Fundamentally, we are all decision makers. Everything we do, consciously or not, is the result of a decision-making process, involving various information that must be evaluated to provide greater transparency to the process and understanding of the system by the decision maker(s) [1][2][3].

The ordinal methods were the first methods to aid decision-making, developed after the mid-eighteenth century, by studies by Jean-Charles de Borda [4]. These methods offer advantages due to their relative ease of understanding and administration, as well as greater reliability in terms of reducing inconsistencies generated in the preference elicitation process[5]. Due to the need to require less cognitive effort from the decision maker, it is expected that they reflect their preferences more accurately [4][6][7]. This paper presents the method called SAPEVO-M-NC [8]. The method allows to aggregate, through an ordinal process, the preferences of decision makers regarding the importance of the criteria and the performance of the alternatives.

[1] Corresponding Author, Sérgio Mitihiro do Nascimento Maêda, Federal Fluminense University - UFF, 24210-240, Niterói, Brazil; E-mail: sergiomnmaeda@gmail.com.
This research was financially supported by Military Police of the State of Rio de Janeiro.

Although robust, it uses a relatively simple axiomatic base, resulting in low cognitive effort on the part of evaluators.

2. The SAPEVO-M-NC Method

Derived from the SAPEVO-M method [4], the SAPEVO-M-NC method [8], consists of an ordinal method, non-compensatory nature, which aims at the problem of ordering (Pγ), and with the possibility of acting by multiple decision makers. In the method, the evaluation of the performance of the alternatives is carried out directly, with no need to carry out parity comparisons between the alternatives to obtain the modeling of preferences, resulting in a substantial reduction in the cognitive effort on the part of the Decision Makers DMs. The method allows for two assessments, one partial and one global, resulting in a more sensitive analysis of the performance of alternatives, as well as greater transparency about the decision-making process [8].

Unlike Classic Decision Theory, which basically considers two supposedly transitive preference relations, designated by Indifference (I) and by Strict Preference (P), this method is based on the Fundamental System of Preference Relations (FSPR), also incorporating the weak preference relationship (Q). According to [9], there are several reasons why researchers seek to avoid the type of modeling that is based on the axiom of complete comparability and transitivity between alternatives, among which, can be mentioned the fact that the decision maker does not have all the information that allows him to choose one of the alternatives and by forcing a relationship of strict preference, or indifference, could lead to arbitrary and inconsistent errors.

After the DMs establish the criteria and alternatives, the method can be divided into five steps:

Step 1 - Ordinal transformation of the preferences of each DM, in each criterion, which are added at the end of this step, giving rise to a vector (V_i), representing the weights of the criteria. Table 1 shows the relationship of relative importance between the criteria:

Table 1. Relative importance between the criteria

Scale 1	Verbal representation	Scale 2
$\delta(c_i c_j) \lll 1 \leftrightarrow c_i <<< c_j$	Absolutely less important	-3
$\delta(c_i c_j) \preceq 1 \leftrightarrow c_i << c_j$	Much less important	-2
$\delta(c_i c_j) \prec 1 \leftrightarrow c_i < c_j$	Less important	-1
$\delta(c_i c_j) = 1 \leftrightarrow c_i \approx c_j$	Equally important	0
$\delta(c_i c_j) \succ 1 \leftrightarrow c_i > c_j$	More important	1
$\delta(c_i c_j) \succeq 1 \leftrightarrow c_i >> c_j$	Much more important	2
$\delta(c_i c_j) \ggg 1 \leftrightarrow c_i >>> c_j$	Absolutely more important	3

Let DM_k be a set of "k" decision-makers, D = {DM_1, DM_2, ..., DM_k} that express their opinions on the relative importance of the criteria involved. These preferences give rise to the MDM_k preference matrix. The relationship between the two scales of the table allows the transformation of the matrix (1) into (2):

$MDM_k = [\delta(c_i c_j)]$, in a column vector $[V_i]$, where: (1)

$V_i = \sum_{j=1}^{m} \delta(cicj)$ (i = 1, ..., m, and k = 1, ..., n) (2)

After generating the vector V_i, its a_{ij} elements are normalized according to (3):

$$v = (a_{ij} - \min a_{ij}) / (\max a_{ij} - \min a_{ij}) \quad (3)$$

Giving rise to the DM_k preferences vector. If null values occur in this step, they are replaced by 1% of the second lowest value obtained. After all DM's carry out their evaluations, the normalized vectors are added, giving rise to the weight vector that expresses the importance of the criteria [4].

Step 2 - Ordinal classification (Θ_{ij}) of the performance of the alternatives:

In this step, each DM assigns the ratings related to the performance of the alternatives in each criterion (table 2), which are related to their rating ranges $g_{(ij)}$. After all "k" DMs perform their evaluations, the arithmetic mean $\mu_{(ij)}$ of the classification ranges of the performances of the alternatives in each criterion is obtained.

Table 2. Ordinal ratings of performance of alternatives

Ordinal classification (Θ_{ij}) of the performance of alternative i in criterion j	Classification range $g_{(ij)}$
Excellent (E)	1
Very Good (VG)	2
Good (G)	3
Medium (M)	4
Bad (B)	5
Very Bad (VB)	6
Poor (P)	7

Step 3 – Obtaining the fractions of the criteria weights ($\sigma_{j(ab)}$).

For each criterion "j", a parity comparison is made between the alternatives to verify the relative distance between the mean values of the classification ranges (4):

$$\Delta\mu_{i(ab)} = \mu_{(ia)} - \mu_{(ib)} \quad (4)$$

This value allows identifying in the preference modeling (figure 1 and table 3) the weight fraction of criterion "j", obtained by alternative "a" in relation to alternative "b" ($\sigma_{j(ab)}$).

Figure 1. Preference function of a criterion with linear variation.

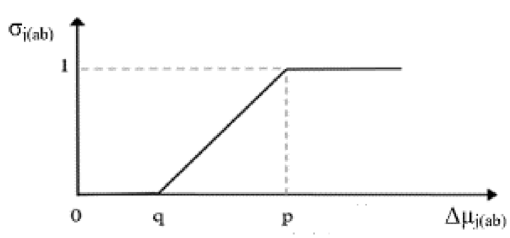


Table 3. Criteria preference modeling

Indifference (I)	$\mu_{(ia)} - \mu_{(ib)} \leq 1 : \sigma_{(ab)} \rightarrow$	0

Weak Preference (Q)	$1 < \mu_{(ia)} - \mu_{(ib)} \leq 3$: $\sigma_{(ab)} \rightarrow$	$\dfrac{(A_{ij} - \min A_{ij})}{(\text{máx } A_{ij} - \min A_{ij})}$
Strong Preference (P)	$3 < \mu_{(ia)} - \mu_{(ib)}$: $\sigma_{(ab)} \rightarrow$	1

Step 4 – Calculation of relative d_{ab} dominance.

Obtained by the weighted sum of the criteria weights (w_j), with the corresponding fraction ($\sigma_{j(ab)}$) verified in the preference modeling (5):

$$d_{ab} = \sum w_j \text{ x } \sigma_{j(ab)} \tag{5}$$

Step 5 - Conducting assessments

5.1 - Partial evaluation: Calculation of Absolute D_{ab} Dominance and Outranking Rate η_{ab}. The difference between the relative dominances d_{ab} - d_{ba} provides information on the absolute dominance D_{ab} between the alternatives (6), where positive values of "D_{ab}", indicate that alternative "a" dominated alternative "b".

$$D_{ab} = d_{ab} - d_{ba} \tag{6}$$

This information allows to identify the existing relationships between the alternatives, and the assembly of a dominance graph. Dividing D_{ab}, by the sum of the weights, the percentage rate of absolute dominance is obtained (7).

$$\eta_{ab} = D_{ab} / (\sum w_j) \tag{7}$$

This information allows the DM more clarity about the partial performances between the alternatives.

5.2 - Global assessment: The method makes it possible to carry out an analysis of the total performance of each of the “n” alternatives, evaluating the Performance Rates (T_a) obtained by each one (8):

$$T_a = \sum d_{ab} /(\sum w_j \text{ X } (n-1)) \tag{8}$$

This information allows greater transparency of the process to the DM, especially in situations where: D_{ab}= D_{ba}= 0. Finally, the results allow the ordering of alternatives.

2.1. Application of SAPEVO-M-NC (Numerical example)

To elucidate the steps presented, consider for example composed of seven alternatives (A_1, A_2, A_3, A_4, A_5, A_6, A_7), which are compared considering four criteria (C_1, C_2, C_3, C_4), by three DM`s = (DM_1, DM_2, DM_3). Table 4 shows as an example the ordinal evaluation of the criteria, by DM_1:

Table 4. Example of ordinal assessment of the importance of attributes by DM1

	C_1	C_2	C_3	C_4
C_1	Equally important	More important	Absolutely more important	Much more important
C_2	Less important	Equally important	Much more important	More important

C_3	Absolutely less important	Much less important	Equally important	Less important
C_4	Much less important	Less important	More important	Equally important

The table 5 presents the values of scale 2 related to the ordinal evaluations, together with the normalized weight vector.

Table 5. Criteria preference modeling

	C_1	C_2	C_3	C_4	V_i	Normalized vector
C_1	0	+1	+3	+2	6	1
C_2	-1	0	+2	+1	2	0,666667
C_3	-3	-2	0	-1	-6	0,003333
C_4	-2	-1	+1	0	-2	0,333333

The table 6 presents the evaluation of the importance of the criteria by all DM`s.

Table 6. Evaluation of the importance of the criteria by the DMs

	DM_1	DM_2	DM_3	Final weights
C_1	1	0,875	1	2,875
C_2	0,6667	1	0,5555	2,222
C_3	0,0033	0,00875	0,2222	0,234
C_4	0,3333	0,125	0,0022	0,461

After defining the weights of the criteria, the next step is to classify the performance of the alternatives by the DMs. The table 7 presents the performance ratings of the alternatives with the corresponding cardinal values:

Table 7. Evaluation of the importance of the criteria by the DMs

DM_1					DM_2					DM_3					Arithmetic mean $\mu_{(ij)}$				
	C_1	C_2	C_3	C_4		C_1	C_2	C_3	C_4		C_1	C_2	C_3	C_4		C_1	C_2	C_3	C_4
A_1	1	2	1	1	A_1	1	3	1	1	A_1	1	2	1	1	A_1	1	2,33	1	1
A_2	6	5	3	3	A_2	6	6	3	3	A_2	6	6	3	3	A_2	6	5,66	3	3
A_3	6	4	6	5	A_3	6	5	6	5	A_3	6	5	5	6	A_3	6	4,66	5,66	5,33
A_4	3	1	2	1	A_4	2	1	2	1	A_4	2	1	2	1	A_4	2,33	1	2	1
A_5	5	5	6	2	A_5	5	6	6	2	A_5	4	6	5	2	A_5	4,66	5,66	5,66	2
A_6	3	4	6	2	A_6	2	5	5	2	A_6	2	5	5	2	A_6	2,33	4,66	5,33	2
A_7	4	3	3	4	A_7	4	4	3	4	A_7	3	4	3	4	A_7	3,66	3,66	3	4

Table 8 presents, respectively, the difference between the performance evaluations of the alternatives $\Delta\mu_{j(ab)}$, the values obtained in the preference modeling $\sigma_{j(ab)}$, the calculation of the relative dominance dab, the absolute dominance D_{ab} and the outranking rate η_{ab}.

Table 8. Values of $\Delta\mu_{j(ab)}$, $\sigma_j(ab)$, d_{ab}, D_{ab} e η_{ab}

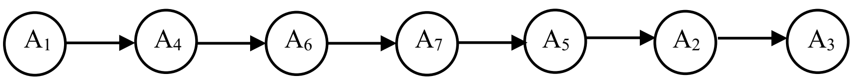
	$\Delta\mu_{j(ab)} = \mu_{j(a)} - \mu_{j(b)}$				$\sigma_{i(ab)}$				$d_{ab} = \sum w_j \times \sigma_{j(ab)}$					D_{ab}	η_{ab}
	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4	Total	Total	Total
A_1-A_2	5,00	3,33	2,00	2,00	1,00	1,00	0,50	0,50	2,88	2,22	0,12	0,23	5,44	5,44	0,94
A_1-A_3	5,00	2,33	4,67	4,33	1,00	0,67	1,00	1,00	2,88	1,48	0,23	0,46	5,05	5,05	0,87
A_1-A_4	1,33	-1,33	1,00	0,00	0,17		0,00	0,00	0,48	0,00	0,00	0,00	0,48	0,11	0,02
A_1-A_5	3,67	3,33	4,67	1,00	1,00	1,00	1,00	0,00	2,88	2,22	0,23	0,00	5,33	5,33	0,92
A_1-A_6	1,33	2,33	4,33	1,00	0,17	0,67	1,00	0,00	0,48	1,48	0,23	0,00	2,19	2,19	0,38
A_1-A_7	2,67	1,33	2,00	3,00	0,83	0,00	0,50	1,00	2,40	0,00	0,12	0,46	2,97	2,97	0,51

A_2-A_3	0,00	-1,00	2,67	2,33	0,00		0,83	0,67	0,00	0,00	0,20	0,31	0,50	0,50	0,09
A_2-A_4	-3,67	-4,67	-1,00	-2,00					0,00	0,00	0,00	0,00	0,00	-5,33	-0,92
A_2-A_5	-1,33	0,00	2,67	-1,00		0,00	0,83		0,00	0,00	0,20	0,00	0,20	-0,28	-0,05
A_2-A_6	-3,67	-1,00	2,33	-1,00			0,67		0,00	0,00	0,16	0,00	0,16	-2,72	-0,47
A_2-A_7	-2,33	-2,00	0,00	1,00			0,00	0,00	0,00	0,00	0,00	0,00	0,00	-3,03	-0,52
A_3-A_4	-3,67	-3,67	-3,67	-4,33					0,00	0,00	0,00	0,00	0,00	-5,79	-1,00
A_3-A_5	-1,33	1,00	0,00	-3,33		0,00	0,00		0,00	0,00	0,00	0,00	0,00	-0,46	-0,08
A_3-A_6	-3,67	0,00	-0,33	-3,33		0,00			0,00	0,00	0,00	0,00	0,00	-3,34	-0,58
A_3-A_7	-2,33	-1,00	-2,67	-1,33					0,00	0,00	0,00	0,00	0,00	-2,19	-0,38
A_4-A_5	2,33	4,67	3,67	1,00	0,67	1,00	1,00	0,00	1,92	2,22	0,23	0,00	4,37	4,37	0,75
A_4-A_6	0,00	3,67	3,33	1,00	0,00	1,00	1,00	0,00	0,00	2,22	0,23	0,00	2,46	2,46	0,42
A_4-A_7	1,33	2,67	1,00	3,00	0,17	0,83	0,00	1,00	0,48	1,85	0,00	0,46	2,79	2,79	0,48
A_5-A_6	-2,33	-1,00	-0,33	0,00				0,00	0,00	0,00	0,00	0,00	0,00	-1,92	-0,33
A_5-A_7	-1,00	-2,00	-2,67	2,00				0,50	0,00	0,00	0,00	0,23	0,23	-1,08	-0,19
A_6-A_7	1,33	-1,00	-2,33	2,00	0,17			0,50	0,48	0,00	0,00	0,23	0,71	0,55	0,10

The positive values for D_{ab}, indicates that the first alternative dominates the second one of the analyzed pair. Through the results obtained, we verified the following overcoming relationship: $A_1 > A_4 > A_6 > A_7 > A_5 > A_2 > A_3$.

And the graph with the dominance relationships can be constructed (figure 2):

Figure 2. Graph representing the dominance relationships between the alternatives.



It is observed that with the D_{ab} values, it was possible to establish all the dominance relationships and order the alternatives, however, for a deeper analysis, the performance rates (T_a) were obtained by each alternative.

A_1 = 0,6179; A_4 = 0,6074; A_6 = 0,2542; A_7 = 0,1921; A_5 = 0,0336; A_2 = 0,0245; A_3 = 0,0000

3. Analysis of results and Conclusion

Ordinal methods have a wide field of application and, due to their nature, are closer to the way people make their decisions in the face of processes that deal with qualitative variables, partial or incomplete information.

Through the numerical example presented, it was verified that the SAPEVO-M-NC method: allowed to aggregate, through an ordinal process, the preferences of the decision makers regarding the importance of the criteria and the performance of the alternatives; although robust, it uses a relatively simple axiomatic base; the fact of not exploring parity comparisons between alternatives to perform preference modeling results in less cognitive effort on the part of evaluators; the sensitivity analysis on the performance of the alternatives allows for a more in-depth assessment, resulting in greater transparency to the decision maker(s) about the decision-making process developed.

Due to the relative ease of application, associated with a low cognitive effort on the part of the evaluators, it is concluded that this methodology can provide great gains, not only for the academic community, but also for society as a whole, presenting itself as an alternative tool for multicriteria decision support, of an ordinal, non-compensatory nature and with the possibility of supporting multiple decision makers.

As a proposal for future work, this method can be better explored by being approached in case studies comparing the results with those obtained through other methods in the literature.

References

[1] Saaty TL. Decision making with the analytic hierarchy process. Int J Serv Sci [Internet]. 2008;1(1):83. Available at: http://www.inderscience.com/link.php?id=17590

[2] Moreira MÂL, Costa IP de A, Pereira MT, dos Santos M, Gomes CFS, Muradas FM. PROMETHEE-SAPEVO-M1 a Hybrid Approach Based on Ordinal and Cardinal Inputs: Multi-Criteria Evaluation of Helicopters to Support Brazilian Navy Operations. Algorithms [Internet]. 2021;14(5):140. Available at: https://www.mdpi.com/1999-4893/14/5/140, DOI: 10.3390/a14050140

[3] Santos M dos, Costa IP de A, Gomes CFS. Multicriteria Decision-Making In The Selection Of Warships: A New Approach To The Ahp Method. Int J Anal Hierarchy Process. 2021;13(1):147–6, DOI: 10.13033/ijahp.v13i1.833

[4] Gomes CFS, Santos M dos, Teixeira LFH de S de B, Sanseverino AM, Barcelos MR dos S. SAPEVO-M: A group multicriteria ordinal ranking method. Pesqui Operacional [Internet]. 2020;40, DOI: 10.1590/0101-7438.2020.040.00226524

[5] Moreira MÂL, Gomes CFS, dos Santos M, do Carmo Silva M, Araujo JVGA. PROMETHEE-SAPEVO-M1 a Hybrid Modeling Proposal: Multicriteria Evaluation of Drones for Use in Naval Warfare. In: International Joint conference on Industrial Engineering and Operations Management [Internet]. Military Institute of Engineering (IME), Urca, RJ 22290-270, Brazil: Springer; 2020. p. 381–93. Available at: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85097128127, DOI: 10.1007/978-3-030-56920-4_31

[6] Rodrigues MVG, Duarte TE, dos Santos M, Gomes CFS. Prospective scenarios analysis: Impact on demand for oil and its derivatives after the COVID-19 pandemic. Brazilian J Oper Prod Manag. 2021;18(2), DOI: 10.14488/BJOPM.2021.039.

[7] Costa IP de A, Sanseverino AM, Barcelos MR dos S, Belderrain MCN, Gomes CFS, dos Santos M. Choosing flying hospitals in the fight against the COVID-19 pandemic: structuring and modeling a complex problem using the VFT and ELECTRE-MOr methods. IEEE Lat Am Trans. 2021;19(6):1099–106, DOI: 10.1109/TLA.2021.9451257

[8] Maêda SM do N, Costa IP de A, Santos M dos., Gomes CFS. Economic and edaphoclimatic evaluation of Brazilian regions for African mahogany planting - an approach using the SAPEVO-M-NC ordinal method. The International Conference on Information Technology and Quantitative Management (ITQM), organizador. Chengdu: The International Conference on Information Technology and Quantitative Management (ITQM), Elsevier - Procedia Computer ScienceProcedia Computer Science; 2021.

[9] GOMES LFAM, GOMES CFS. Princípios e métodos para a tomada de decisão: Enfoque multicritério. 6º. São Paulo: Atlas; 2019.