Origin and Evolution of Conceptual Differences Between Two Measurement Theories

Shi Huisheng^a, Ye Xiaoming^{b1}, Xing Cheng^b, Ding Shijun^b

^a*AVIC Changcheng Institute of Metrology & Measurement, Beijing, 100095, China*

^b*School of Geodesy and Geomatics, Wuhan University, Wuhan, Hubei, 430079, China*

Abstract. In several published literatures, some new measurement concepts have been proposed to reinterpret measurement theory, which leads to a new interpretation method of measurement theory. By directly comparing the measurement concept differences between the new theory and the traditional theory, this paper reveals that the root of these differences is due to the different understanding of the mathematical concept of random variable, and clarifies the evolution process of other conceptual differences. Also, by reviewing the mathematical concept, it points out that the traditional theory has been gone astray, and the concept of error classification is actually the product of going astray, while the new concept theory should get active attention and research.

Keywords. Measurement; measurement error; random variable; variance; uncertainty

1. Introduction

Traditional measurement theory interprets its measurement concepts according to the logic of error classification, which is showed in various measurement specifications [1][2][3][4][5] and textbooks[6][7][8]. However, in several published literatures, some new measurement concepts have been proposed to reinterpret the measurement theory, which leads to a new interpretation method of measurement theory in which error has no classification[9][10][11][12][13][14]. Reference [9] is the first to criticize the error classification philosophy in a negative way. Reference [10] proposes that there are no systematic and random categories of errors, any error has its variance which is used to evaluate its probability interval, the change of measurement conditions is the source of the dispersion of repeated measured values, and so on. Reference [11] further pointed out that the regularity and randomness of errors are due to different perspectives of observing errors, and both functional model and random model can be used to handle errors. Reference [12] makes a comparative discussion on the explanation of the concept of variance, which proves that the concept of dispersion of measured value in traditional theory is incorrect. References [13][14] provide a relatively complete

¹ Corresponding Author: Ye Xiaoming, School of Geodesy and Geomatics, Wuhan University, Wuhan, Hubei, 430079, China; Email: xmye@sgg.whu.edu.cn

interpretation of measurement theory mathematically based on these new concepts.

However, because the traditional measurement concepts have been deeply rooted in people's minds and many measurement scientists are also difficult to return to the probability theory and rethink the measurement concepts, few people believe that there are conceptual troubles in traditional theories, so that people usually despise these new concepts. Therefore, for the two measurement theories with opposite concepts, it is necessary to dig out the core difference between the two theories and clarify the evolution of the conceptual differences between them, so as to demonstrate which theory is correct.

From the mathematical concept, this paper will conduct a direct comparative study on the source of the basic concepts of the two theories, and show the complete evolution process of the conceptual differences between them, so that readers can have a complete understanding of the formation process of the concept logic of measurement theory. Besides, by reviewing the mathematical concepts, the paper will make readers understand that the traditional theory has been gone astray in the mathematical concept, and the concept of error classification is actually the product of going astray, while the new concept theory should be a new topic that needs active attention and research.

It should be noted that the traditional concepts discussed in this article refer to the mainstream concepts recognized by various measurement standards [1][2][3][4][5]. Although there are actually many literatures that have put forward some concepts beyond the mainstream, because they are not universally recognized and are also different from the new concept theory, this article will not discuss them.

2. Difference in Understanding of the Concepts of Constant and Random Variable

The conceptual differences between the two theories originate from different understandings of mathematical concepts. That is, they have a completely different understanding of the concept of constant and random variable. See Table 1.

Table 1 Different understanding of mathematical concepts in two theories

	Traditional theory	New concept theory
Constant	True value and systematic error	Measured value
Random variable	Random error and measured value	Error and true value

The traditional measurement theory uses Fig 1 to interpret its measurement concepts.

Because people notice that measured values are in a state of random change in repeated measurement, the measured value and random error are considered as random variables, and the variance is assigned to the measured value and random error. Besides, it is noted that the systematic error and the true value are constant in repeated measurement, so the systematic error and the true value are considered as constants and are considered to have no variance (or the variance is zero).

Fig 1. Conceptual sketch of traditional theory

Fig 2. Conceptual sketch of new concept theory

However, this understanding is inconsistent with the concept of probability theory. Therefore, the traditional measurement theory has two troubles in mathematical concepts.

At first, the measured value or observed value in any measurement is expressed by a numerical value, so classifying the measured value or observed value as a random variable actually violates a basic mathematical concept that the variance of a constant is 0.

For example, China National Bureau of Surveying and Mapping announced that the height of Everest is $x = 8844.43m$, and its precision was $u(x) = \pm 0.21m$. However, because equation $x = 8844.43$ clearly indicates that x is a constant 8844.43 instead of a random variable, when we substitute equation $x = 8844.43$ into equation $u(x) = \pm 0.21$, we will find that the equation $u(x) = \pm 0.21$ actually expresses a wrong equation $u(8844.43) = \pm 0.21$. Even though repeated measurements may obtain many other measured values $x_1, x_2, \dots$, and there can be $x \neq x_1 \neq x_2 \neq \dots$, the equations $x = 8844.43m$ and $u(x) = u(8844.43) = \pm 0.21$ still exist and cannot be explained.

Another example, in Section 5.17 of reference [2] and Section 4.2 of reference [3], a method process is described as follows:

When there are n observed values x_k , the final measured value is

$$\bar{x} = \frac{1}{n} \sum_{k=1}^n x_k \quad (2-1)$$

The variance of each measured value x_k is

$$u^2(x_k) = \frac{1}{n-1} \sum_{j=1}^n (x_j - \bar{x})^2 \quad (2-2)$$

The variance of the final measured value is

$$u^2(\bar{x}) = \frac{u^2(x_k)}{n} \quad (2-3)$$

Similarly, although formula (2-1) has clearly defined that x_k and $\bar{x}$ are numerical values and are constants, formulas (2-2) and (2-3) still assigns them a variance that is not zero.

In probability theory, a constant is a numerical value, such as 20, 300, $x=8844.43$, and so on. Unlike constants, the random variable has no a certain value, but all its possible values (sample space) exist dispersedly within an interval on the number axis. In order to describe and study random variables, people define two parameters: mathematical expectation and variance.

For a random variable $L \in \{L_i\}$, its mathematical expectation is the average of all its possible values $\{L_i\}$. That is:

$$E(L) = \sum_{i=1}^n P_i L_i \text{ or } E(L) = \int_{-\infty}^{+\infty} L P(L) dL \quad (2-4)$$

And its variance is the dispersion of all its possible values $\{L_i\}$. That is:

$$u^2(L) = E[L - E(L)]^2 \quad (2-5)$$

The meaning of definitions (2-4) and (2-5) is that although the value of random variable L is unknown, it exists within a probability interval with $E(L)$ as the center and $u^2(L)$ as the width evaluation. In other words, mathematical expectation $E(L)$ and variance $u^2(L)$ are the evaluation values of the probability interval of the random variable L .

Because all possible values of a constant C (a numerical value) are itself, there are $E(C) = C$ and $u^2(C) = 0$. Obviously, it is incorrect that traditional measurement theory assigns a non-zero variance to a numerical value, and its root cause is that the relationship $L \in \{L_i\}$ between the random variable L and the sample L_i is ignored, so that $u^2(L) = E[L - E(L)]^2$ is written as $u^2(L_i) = E[L - E(L)]^2$.

Secondly, according to probability theory, as the numerical description of a random variable, mathematical expectation and variance are two indispensable parameters, and a constant can also be expressed by them. However, the traditional measurement theory cannot give the mathematical expectations of the measured value, systematic error and true value (Shown in Table 2 [13]), which actually means that the probability evaluation of error or true value is not completely described.

Table 2. The conceptual troubles in the traditional theory

	Measured value x	Random error $x - E(x)$	Systematic error $E(x) - x_T$	True value x_T
Mathematical expectation	Absent	0	Absent	Absent
Variance	$u^2(x)$	$u^2(x)$	0	0

Different from the traditional theory, the new concept theory uses Fig. 2 to interpret its measurement concepts.

In this conceptual interpretation, the constant is a known numerical value which represents a unique point on the number axis, but the random variable is an unknown value which exists within an interval on the number axis. Also, the mathematical expectation and variance are the average and dispersion of all possible values of a random variable respectively, express the probability interval in which the value of the

random variable exists, and are the parameters that express the uncertainty of the random variable. In this way, as a numerical value, the measured value x_0 is a constant without dispersion, the random variables are error and true value (Shown in Table 3 [12] [13]). Also, the mathematical expectation and variance of error are the average and dispersion of all possible values of error respectively.

Table 3. The conceptual interpretations in new concept theory

	Measured value x_0	Error Δ	True value x_T
Mathematical expectation	x_0	0	x_0
Variance	0	$u^2(\Delta)$	$u^2(\Delta)$

In this way, the expressions of the above Everest elevation case are $x_0 = 8844.43m$ and $u(\Delta) = \pm 0.21m$, and the formulas (2-2) and (2-3) are proved to be

$$u^2(\Delta x_k) = \frac{1}{n-1} \sum_{j=1}^n (x_j - \bar{x})^2 \quad \text{and} \quad u^2(\Delta \bar{x}) = \frac{u^2(\Delta x_k)}{n}, \quad \text{respectively (where}$$

$$\Delta x_k = x_k - E(x), \Delta \bar{x} = \bar{x} - E(x)).$$

The keys of this interpretation are that the current measured value x_0 and all possible measured values $\{x_i\}$ are two different concepts. That is, although the current measured value x_0 is a sample within all possible measured values $\{x_i\}$, it is still a constant because it is a numerical value, and only unknown quantities, such as error and true value, need to be expressed with probability. In this way, all possible measured values $\{x_i\}$ are only used to evaluate the error, not need to be used to "evaluate" the current measured value x_0 , and there is no such trouble as equation $u(8844.43) = \pm 0.21$.

3. Difference in Understanding of Error Classification

Because of different understandings for the concepts of the constant and random variable, other conceptual differences arrive one after another. The first conceptual difference is that the traditional theory classifies errors as systematic errors and random errors, but the new concept theory holds that the error has no such classifications.

It can be seen from Fig. 1 that because it is noticed that the measured values are dispersed around the mathematical expectation but the mathematical expectation deviates from the true value, the traditional theory develops its theory according to the conceptual logic of error classification.

Because the random error can be evaluated by the variance, but the systematic error has no variance, people make two concepts of precision and trueness to evaluate them respectively. Because the value of the systematic error is unknown, the trueness can be only a qualitative concept. Besides, because the trueness and precision cannot be combined, as their general name, the accuracy is also a qualitative concept[1][2]. In other words, the total error of a measured value cannot be evaluated quantitatively (as shown in Table 2).

However, from Fig. 3, because the measured value x_0 is a constant, not only the mathematical expectation $E(x)$ deviates from the true value x_T , but also the measured value x_0 deviates from the mathematical expectation $E(x)$. That is, both Δ_A and Δ_B

are unknown deviations. Also, not only Δ_A have all its possible values to obtain its variance, but also Δ_B too, so they are considered to have no systematic/random classifications. Therefore, there is no conceptual difference between the precision and trueness. Only the uncertainty is needed to describe the probability interval of error in error evaluation [10][12][14].

In this way, the conceptual logic of the two theories develops in two completely different directions.

4. The Regularity and Randomness of Error

As we all know, the traditional theory holds that the systematic error has certain regularity, but the random error is random.

However, for the regularity and randomness of errors, the interpretation of the new theory is also different from that of the traditional theory [10][11][12][13][14]. That is, the regularity and randomness come from different perspectives of observing all possible values of error. The regularity refers to the functional relationship between all possible values of error and measurement conditions, but the randomness refers to the density distribution of all possible values of error.

Fig. 3, Fig.4, Fig.5 and Fig.6 are schematic diagrams of the regularity and randomness of four typical errors. In this way, any error has variance to evaluate its probability range. It is all correct to use the function model and the random model to deal with the error [11][14], which completely negates the error classification theory.

Fig 3. Regularity and randomness of periodic error in geodimeter

Fig 4. Regularity and randomness of rounding error

Fig 5. Regularity and randomness of AC interference error in DC voltage measurement

Fig 6. Regularity and randomness of noise error

5. The Principal Interpretation of Deviation, Dispersion and Outlier

The traditional theory holds that the systematic error leads to the deviation, the random error leads to the dispersion, and gross error leads to the outlier. However, on the contrary, the new concept theory also gives a completely different interpretation. [10][11][13][14]

First of all, the new concept theory distinguishes the influence of error on the repeated observed values from that on the final measured value. Because the final measured value is a constant, the influence of any error on it is a deviation.

Secondly, the influence of errors on repeated observations is determined by the change characteristics of measurement conditions, because there are always some changes of measurement conditions in repeated measurements. When the measurement conditions associated with error vary with the repeated measurements, the corresponding error must change and lead to the dispersion; when the measurement conditions associated with error remain unchanged, the corresponding error must also remain unchanged and lead to the deviation; when the measurement conditions associated with error change extremely unbalanced, the corresponding error must change unevenly and lead to the outliers.

We can take the errors in Fig 3 and Fig 4 as examples. If the range condition remains unchanged in the repeated measurements, the two errors must remain constant and contribute to the deviation; if the range conditions vary with the repeated measurements, the two errors will inevitably change and contribute to the dispersion; if the range condition changes unevenly in repeated measurements, the two errors will inevitably change unevenly and contribute to the outliers.

The errors in Fig 5 and Fig 6 can be taken as other examples. If the time remains unchanged in the repeated measurements, the two errors must remain constant and contribute to the deviation; if the time varies with the repeated measurements, the two errors will inevitably change and contribute to the dispersion; if the time changes unevenly in the repeated measurements, the two errors will inevitably change unevenly and can contribute to the outliers.

In addition, there is also a form of influence in which the systematic and random influence coexist. Also, the degree of influence can be divided into strength and weakness, as well as non-influence.

For example, in Fig. 3 and Fig. 5, if the repeated measurements only lead to the phase changes among $0 \sim \pi$, these two errors not only cause the dispersion, but also cause the deviation.

Therefore, the reason why the repeated test values in Fig 1 and Fig 2 present density distribution is actually due to the change of repeated measurement conditions, rather than the interpretation of error classification in traditional theory. The most common measurement conditions are: time, range, temperature, instrument, internal working state of the instrument, measuring personnel, instrument placement, route, alignment and so on. There is no "the same condition" for the repeated measurements, and the statistical evaluation of error requires to collect error samples equally under all possible measurement conditions.

6. Covariance Propagation Law

The traditional theory holds that only random errors have the variance and covariance, and the covariance propagation law is interpreted as the propagation relationship between the dispersions of the measured values.

However, according to the new concept theory, the variance is the evaluation value of probability interval of error, which is expressed by the dispersion of all possible values of error. Because any error has all its possible values to obtain its variance, there must be a covariance between any two errors to describe the correlation between all possible values of them. That is, $u(\Delta x_i \Delta x_j) = E(\Delta x_i \Delta x_j)$, which actually comes from the variance of common component contained in two errors.

In this way, for an error sequence $\Delta \mathbf{X} = (\Delta x_1 \quad \Delta x_2 \quad \cdots \quad \Delta x_t)^T$, the definition of the variance is

$$\mathbf{U}(\Delta \mathbf{X}) = E(\Delta \mathbf{X})(\Delta \mathbf{X})^T \quad (6-1)$$

In this way, for a linear error propagation equation $\Delta \mathbf{Z} = \mathbf{K} \cdot \Delta \mathbf{X}$, according to the definition of the variance, there is:

$$\mathbf{U}(\Delta \mathbf{Z}) = \mathbf{K} \cdot \mathbf{U}(\Delta \mathbf{X}) \cdot \mathbf{K}^T \quad (6-2)$$

That is, the covariance propagation law is the propagation law of probability intervals between the errors, instead of the propagation law of dispersions between the measured values interpreted by the traditional theory.

7. Uncertainty Concept

Although the concept of uncertainty has been applied to the interpretation of traditional measurement theory [3] [4], it is also defined as the dispersion of the measured values (similar to the precision) because it is derived from analysis of variance. Therefore, the logical relationship between it and other measurement concepts has become a difficult problem.

The concept of uncertainty interpreted by the new concept theory is also completely different from that interpreted by traditional theory.

$$\text{In Fig 2, there is} \quad \Delta = \Delta_A + \Delta_B \quad (7-1)$$

According to the covariance propagation law (6-2), there is

$$u^2(\Delta) = u^2(\Delta_A) + u^2(\Delta_B) \quad (7-2)$$

The standard deviation $u(\Delta)$ is the uncertainty of error Δ , which is the evaluation value of probability interval of deviation Δ , and is used to describe the degree we cannot know the value of deviation Δ . Obviously, this uncertainty concept is totally different from the dispersion of measured value interpreted by the traditional measurement theory.

Formula (7-2) is actually a special case of applying the law of covariance propagation to direct repeated measurements. For various indirect measurements, this A/B classification interpretation is not very practical, but it will be simple to use the law of covariance propagation directly [13] (Shown in the case in the next section).

8. Case Comparison

Here, a simple practical case is used to compare the difference between the two theories.

Fig 7. Distances measurement

For example, four points A, B, C and D are located on a straight line, as shown in Fig 7, and the distances obtained by a geodimeter [15] are shown in Table 4. Please solve the final measured values of each baseline and their quality evaluation.

Table 4. Observation values

	Line segment	Observation value
1	AB	x_1
2	BC	x_2
3	CD	x_3
4	AC	x_4
5	BD	x_5
6	AD	x_6

Using y_1 , y_2 and y_3 to express the final measured values of AB, BC and CD respectively, and using k to express the measured value of the additive constant error of geodimeter, the error equations are as follows:

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ k \end{pmatrix} \quad (8-1)$$

According to the least square method, the final measured values are:

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ k \end{pmatrix} = \left[\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \right]^{-1} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} \\ = \frac{1}{4} \times \begin{pmatrix} 1 & -2 & -1 & 1 & -1 & 2 \\ -2 & 1 & -2 & 1 & 1 & 1 \\ -1 & -2 & 1 & -1 & 1 & 2 \\ 2 & 2 & 2 & 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{pmatrix} \quad (8-2)$$

For the two measurement theories, the above processes are the same, but the difference between them lies in the submission process of the variance.

According to the concept in the traditional theory, the next steps are: 1. the values of each residual v_i are obtained by substituting equation (8-2) into equation (8-1); 2.

$u(x)$ is obtained by using Bessel formula $u(x) = \sqrt{\frac{\sum_{i=1}^n v_i^2}{n-t}}$; 3. the precisions $u(y_1)$, $u(y_2)$ and $u(y_3)$ are calculated by using the covariance propagation law.

However, according to the new concept theory, there are three conceptual troubles in the above process: 1. Bessel formula cannot be used, because the value of freedom degree $n-t$ is too small. 2. The variances of the observed value x_i and the measured value y_j should be zero, because they are all constants. 3. The correlation (covariance) between observation errors is not considered at all.

The variance submission method in the new concept theory is as follows.

Taking total differential of equation (8-2), a complete error propagation equation is:

$$\begin{pmatrix} \Delta y_1 \\ \Delta y_2 \\ \Delta y_3 \\ \Delta k \end{pmatrix} = \frac{1}{4} \times \begin{pmatrix} 1 & -2 & -1 & 1 & -1 & 2 \\ -2 & 1 & -2 & 1 & 1 & 1 \\ -1 & -2 & 1 & -1 & 1 & 2 \\ 2 & 2 & 2 & 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_6 \end{pmatrix} \quad (8-3)$$

Applying the law of covariance propagation to equation (8-3), a complete covariance propagation equation is:

$$\begin{pmatrix} u_{\Delta y_1}^2 & u_{\Delta y_1 \Delta y_2} & u_{\Delta y_1 \Delta y_3} & u_{\Delta y_1 \Delta k} \\ u_{\Delta y_1 \Delta y_2} & u_{\Delta y_2}^2 & u_{\Delta y_2 \Delta y_3} & u_{\Delta y_2 \Delta k} \\ u_{\Delta y_1 \Delta y_3} & u_{\Delta y_2 \Delta y_3} & u_{\Delta y_3}^2 & u_{\Delta y_3 \Delta k} \\ u_{\Delta y_1 \Delta k} & u_{\Delta y_2 \Delta k} & u_{\Delta y_3 \Delta k} & u_{\Delta k}^2 \end{pmatrix} = \frac{1}{16} \times \begin{pmatrix} 1 & -2 & -1 & 1 & -1 & 2 \\ -2 & 1 & -2 & 1 & 1 & 1 \\ -1 & -2 & 1 & -1 & 1 & 2 \\ 2 & 2 & 2 & 0 & 0 & -2 \end{pmatrix} \mathbf{U}(\Delta \mathbf{X}) \begin{pmatrix} 1 & -2 & -1 & 2 \\ -2 & 1 & -2 & 2 \\ -1 & -2 & 1 & 2 \\ 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 0 \\ 2 & 1 & 2 & -2 \end{pmatrix} \quad (8-4)$$

Next, the acquisition method of $\mathbf{U}(\Delta \mathbf{X})$ is as follows.

For the observation x_i , its error Δx_i consists of three components: the additive constant error K , the multiplicative constant error R and the non-uniform indexing error c_i . That is:

$$\Delta x_i = K + R \cdot x_i + c_i \quad (8-5)$$

Its variance is

$$u_{\Delta x_i}^2 = u_K^2 + x_i^2 \cdot u_R^2 + u_{c_i}^2 \quad (8-6)$$

Among equation (8-6), the values of u_K , u_R and u_{c_i} , which are the dispersions of all possible values of errors K , R and c_i respectively, come from the tolerance standard in the instrument specification or instrument instruction. Further, according to the definition of variance, there is:

$$\begin{aligned}
 \mathbf{U}(\Delta\mathbf{X}) &= E \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_6 \end{pmatrix} \begin{pmatrix} \Delta x_1 & \Delta x_2 & \cdots & \Delta x_6 \end{pmatrix} \\
 &= \begin{pmatrix} u_K^2 + x_1^2 \cdot u_R^2 + u_c^2 & u_K^2 + x_2 x_1 \cdot u_R^2 & \cdots & u_K^2 + x_6 x_1 \cdot u_R^2 \\ u_K^2 + x_1 x_2 \cdot u_R^2 & u_K^2 + x_2^2 \cdot u_R^2 + u_c^2 & \cdots & u_K^2 + x_6 x_2 \cdot u_R^2 \\ \vdots & \vdots & \ddots & \vdots \\ u_K^2 + x_1 x_6 \cdot u_R^2 & u_K^2 + x_2 x_6 \cdot u_R^2 & \cdots & u_K^2 + x_6^2 \cdot u_R^2 + u_c^2 \end{pmatrix} \quad (8-7)
 \end{aligned}$$

Finally, substituting equation (8-7) into equation (8-4), the $\mathbf{U}(\Delta\mathbf{Y})$ is calculated. Among them, the $u(\Delta y_j)$ is called as error's uncertainty, which is the evaluation of probability interval of error Δy_j .

It can be seen that the so-called systematic error without variance does not exist indeed according to the conceptual logic of the new theory. Also, if we adopt the principle of the combination of Type A and Type B uncertainties in the traditional theory, the uncertainty analysis in this case is almost impossible to complete, which is also because the traditional concept of uncertainty has no mathematical logic.

9. Conclusion

Due to the different understanding of concepts of the constant and random variable, the conceptual logic of two measurement error theories has developed in two completely different directions.

Traditional measurement theory pays attention to the dispersion of repeated observations, and thus regards each observation as a random variable, so that the variance is assigned to each specific numerical value, the overall conceptual logic is in a plight, and the quantitative evaluation of error or true value cannot be realized. The traditional theory obviously ignores that the numerical value belongs to a constant, and ignores the conceptual distinction between random variable and sample.

The new concept theory clarifies that the measured value belongs to a constant without the variance, and that the error's variance is the dispersion of all possible values of error (the evaluation of probability interval of error). Recognizing that any error has all its possible values and has its variance, the measurement theory is developed according to the logic that error has no systematic and random category.

By comparison, the traditional theory has obvious deficiency in explaining the concept of probability theory, and other concepts deduced from this are naturally not correct, while the new concept theory is more in line with the concept principle of probability theory and completely solved the issue of quantitative evaluation of error.

In this way, according to the logic of the new concept theory, the error classification concepts such as systematic error, random error, precision, trueness and accuracy will not be needed in measurement practice, and the error evaluation concept is only the uncertainty concept: the evaluation of the probability interval of error (The dispersion of all possible values of the error). Therefore, the revision of metrology terminology and measurement textbooks has become a topic that needs to be discussed

again. Moreover, because the deviation, dispersion and outlier phenomena of repeated observations come from the changing rules of repeated measurement conditions, and especially many outliers are regarded as an unbalanced dispersion instead of gross errors, there is still room for further research on error processing methods.

References

- [1] JCGM 200:2012, International vocabulary of metrology — Basic and general concepts and associated terms (VIM). BIPM, Sèvres, France, 2012.
- [2] JJF1001-2011, General Terms in Metrology and Their Definitions. China Quality Inspection Press, Beijing, China, 2011.
- [3] JCGM 100:2008, Guide to the Expression of Uncertainty in Measurement(GUM). BIPM, Sèvres, France, 2008.
- [4] JJF1059-2012, Evaluation and Expression of Uncertainty in Measurement. China Quality Inspection Press, Beijing, China, 2012.
- [5] GB/T14911-2008, Basic Terms of Surveying and Mapping. China Standard Press, Beijing, China, 2008.
- [6] School of Geodesy and Geomatics, Wuhan University. Error Theory and Foundation of Surveying Adjustment [M]. Wuhan University Press, 2016 6.
- [7] Fei Yetai. Error Theory and Data Processing. Machinery Industry Press, 2016, 3.
- [8] Ye Depei. Understanding, Evaluation and Application of Measurement Uncertainty. China Metrology Publishing House, 2007 10.
- [9] Ye Xiao-ming, Ling Mo, Zhou Qiang, Wang Wei-nong, Xiao Xue-bin. The New Philosophical View about Measurement Error Theory. *Acta Metrologica Sinica*, 2015, 36(6): 666-670.
- [10] Ye Xiao-ming, Xiao Xue-bin, Shi Jun-bo, Ling Mo. The new concepts of measurement error theory. *Measurement*, 2016 April, Volume 83, Pages 96-105.
- [11] Ye Xiao-ming, Liu Hai-bo, Ling Mo, Xiao Xue-bin. The new concepts of measurement error's regularities and effect characteristics. *Measurement*, 2018, October, Volume 126, Pages 65-71.
- [12] Ye Xiao-ming, Ding Shi-jun. Comparison of variance concepts interpreted by two measurement theories. *Journal of Nonlinear and Convex Analysis*, 2019, Volume 20, Number 7, Pages 1307-1316.
- [13] Shi Huisheng, Ye Xiao-ming, Xing Cheng, Ding Shijun. A New Theoretical Interpretation of Measurement Error and Its Uncertainty. *Discrete Dynamics in Nature and Society*, Volume 2020, Article ID 3864578, <https://doi.org/10.1155/2020/3864578>
- [14] Ye Xiao-ming. Measurement error theory based on new concepts [M]. Hubei Science and Technology Press, 2017, 11.
- [15] ISO 17123-4:2012, Optics and optical instruments -- Field procedures for testing geodetic and surveying instruments -- Part 4: Electro-optical distance meters (EDM measurements to reflectors), ISO, Geneva, Switzerland, 2012.