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Semantics Hierarchy in Preference-Based Argumentation Frameworks

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Abstract. We define the pref-complete semantics for the Preference-Based Argumentation Frameworks (PAFs) of Amgoud and Vesic. The new semantics generalizes Dung's complete semantics for Argumentation Frameworks (AFs) in the same way that their original semantics (called pref-grounded, pref-stable, pref-preferred) respectively generalize the grounded, stable and preferred semantics for AFs. Additionally, we show that the pref-grounded/stable/preferred semantics are particular cases of the newly defined pref-complete semantics, therefore preserving the semantic hierarchy observed for AF semantics. This yields new ways for computing the semantics of PAFs, since the particular cases can be obtained from the pref-complete semantics with straightforward operations. Our contributions reinforce their thesis of backwards compatibility towards Dung's AF semantics.

Keywords. Abstract Argumentation, Preferences, Argumentation Semantics

1. Introduction

This work contributes to the thesis that the *Preference-Based Argumentation Frameworks* (PAFs) of Amgoud and Vesic [1] are backwards compatible to Dung's *abstract argumentation frameworks* (AFs) [2] concerning semantics. Their work can be found among several others [1,3,4,5,6,7,8,9] advocating that arguments do not always have the same strength and that, in some cases, the confidence one has in an argument could be enough to accept it despite reasons not to. In each case, these works (as well as [10,11]) approached how preferences over arguments in an AF should affect their evaluation, leading to different results.

To their advantage, [1] only retrieves *conflict-free* [2] sets of arguments in their preferential semantics. To that matter, they developed three preferential semantics respectively called *pref-grounded*, *pref-stable* and *pref-preferred* semantics, which respectively retrieve Dung's *grounded*, *stable* and *preferred* semantics [2] when the preferences over arguments cope with the attacks, but the *complete* semantics [2], commonly understood as the core AF semantics, was not addressed. The missing semantics is known to subsume the ones they approached, in the sense that the grounded, stable and preferred semantics are all particular cases of the complete semantics [12] for AFs. For this reason, had they defined the pref-complete semantics in [1], one would expect it to subsume the pref-grounded, pref-stable and pref-preferred semantics, and also to coincide with

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Dung's complete semantics when the preferences cope with the attacks. Our work means to close that gap, therefore we start by properly defining the *pref-complete* semantics for PAFs. Based on the new definition, we will prove that our preferential semantics generalizes Dung's complete semantics (following criteria from [1]) as expected. Further, we confirm that the pref-complete, pref-grounded, pref-stable and pref-preferred semantics for PAFs preserve the exact same hierarchy found between their corresponding AF semantics, even when the preferences influence the attack relation. Our results would also allow the results for the pref-grounded, pref-stable and pref-preferred semantics to be computed from the pref-complete extensions with straightforward operations, based on the confirmed hierarchy.

2. Preliminaries

We briefly review Argumentation Frameworks [2] and Preference-based Argumentation Frameworks as in [1] along with their semantics.

Definition 1 (Dung's framework)**.** *[2] An argumentation framework (AF) is a pair* $(\mathtt{Ar}, att)$ *where* $\mathtt{Ar}$ *is a set of arguments and* $att \subseteq \mathtt{Ar} \times \mathtt{Ar}$.

Arguments are related to others by the attack relation *att*: an argument a attacks b iff $(a,b) \in att$. An argumentation framework can be seen as a directed graph where the arguments are nodes and each attack is an arrow.

Definition 2 (defense/conflict-free)**.** *[2] Let* $\mathscr{F} = (\mathtt{Ar}, att)$ *be an AF and* $\mathtt{E} \subseteq \mathtt{Ar}$. *We say* $\mathtt{E}$ *is conflict-free iff* $\nexists a, b \in \mathtt{E}$ *such that* $(a,b) \in att$. *We will refer to* $\mathtt{CF}(\mathscr{F}) = \{\mathtt{E} \subseteq \mathtt{Ar} \mid \mathtt{E}$ *is a conflict-free set of arguments w.r.t.* $\mathscr{F}\}$ *as the set of all conflict-free sets of arguments w.r.t.* $\mathscr{F}$. *We say* $\mathtt{E}$ defends *a iff every argument attacking a is attacked by some argument in* $\mathtt{E}$. *We define the characteristic function* $f : 2^{\mathtt{Ar}} \to 2^{\mathtt{Ar}}$ *of* $\mathscr{F}$ *as* $f(\mathtt{E}) = \{a \in \mathtt{Ar} \mid \forall b \in \mathtt{Ar},$ *if* $(b,a) \in att,$ *then* $\exists c \in \mathtt{E}$ *such that* $(c,b) \in att\}$ *to determine the set of all arguments defended by* $\mathtt{E}$. *We write* $\mathtt{E}^+ = \{a \in \mathtt{Ar} \mid \exists b \in \mathtt{E}$ *such that* $(b,a) \in att\}$ *to refer to the set of arguments attacked by* $\mathtt{E}$.

Traditional approaches to argumentation semantics are based on extensions of arguments. Some of the mainstream approaches are summarised below:

Definition 3 (Argumentation Semantics)**.** *[2,13] Let* $\mathscr{F} = (\mathtt{Ar}, att)$ *be an AF,* $\mathtt{E}$ *be a conflict free subset of* $\mathtt{Ar}$, *and f the characteristic function of* $\mathscr{F}$. *Then*

- $\mathtt{E}$ *is a complete extension of* $\mathscr{F}$ *iff* $f(\mathtt{E}) = \mathtt{E}$.
- $\mathtt{E}$ *is the grounded extension of* $\mathscr{F}$ *iff* $\mathtt{E}$ *is the* $\subseteq$*-minimal complete extension of* $\mathscr{F}$.
- $\mathtt{E}$ *is a preferred extension of* $\mathscr{F}$ *iff* $\mathtt{E}$ *is a* $\subseteq$*-maximal complete extension of* $\mathscr{F}$.
- $\mathtt{E}$ *is a stable extension of* $\mathscr{F}$ *iff* $\mathtt{E}$ *is a complete extension of* $\mathscr{F}$ *s.t.* $\mathtt{E} \cup \mathtt{E}^+ = \mathtt{Ar}$.

Several works generalizing Dung's *AF* to handle preferences over arguments have been proposed [14,3,4,15,16,5,1]. In the so-called Preference-based Argumentation Frameworks (*PAF*s), preferences are used to represent the comparative strength of arguments. In *PAF*s, a critical scenario is what to do when the attacked argument b is stronger than its attacker a. In [16,7,9], they ignore those attacks and evaluate the arguments of *PAF* based only on the remaining attacks. This approach has been criticised by Amgoud and Vesic [1] as it leads to non-conflict-free extensions. As an alternative, they propose that the frameworks should instead be repaired by reversing the direction of those attacks.

Definition 4 (*PAF*)**.** *[1] A Preference-based Argumentation Framework* (PAF) *is a tuple* $(\mathtt{A}r, att, \geq)$ *s.t.* $\mathtt{A}r$ *is a set of arguments,* $att \subseteq \mathtt{A}r \times \mathtt{A}r$*, and* $\geq$ *is a (partial/total) preorder.*

As in [1], we assume in this paper and without loss of generality that for a *PAF* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$, $\mathtt{A}r$ is finite and *att* does not contain self-attacking arguments. By $\mathtt{CF}(\mathfrak{T}) = \{\mathtt{E} \subseteq \mathtt{A}r \mid \mathtt{E}$ *is conflict-free*$\}$, we denote the set of all conflict-free sets of arguments in $\mathfrak{T}$.

A distinguishing aspect of this approach is how a set of arguments defends an argument from other sets of arguments.

Definition 5 (Defense)**.** *Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF and* $\mathtt{E}, \mathtt{E}' \subseteq \mathtt{A}r$*. We say* $\mathtt{E}$ *defends* $a \in \mathtt{A}r$ *from* $\mathtt{E}'$*, denoted by* $d(a, \mathtt{E}, \mathtt{E}')$*, iff* $\forall b \in \mathtt{E}'$ *if* $((b,a) \in att$ *and* $a \not> b)$ *or* $((a,b) \in att$ *and* $b > a)$*, then* $\exists c \in \mathtt{E}$ *s. t.* $((c,b) \in att$ *and* $b \not> c)$ *or* $((b,c) \in att$ *and* $c > b)$*.*

Still in [1], a semantics for evaluating arguments of a *PAF* is defined as a dominance relation $\succeq$ on $2^{\mathtt{A}r}$. For $\mathtt{E}, \mathtt{E}' \subseteq \mathtt{A}r$, writing $\mathtt{E} \succeq \mathtt{E}'$ means that $\mathtt{E}$ is at least as good as $\mathtt{E}'$. By $\mathtt{E} \succ \mathtt{E}'$ we say that $\mathtt{E}$ is strictly better than $\mathtt{E}'$, i.e., that $\mathtt{E} \succeq \mathtt{E}'$ and $\mathtt{E}' \not\succeq \mathtt{E}$.

An *acceptability* semantics for a *PAF* $(\mathtt{A}r, att, \geq)$ is defined by a dominance relation $\succeq \subseteq 2^{\mathtt{A}r} \times 2^{\mathtt{A}r}$ satisfying the postulates P_1, P_2 and P_3 that follow. Below, $\mathtt{E}, \mathtt{E}' \subseteq \mathtt{A}r$ and $a, a' \in \mathtt{A}r$ and $\frac{X_1, \ldots, X_n}{Y}$ means that whenever $X_1, \ldots$, and X_n hold, Y holds.

$$\frac{\mathtt{E} \in \mathtt{CF}(\mathfrak{T}) \quad \mathtt{E}' \notin \mathtt{CF}(\mathfrak{T})}{\mathtt{E} \succeq \mathtt{E}'} \qquad \frac{(a,a') \in att \quad (a',a) \notin att \quad \neg(a' > a)}{\{a\} \succ \{a'\}} \qquad \frac{(a,a') \in att \quad (a' > a)}{\{a'\} \succ \{a\}}$$

Postulate P_1 **Postulate P_2** **Postulate P_3**

The authors also defined three semantics for *PAF*s in [1]: pref-grounded, pref-stable, pref-preferred. We proceed by recalling the notion of strong defense, which will be employed in the characterisation of pref-grounded:

Definition 6 (Strong Defense)**.** *[1] Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF and* $\mathtt{E} \subseteq \mathtt{A}r$*. We say* $\mathtt{E}$ *strongly defends an argument* a *from attacks of a set* $\mathtt{E}'$*, denoted by* $sd(a, \mathtt{E}, \mathtt{E}')$*, iff* $\forall b \in \mathtt{E}'$ *if* $((b,a) \in att$ *and* $a \not> b)$ *or* $((a,b) \in att$ *and* $b > a)$*, then* $\exists c \in \mathtt{E} \backslash \{a\}$ *such that* $((c,b) \in att$ *and* $b \not> c)$ *or* $((b,c) \in att$ *and* $c > b)$ *and* $sd(c, \mathtt{E} \backslash \{a\}, \mathtt{E}')$*.*

Intuitively, an argument is strongly defended when it is preferred to its attackers or it is defended by another argument that is strongly defended without the argument in question. In [1], the extensions of a semantics $\succeq$ are then given by its maximal elements:

Definition 7 ((Maximal) Upper Bounds)**.** *Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF,* $\mathtt{E} \subseteq \mathtt{A}r$ *and* $\succeq \subseteq 2^{\mathtt{A}r} \times 2^{\mathtt{A}r}$ *a semantics for PAF. We say* $\mathtt{E}$ *is an upper bound wrt* $\succeq$ *iff* $\forall \mathtt{E}' \in 2^{\mathtt{A}r}, \mathtt{E} \succeq \mathtt{E}'$*. Besides, if no strict superset of* $\mathtt{E}$ *is an upper bound wrt* $\succeq$*, then* $\mathtt{E}$ *is a maximal wrt* $\succeq$*. Let* $\succeq_{ub}$ *and* $\succeq_{max}$ *denote respectively the set of upper bound and maximal sets w.r.t.* $\succeq$*.*

We are ready to define the pref-grounded, pref-stable and pref-preferred semantics:

Definition 8 (Pref-grounded semantics)**.** *[1] Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF and* $\mathtt{E}, \mathtt{E}' \subseteq \mathtt{A}r$*. It holds that* $\mathtt{E} \succeq_g \mathtt{E}'$ *iff a)* $\mathtt{E} \in \mathtt{CF}(\mathfrak{T})$ *and* $\mathtt{E}' \notin \mathtt{CF}(\mathfrak{T})$*, or b)* $\forall a \in \mathtt{E}$*, it holds* $sd(a, \mathtt{E}, \mathtt{E}')$*.*

Definition 9 (Pref-stable semantics). *[1] Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF and* $\mathtt{E}, \mathtt{E}' \subseteq \mathtt{A}r$. *It holds that* $\mathtt{E} \succeq_s \mathtt{E}'$ *iff a)* $\mathtt{E} \in \mathtt{CF}(\mathfrak{T})$ *and* $\mathtt{E}' \notin \mathtt{CF}(\mathfrak{T})$, *or b)* $\mathtt{E}, \mathtt{E}' \in \mathtt{CF}(\mathfrak{T})$ *and* $\forall b \in \mathtt{E}' \backslash \mathtt{E}$, $\exists a \in \mathtt{E} \backslash \mathtt{E}'$ *s.t.* $((a,b) \in att$ *and* $b \not> a)$ *or* $(a > b)$.

Definition 10 (Pref-preferred semantics). *[1] Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF and* $\mathtt{E}, \mathtt{E}' \subseteq \mathtt{A}r$. *It holds that* $\mathtt{E} \succeq_p \mathtt{E}'$ *iff a)* $\mathtt{E} \in \mathtt{CF}(\mathfrak{T})$ *and* $\mathtt{E}' \notin \mathtt{CF}(\mathfrak{T})$, *or b)* $\mathtt{E}, \mathtt{E}' \in \mathtt{CF}(\mathfrak{T})$ *and* $\forall a \in \mathtt{E}, \forall b \in \mathtt{E}'$, *if* $((b,a) \in att$ *and* $a \not> b)$ *or* $((a,b) \in att$ *and* $b > a)$, *then* $\exists c \in \mathtt{E}$ *such that* $((c,b) \in att$ *and* $b \not> c)$ *or* $((b,c) \in att$ *and* $c > b)$.

We say $\mathtt{E} \subseteq \mathtt{A}r$ is a pref-grounded, pref-stable or a pref-preferred extension iff it is respectively maximal (Definition 7) with respect to $\succeq_g$, $\succeq_s$ and $\succeq_p$. By $\succeq_{g,max}$, $\succeq_{s,max}$ and $\succeq_{p,max}$, we denote respectively the set of maximal sets w.r.t. $\succeq_g$, $\succeq_s$ and $\succeq_p$.

Example 1. *Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF such that* $\mathtt{A}r = \{a,b,c,d,e,f\}$, $att = \{(a,b), (b,c), (c,a), (d,e), (d,f), (e,a), (f,d)\}$ *and* $(a > e)$. *Its sets of extensions are* $\succeq_{g,max} = \{\emptyset\}$, $\succeq_{s,max} = \emptyset$ *and* $\succeq_{p,max} = \{\{d\}, \{f\}\}$.

According to [1], pref-grounded, pref-stable and pref-preferred coincide respectively with grounded, stable and preferred when the available preferences do not conflict with the attacks. Note also that instead of partitioning the powerset of the set of arguments into extensions and non-extensions as usual in the definition of the semantics for *AF*, this approach is more informative as it compares all the subsets of arguments.

Next, we define a new semantics for *PAF*, namely the *pref-complete* semantics. We will proceed to show that the relations between the pref-complete extensions and the pref-grounded, pref-stable and pref-preferred extensions are respectively the same as the relations between complete extensions and grounded, stable and preferred extensions.

3. Complete Semantics for *PAF*s

In this section, we will define the pref-complete semantics $\succeq_c$ for *PAF*s, designed to coincide with the complete semantics for *AF* when preferences are ignored. The challenge behind this goal is that, differently from $\succeq_g$, $\succeq_p$, and $\succeq_s$, the extensions of $\succeq_c$ cannot be defined in terms of only its maximal elements. For instance, the complete extensions of $AF = (\{a,b\}, \{(a,b),(b,a)\})$ are $\emptyset$, $\{a\}$, $\{b\}$, amongst which $\emptyset$ fails maximality. As we will show, the extensions of $\succeq_c$ are instead characterized by its upper bounds.

Definition 11 (Pref-complete semantics). *Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF and* $\mathtt{E}, \mathtt{E}' \subseteq \mathtt{A}r$. *It holds that* $\mathtt{E} \succeq_c \mathtt{E}'$ *iff a)* $\mathtt{E} \in \mathtt{CF}(\mathfrak{T})$ *and* $\mathtt{E}' \notin \mathtt{CF}(\mathfrak{T})$ *or b)* $\mathtt{E}, \mathtt{E}' \in \mathtt{CF}(\mathfrak{T})$ *and* $\mathtt{E} \subseteq \{a \in \mathtt{A}r | d(a, \mathtt{E}, \mathtt{E}')\}$ *and if* $\mathtt{E} \subseteq \mathtt{E}'$, *then* $(\{a \in \mathtt{A}r | d(a, \mathtt{E}, \mathtt{A}r)\} - \mathtt{E}) \subseteq (\{a \in \mathtt{A}r | d(a, \mathtt{E}', \mathtt{A}r)\} - \mathtt{E}')$.

We define $\succeq_{c,ub} = \{\mathtt{E} \subseteq \mathtt{A}r \mid \mathtt{E}$ *is an upper bound w.r.t.* $\succeq_c\}$. *A set* $\mathtt{E}$ *is a pref-complete extension of* $\mathfrak{T}$ *iff* $\mathtt{E} \in \succeq_{c,ub}$.

Note that when $\mathtt{E}, \mathtt{E}' \in \mathtt{CF}(\mathfrak{T})$, it holds $\mathtt{E} \succeq_c \mathtt{E}'$ iff $\mathtt{E}$ defends all its elements from the attacks of $\mathtt{E}'$, and if $\mathtt{E} \subseteq \mathtt{E}'$, those extra elements defended by $\mathtt{E}$ beyond the elements in $\mathtt{E}$ are also defended by $\mathtt{E}'$. In particular, if $\mathtt{E}$ is conflict-free and the set of elements it defends is exactly $\mathtt{E}$, then $\mathtt{E}$ is a pref-complete extension. Recalling the *PAF* $\mathfrak{T}$ in Example 1, we obtain the set of its pref-complete extensions is $\succeq_{c,ub} = \{\emptyset, \{d\}, \{f\}\}$.

It is clear that $\succeq_c$ is an acceptability semantics.

Proposition 1. *The relation $\succeq_c$ satisfies postulates P_1, P_2 and P_3.*

The next definition describes semantics generalization. We will employ it to prove that the pref-complete semantics generalizes Dung's complete semantics.

Definition 12 (Generalizing a semantics)**.** *A semantics $\succeq$ for PAF generalizes a semantics S for AF iff for all PAF $(\mathtt{Ar}, att, \geq)$, such that $\nexists a, b \in \mathtt{Ar}$ with $(a,b) \in att$ and $b > a$, it holds $\mathtt{E} \in \succeq_{ub}$ iff $\mathtt{E}$ is an extension of $\mathscr{F} = (\mathtt{Ar}, att)$ according to S.*

As expected, Proposition 2 guarantees pref-complete extensions are conflict-free:

Proposition 2. *Let $\mathfrak{T} = (\mathtt{Ar}, att, \geq)$ be a PAF and $\mathtt{E} \subseteq \mathtt{Ar}$. If $\mathtt{E} \in \succeq_{c,ub}$, then $\mathtt{E} \in \mathtt{CF}(\mathfrak{T})$.*

The next result will help us prove that the pref-complete semantics generalizes Dung's complete semantics.

Lemma 1. *Let $\mathfrak{T} = (\mathtt{Ar}, att, \geq)$ be a PAF, in which $\nexists a, b \in \mathtt{Ar}$ such that $(a,b) \in att$ and $b > a$. For any $\mathtt{E} \subseteq \mathtt{Ar}$, it holds $\{a \in \mathtt{Ar} \mid d(a, \mathtt{E}, \mathtt{Ar})\} = f(\mathtt{E})$. Besides, for each $\mathtt{E}' \subseteq \mathtt{Ar}$, it holds $f(\mathtt{E}) \subseteq \{a \in \mathtt{Ar} \mid d(a, \mathtt{E}, \mathtt{E}')\}$.*

Theorem 1. *The relation $\succeq_c$ generalises complete semantics.*

Proof. (*sketch*) Let $\mathfrak{T} = (\mathtt{Ar}, att, \geq)$ be a *PAF*, in which $\nexists a, b \in \mathtt{Ar}$ such that $(a,b) \in att$ and $b > a$. We will prove $\mathtt{E}$ is a complete extension of $\mathscr{F} = (\mathtt{Ar}, att)$ iff $\mathtt{E} \in \succeq_{c,ub}$: it holds $\mathtt{E}$ is a complete extension of $\mathscr{F}$ iff $\mathtt{E} \in \mathtt{CF}(\mathscr{F})$ and $f(\mathtt{E}) = \mathtt{E}$ iff (Lemma 1) $\mathtt{E} \in \mathtt{CF}(\mathfrak{T})$ and $\forall \mathtt{E}' \in \mathtt{CF}(\mathfrak{T}), \mathtt{E} \subseteq \{a \in \mathtt{Ar} | d(a, \mathtt{E}, \mathtt{E}')\}$ and $\{a \in \mathtt{Ar} | d(a, \mathtt{E}, \mathtt{Ar})\} = \mathtt{E}$ iff $\mathtt{E} \in \mathtt{CF}(\mathfrak{T})$ and $\forall \mathtt{E}' \in \mathtt{CF}(\mathfrak{T}), \mathtt{E} \subseteq \{a \in \mathtt{Ar} | d(a, \mathtt{E}, \mathtt{E}')\}$ and if $\mathtt{E} \subseteq \mathtt{E}'$, then $(\{a \in \mathtt{Ar} | d(a, \mathtt{E}, \mathtt{Ar})\} - \mathtt{E}) = \emptyset \subseteq (\{a \in \mathtt{Ar} | d(a, \mathtt{E}', \mathtt{Ar})\} - \mathtt{E}')$ iff $\mathtt{E} \in \mathtt{CF}(\mathfrak{T})$ and $\forall \mathtt{E}' \subseteq \mathtt{Ar}$, $\mathtt{E} \succeq_c \mathtt{E}'$ iff $\mathtt{E} \in \succeq_{c,ub}$. □

4. The pref-Semantics Satisfies the Classical AF Semantics Hierarchy

In this section, we show that the pref-grounded, pref-stable and pref-preferred semantics are particular cases of the pref-complete semantics in the same way that the grounded, stable and preferred AF semantics are particular cases of the complete AF semantics. Therefore, we show that the semantic hierarchy of AFs is entirely preserved by the semantics defined for PAFs. Timely, we highlight this result holds for all PAFs, independently of what preferences one has over arguments.

Regarding the successful attacks (defeats), we have the *AF* corresponding to a *PAF*:

Definition 13 (Defeat)**.** *[1] Let $\mathfrak{T} = (\mathtt{Ar}, att, \geq)$ be a PAF and $a, b \in \mathtt{Ar}$. We say a defeats b in $\mathfrak{T}$ if $((a,b) \in att$ and $b \not> a)$ or $((b,a) \in att$ and $a > b)$. We will refer to $(\mathtt{Ar}, \mathscr{D})$ as the AF corresponding to $\mathfrak{T}$, in which $\mathscr{D} = \{(a,b) | a, b \in \mathtt{Ar}$ and a defeats b in $\mathfrak{T}\}$.*

We show the arguments defended by a set of arguments $\mathtt{E}$ via d operator in a *PAF* $\mathfrak{T}$ are the same as those defended by $\mathtt{E}$ via f operator in the *AF* corresponding to $\mathfrak{T}$:

Lemma 2. *Let $\mathfrak{T} = (\mathtt{Ar}, att, \geq)$ be a PAF, $a \in \mathtt{Ar}$ and $(\mathtt{Ar}, \mathscr{D})$ the corresponding argumentation framework to $\mathfrak{T}$. We have $d(a, \mathtt{E}, \mathtt{Ar})$ in $(\mathtt{Ar}, att, \geq)$ iff $a \in f(\mathtt{E})$ in $(\mathtt{Ar}, \mathscr{D})$.*

Lemma 3 shows a pref-complete extension is equal the set of arguments it defends:

Lemma 3. *Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF and* $\mathtt{E} \in \mathtt{CF}(\mathfrak{T})$*. It holds* $\mathtt{E} \in \succeq_{c,ub}$ *if and only if* $\{a' \in \mathtt{A}r | d(a', \mathtt{E}, \mathtt{A}r)\} = \mathtt{E}$.

Now we ensure the pref-complete extensions of a *PAF* are the complete extensions of the corresponding *AF*:

Theorem 2. *Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF and* $\mathscr{T}^d = (\mathtt{A}r, \mathscr{D})$ *the corresponding argumentation framework. We have that* $\mathtt{E} \in \succeq_{c,ub}$ *iff* $\mathtt{E}$ *is a complete extension of* $\mathscr{T}^d$*.*

Proof. $\mathtt{E}$ is a complete extension of $\mathscr{T}^d$ iff $f(\mathtt{E}) = \mathtt{E}$ in $\mathscr{T}^d$ and $\mathtt{E} \in \mathtt{CF}(\mathscr{T}^d)$ iff (Lemma 2) $\mathtt{E} = \{a \in \mathtt{A}r \mid d(a, \mathtt{E}, \mathtt{A}r)\}$ and $\mathtt{E} \in \mathtt{CF}(\mathscr{T}^d)$ iff (Lemma 3) $\mathtt{E} \in \succeq_{c,ub}$ and $\mathtt{E} \in \mathtt{CF}(\mathscr{T}^d)$ iff (Proposition 2) $\mathtt{E} \in \succeq_{c,ub}$. □

Theorems 3, 4 and 5 show respectively pref-grounded, pref-stable and pref-preferred extensions can be depicted via pref-complete extensions in the same way grounded, stable and preferred extensions can be depicted via complete extensions. Theorem 3 follows immediately from Theorem 2 and the fact $\mathtt{E}$ is the pref-grounded extension of a *PAF* iff $\mathtt{E}$ is the grounded extension of the corresponding *AF* (see [1]):

Theorem 3. *Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF. It holds* $\mathtt{E}$ *is the minimal (w.r.t. set inclusion) pref-complete extension of* $\mathfrak{T}$ *iff* $\mathtt{E}$ *is the pref-grounded extension of* $\mathfrak{T}$*.*

In the remaining of this section, for a dominance order $\succeq$ in the context of a *PAF* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$, we will write $\succeq^{\mathfrak{T}}$ to indicate the reference framework. By $\mathscr{T}^d = (\mathtt{A}r, \mathscr{D})$ we mean the corresponding argumentation framework and we assume $\mathfrak{T}^r = (\mathtt{A}r, \mathscr{D}, \geq)$.

Theorem 4. *Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF,* $\mathscr{T}^d = (\mathtt{A}r, \mathscr{D})$ *be the corresponding argumentation framework,* $\mathtt{E} \subseteq \mathtt{A}r$ *and* $\mathtt{E}^+ = \{a \in \mathtt{A}r \mid \exists b \in \mathtt{E} \text{ s. t. } (b, a) \in \mathscr{D}\}$*. It holds* $\mathtt{E}$ *is a pref-complete extension of* $\mathfrak{T}$ *such that* $\mathtt{E} \cup \mathtt{E}^+ = \mathtt{A}r$ *iff* $\mathtt{E}$ *is a pref-stable extension of* $\mathfrak{T}$*.*

Proof. It holds $\mathtt{E} \in \succeq_{s,max}$ iff (Theorem 11 in [1]) $\mathtt{E}$ is stable in $\mathscr{T}^d$ iff (according to [2]) $\mathtt{E}$ is complete in $\mathscr{T}^d$ and $\mathtt{E} \cup \mathtt{E}^+ = \mathtt{A}r$ iff (Theorem 2) $\mathtt{E} \in \succeq_{c,ub}$ and $\mathtt{E} \cup \mathtt{E}^+ = \mathtt{A}r$. □

The next lemmas are employed to prove Theorem 5:

Lemma 4. *Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF and* $\mathtt{E}, \mathtt{E}' \subseteq \mathtt{A}r$*. Then* $\mathtt{E} \succeq_p^{\mathfrak{T}} \mathtt{E}'$ *iff* $\mathtt{E} \succeq_p^{\mathfrak{T}^r} \mathtt{E}'$*.*

Proof. As $\mathtt{E} \in \mathtt{CF}(\mathfrak{T})$ iff $\mathtt{E} \in \mathtt{CF}(\mathfrak{T}^r)$, it is sufficient to consider the case where $\mathtt{E}, \mathtt{E}' \in \mathtt{CF}(\mathfrak{T})$. We have $\mathtt{E} \succeq_p^{\mathfrak{T}} \mathtt{E}'$ iff $\forall a \in \mathtt{E}, \forall b \in \mathtt{E}'$ if $((b,a) \in att$ and $a \not> b)$ or $((a,b) \in att$ and $b > a)$, then $\exists c \in \mathtt{E}$ such that $((c,b) \in att$ and $b \not> c)$ or $((b,c) \in att$ and $c > b)$ iff $\forall a \in \mathtt{E}, \forall b \in \mathtt{E}'$, if $(b,a) \in \mathscr{D}$ then $\exists c \in \mathtt{E}$ such that $(c,b) \in \mathscr{D}$ iff $\forall a \in \mathtt{E}, \forall b \in \mathtt{E}'$ if $((b,a) \in \mathscr{D}$ and $a \not> b)$ (or the impossible case where $(a,b) \in \mathscr{D}$ and $b > a)$ then $\exists c \in \mathtt{E}$ such that $((c,b) \in \mathscr{D}$ and $b \not> c)$ (or the impossible case where $(b,c) \in \mathscr{D}$ and $c > b)$ iff $\mathtt{E} \succeq_p^{\mathfrak{T}^r} \mathtt{E}'$. □

Lemma 5. *Let* $\mathfrak{T} = (\mathtt{A}r, att, \geq)$ *be a PAF and* $\mathtt{E} \subseteq \mathtt{A}r$*. We have* $\mathtt{E}$ *is a preferred extension of* $\mathscr{T}^d$ *iff* $\mathtt{E} \in \succeq_{p,max}^{\mathfrak{T}^r}$ *iff* $\mathtt{E} \in \succeq_{p,max}^{\mathfrak{T}}$*.*

Proof. We have $\mathtt{E}$ is a preferred extension of $\mathscr{T}^d$ iff (Theorem 3 from [1]) $\mathtt{E} \in \succeq_{p,max}^{\mathfrak{T}^r}$ iff (Lemma 4) $\mathtt{E} \in \succeq_{p,max}^{\mathfrak{T}}$. □

Theorem 5. *Let $\mathfrak{T} = (\mathtt{Ar}, att, \geq)$ be a PAF. A pref-complete extension* $\mathtt{E}$ *of* $\mathfrak{T}$ *is $\subseteq$-maximal among all* $\mathtt{E} \in \succeq_{c,ub}$ *iff* $\mathtt{E}$ *is a pref-preferred extension of* $\mathfrak{T}$.

Proof. Let $\mathscr{T}^d = (\mathtt{Ar}, \mathscr{D})$ be the corresponding argumentation framework to $\mathfrak{T}$. We have $\mathtt{E}$ is a $\subset$-maximal pref-complete extension in $\mathfrak{T}$ iff (Theorem 2) $\mathtt{E}$ is a $\subset$-maximal complete extension of $\mathscr{T}^d$ iff (according to [2]) $\mathtt{E}$ is a preferred extension of $\mathscr{T}^d$ iff (Lemma 5) $\mathtt{E}$ is a pref-preferred extension of $\mathfrak{T}$. □

5. Conclusion

The literature on preferences in argumentation is rich with different approaches, lacking consensus on a standard. The disagreement can be backtracked to a critical scenario where an attacked argument (in the sense of [2]) is deemed stronger or preferred over its attackers. Here, we contributed to the debate showing that a prominent approach, namely that of Amgoud and Vesic [1], retains the hierarchy of admissibility-based semantics established in [12]. This result is not straightforward, since [1] did not provide a preferential semantics corresponding to Dung's complete semantics, which is often considered the core AF semantics. For this reason, we started by defining the pref-complete semantics $\succeq_c$ for the Preference-based Argumentation Frameworks (*PAF*s) of [1]. Here, we associated the pref-complete extensions with the upper bounds of $\succeq_c$ and showed that it adequately generalizes Dung's complete semantics for *AF*s. The new semantics allowed us to establish a proper hierarchy among preferential semantics for *PAF*s from [1], showing they preserve the same subsumption relations as the *AF* semantics.

While there is a general agreement that conflict-freeness should be respected by the semantics of AFs with preferences, works such as [11,17,10] criticized the solution of [1]. In [11], Kaci et. al. propose that the attack (A,B) should be ignored only if it is symmetric, i.e., if B also attacks A, otherwise it should remain unchanged. This choice leaves room for an attack from a less preferred argument to still be successful, which is debatable. For comparison, the PAF $(\{A,B\}, \{(A,B)\}, \{(B,A)\})$, has the unique complete extension $\{A\}$ according to [11] and $\{B\}$ according to [1]. In [17], Wakaki ensures that extensions of a *PAF* $(\mathtt{Ar}, att, \geq)$ are extensions of its base *AF* $(\mathtt{Ar}, att)$. Instead of changing the attack relation, they simply select what extensions of *AF* respect the preferences. For comparison, the *PAF* $(\{A,B\}, \{(A,B),(B,A)\}, \{(B,A)\})$ has the complete extensions $\emptyset$ and $\{B\}$ according to [17] (notice $\emptyset$ is grounded) and only $\{B\}$ according to [1]. In [10], Modgil and Prakken focused on preferences in ASPIC$^+$ [18]. They argue the structure of arguments and the nature of attacks should be considered when applying preferences, adding more conditions to the reversal of the attacks that do not satisfy preferences.

Here, we do not advocate that Amgoud and Vesic's approach [1] would be the best available, but instead that the special cases of [11,17,10] are also worthwhile investigating. In our view, the divergences between them occur simply because they model different notions of preferences, each deserving attention on its own. In future works we will extend our investigation to verify whether other proposals of preference-based argumentation also preserve the semantic hierarchy observed among Dung's semantics. Another promising venture inspired by Wakaki's work [17] involves adapting other approaches of preferences from logic programming (see [19] for a survey) based on the mappings between abstract argumentation frameworks and logic programs found in [20].

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