

Remaining Useful Life Curve Prediction of Rolling Bearings Under Defect Progression Based on Hierarchical Bayesian Regression

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Abstract. In order to improve Remaining Useful Life (RUL) prediction accuracy for rolling bearings under defect progressing, robustness for individual difference and fluctuation of vibration features are challenging issues. In this research, we propose a novel RUL prediction method that uses a hierarchical Bayesian method to consider the individual difference of RUL, and uses an intermediate variable indicating the defect condition instead of predicting RUL directly from vibration features. The proposed method can perform a monotonous RUL prediction curve and improved prediction accuracy especially for early stage of defect progression.

1 INTRODUCTION

Rolling bearings are one of the essential mechanical elements in rotating machinery. In general, the Remaining Useful Life (RUL) of a rolling bearing is regarded as the operating time until some kind of defect occurs on the raceway surface. However, in situations where it is not easy to replace the rolling bearing or where maintenance costs are high, the bearing may be used even after defects have occurred. There is a need for a method to estimate the RUL under progressive defects.

There are two main approaches for predicting RUL of rolling bearings; Time Based Maintenance (TBM) and Condition Based Maintenance (CBM). TBM is based on the concept of performing maintenance on a time basis. For TBM, L_{10} life is generally used for RUL prediction of rolling bearings[5]. It is known that the RUL of rolling bearings varies widely[9]. Considering this variation, the L_{10} life is calculated based on Weibull distribution and it is defined as the total rotation cycles or total operating time that 10 % of the rolling bearings were damaged when many rolling bearings (same type) were operated under the same conditions. TBM may require rolling bearings to be replaced that are perfectly functional, or serious defects may occur before the periodic inspection, increasing the cost of maintenance.

On the other hand, CBM has recently attracted attention as a diagnostic method for rolling bearings. Traditional CBM for rolling bearings calculates the RUL by extracting the degradation index from the features of the vibration data and estimates the remaining operating time until the trend of the degradation index exceeds the

threshold[10]. In recent years, there are several research reports on CBM using vibration signals, foreign matter in the lubricant, as well as temperature and acoustic emissions for rolling bearings[6],[7]. Most of these methods target early stage defect progression. For a CBM under defect progression, two methods for estimating the defect state during operation have been proposed, by expressing the relationship between vibration and flaking surface area as a regression equation[8], and from vibration signals using a Self-Organizing Map[13]. In both cases, there is no mention of RUL based on the useful limit under defect progression.

In this research, we propose a RUL curve prediction method based on Hierarchical Bayesian Regression (HBR)[4] for rolling bearings under defect progression including late stage. The characteristics and advantages of this method are;

- By using a Bayesian Regression (BR) model that inputs circumferential defect length of the rolling bearing (defect size, ds) and outputs an RUL curve, a monotonic decrease in the RUL prediction is guaranteed.
- In order to predict RUL for a rolling bearing sample whose RUL are not known, the inputs to the BR model are estimated from vibration acceleration features using a regression model (pre-regression).
- Considering the individual differences of rolling bearings by hierarchizing the main-regression (HBR) model, the RUL prediction accuracy is improved especially for early stages of defect progression.

The rest of this paper is organized as follows. Section 2 introduces the background of this research. Section 3 describes related research. The details of the proposed RUL prediction method is presented in Section 4. Experimental conditions, evaluation methods and the results are described in Section 5. Future work and conclusions are in Sections 6 and 7 respectively.

2 PROBLEMS OF PREDICTING RUL UNDER DEFECT PROGRESSION

2.1 Defect Progression and Vibration Features

Defects (flaking) often occur on the raceway surface of the fixed ring (in this case, inner ring) of rolling bearings. If operation is continued after the initial defect occurs on the raceway surface, the defect expands in axial and circumferential directions. This causes vibration acceleration to increase as the defect progresses. In addition,

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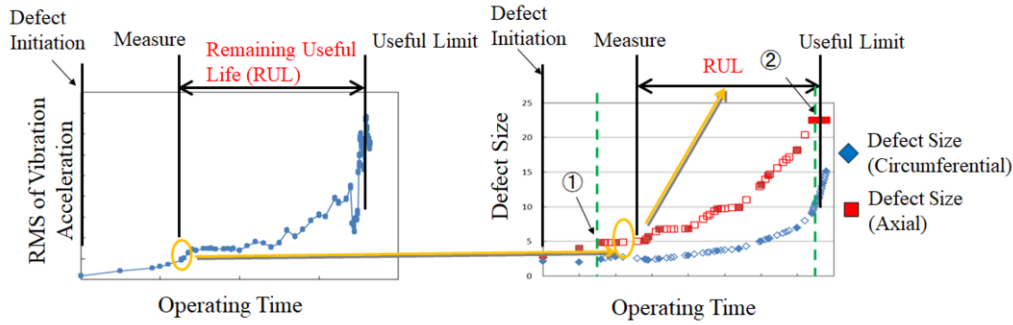


Figure 1: Relationship between operating time and vibration feature, defect size under damage progressing.

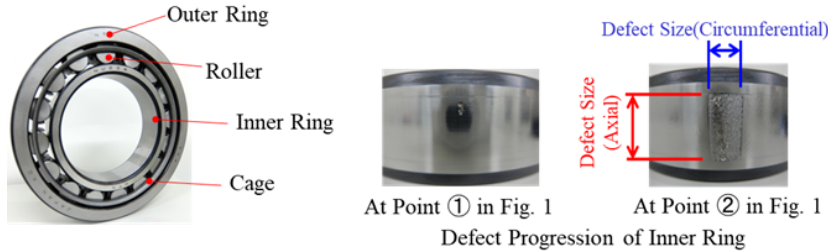


Figure 2: Overview of rolling bearing and defect shape on raceway surface of inner ring.

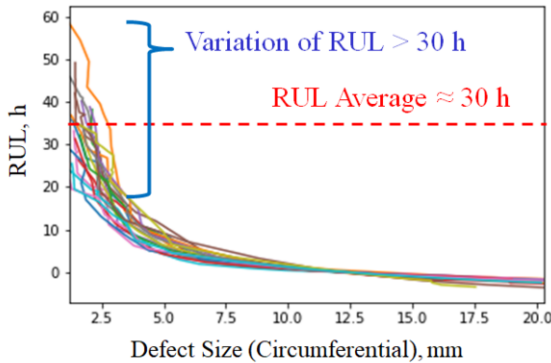


Figure 3: Variation of RUL among samples.

the upward trend of vibration varies depending on the defect state. Figure 1 shows the relationship between the operating time and Root-Mean-Square (RMS) value of vibration acceleration. It also shows the relationship between the operating time and defect length in the axial and circumferential directions. Figure 1 also shows the RUL from the measurement time to the operational limit. The operational limit is assumed to be the time when the defect size reaches 12 mm. Figure 2 shows an overview of the rolling bearing and the defect shape on the raceway of the inner ring. The right two images in Fig. 2 indicate the condition of defect progression at points ① and ② in Fig. 1. The RMS value is generally used for rolling bearing diagnostics[10], but it varies greatly around the end of defect progression, and it is difficult to accurately determine the condition of rolling bearings. On the other hand, the changes in axial and circumferential defect length is a suitable index for predicting the RUL because it shows a monotonous change with respect to operating time. However, it is difficult to measure the defect size while operating be-

cause it is necessary to stop or disassemble the equipment to measure them.

2.2 Variation of rolling bearing RUL

The RUL until a defect occurs has variation among the rolling bearings in general. We need to consider the same problem for predicting RUL under defect progression. Figure 3 shows the relationship between the defect size and RUL until the defect size reaches the specified length for 33 bearing samples. Despite that the multiple rolling bearings shown in Fig. 3 are all measured under the same operating conditions, the variation in RUL are larger than the average RUL. Therefore, an RUL prediction method considering variation is important for improving the prediction accuracy.

3 RELATED WORK

In recent years, diagnostic methods using machine learning have attracted attention for predicting RUL of rolling bearings. For example, methods based on deep learning, Deutsch et al. used deep belief networks and feed-forward networks to predict RUL considering the error of the predict value for each bearing[1]. Guo et al. used recurrent neural network based health indicators for predicting the RUL of rolling bearings[3]. On the other hand, as methods based on Bayesian estimation, Gebräel et al. and Zhou et al. predict the RUL using a BR models[2],[14]. In addition, Mishra et al. predicted the RUL considering the variation of rolling bearings by using HBR[12]. However, references [1], [2] and [14] are intended to cover a monotonic range of feature fluctuation in the relatively early stages of defects, and do not describe the RUL prediction accuracy under defect progression where the fluctuations of the features are unstable. References [1] and [3] do not describe variation of RUL of rolling bearings. Reference [12] predicted the RUL of rolling bearings that have been measured from the early to late stages of defect progression,

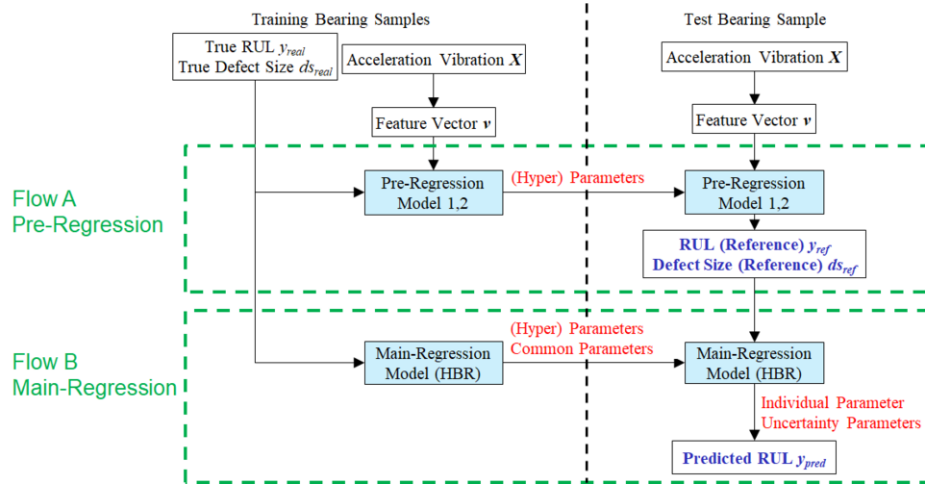


Figure 4: RUL prediction flow.

but does not describe the prediction of RUL for new rolling bearings. In the operation of condition monitoring systems in the field, it is necessary to predict the RUL of new rolling bearings in considering variation from measurement data at the initial stage. In this paper, the proposed method addresses this issue.

4 PROPOSED METHOD

4.1 Flow of the Proposed Method

In this research, the influence of fluctuations in vibration acceleration of rolling bearings under defect progression is suppressed by using defect size as an intermediate variable. In addition, by using HBR as a prediction method, we predict the RUL considering the variation of rolling bearings, and express the RUL curve based on a monotonous regression model. The regression model of the proposed method consists of common parameters, individual parameters and uncertainty parameters. Common parameters indicate invariant characteristics for all bearing samples, individual parameters indicate individual characteristics for each bearing sample, and uncertainty parameters indicate uncertainty of prediction result of RUL. Individual parameters have hyperparameters common for all bearing samples. Each parameter above is expressed as a probability distribution, and as a result, the RUL curve also has a probability distribution.

Figure 4 shows the outline of the RUL prediction flow of the proposed method. We assume that the training bearing samples contain data of vibration acceleration, defect size, and RUL at all measurement times. While, we assume that the test bearing sample has only vibration acceleration data until the measurement time. In the training phase, the main-regression model (HBR) uses the measured value of the defect size (ds_{real}) and the RUL (y_{real}) of the training bearing samples to determine the common parameters and hyperparameter of the individual parameter by the Monte Carlo Markov Chain (MCMC) algorithm. While in the test phase, the main-regression model uses reference values of the defect size (ds_{ref}) and the RUL (y_{ref}) until the measurement time, and the (hyper)parameters obtained above to estimate the individual parameter and uncertainty parameters for the test sample by the MCMC algorithm. Here, these reference values are predicted from the feature vector of vibration acceleration using the pre-regression. Once we estimate the individual parameter and uncertainty parameters for the test sample, we use them to

determine the RUL curve and its probability distribution. RUL reference values (result of pre-regression model) are snapshots and do not consider the continuity of the defect progression trend, so we aim to improve the prediction accuracy by guaranteeing monotonicity of the RUL decrease with the regression model of HBR.

4.2 The Hierarchical Bayesian Regression Model

Equations (1) to (8) show the RUL regression equations and the probability distributions given to each parameter of the regression equation in the proposed HBR model. Level 1 below shows the relationship between the defect size (ds) and the RUL (y_i) of the i^{th} bearing sample. Level 2 shows the probability distributions of the common parameters α and β , individual parameter δ_i for the i^{th} bearing sample, and parameters σ_y and ν_y which indicate uncertainty of the RUL. σ_y and ν_y are the scale and degrees of freedom of the Student's t-distribution, respectively. σ_δ is a hyperparameter of δ_i . And Level 3 shows the hyperprior distribution of σ_δ . δ_i and σ_δ are log-normal distributions since they do not take negative values. α , β are normal distributions. Student's t-distribution parameters σ_y , ν_y are half-Cauchy distribution and exponential distribution, respectively. Equations (3) to (8) indicate prior distributions of each parameter, and we estimate posterior distribution of the parameters by MCMC algorithm in the training or testing phase respectively.

The invariant characteristics for all bearing samples are expressed by α and β , and the individual characteristic for each bearing sample is expressed by δ_i . In addition, by giving σ_δ as a hyperparameter of δ_i and using HBR, the individual difference in the RUL can be expressed by one regression model.

1st Level

$$y_i \sim \text{StudentT}(\mu_i, \sigma_y, \nu_y) \quad (1)$$

$$\mu_i = \delta_i \left(\alpha + \frac{\beta}{ds} \right) \quad (2)$$

2nd Level

$$\alpha \sim \text{Normal}(0, 100) \quad (3)$$

$$\beta \sim \text{Normal}(0, 100) \quad (4)$$

$$\sigma_y \sim \text{HalfCauchy}(5) \quad (5)$$

$$\nu_y \sim \text{Exponential}(0.03) \quad (6)$$

$$\delta_i \sim \text{Lognormal}(0, \sigma_\delta) \quad (7)$$

3rd Level

$$\sigma_\delta \sim \text{Lognormal}(0, 100) \quad (8)$$

The steps for predicting RUL curve are as follows;

Step 1: (Training phase) Training of the pre-regression model

Pre-regression model 1 $f_{pre1}: v \mapsto ds$ is trained by using the feature vector (v) of the vibration acceleration (X) of the training bearing samples as input and the defect size ds of the training bearing samples as output. Pre-regression model 2 $f_{pre2}: v \mapsto y$ is trained by using v of the training bearing samples as input and the RUL (y_{real}) of the training bearing samples as output. The hyperparameters of pre-regression models 1 and 2 were chosen.

Step 2: (Training phase) Training of the main-regression model by using validation data

Using the defect size (ds_{real}) and the RUL (y_{real}) of the training bearing samples as input, the main-regression-model is trained and the posterior distribution of the common parameters and the hyperparameter of main-regression model is calculated by using Eqs. (1) to (8).

Step 3: (Test phase) Calculating ds_{ref} and y_{ref} using the pre-regression model

Using the hyperparameters selected in Step 1 and v of the test bearing sample, the reference value of the defect size (ds_{ref}) and the RUL (y_{ref}) is calculated by the pre-regression models 1 and 2.

Step 4: (Test phase) Calculating the posterior distribution of y_{pred} using the main-regression model

The individual parameter and the uncertainty parameters of the test sample are calculated using the posterior distribution of the common parameters and the hyperparameter of the main-regression model calculated in Step 2 and ds_{ref} and y_{ref} calculated in Step 3. Then, the posterior distribution of the RUL curve (y_{pred}) is estimated by using all parameters of the main-regression model.

5 EXPERIMENT AND RESULT

5.1 Experimental Conditions

Figure 5 shows a schematic diagram of the experimental equipment used to evaluate the RUL prediction accuracy, and Table 1 shows the main experimental conditions. Cylindrical roller bearings (Type: NU224) were used in the experiment, and the defect was considered to be the flaking on the raceway, which is the most common type of defect of rolling bearings.

Vibration acceleration (vertical and horizontal directions) and defect size were measured every 20 minutes on average for 33 bearing samples. Data from the bearing samples were measured from the normal condition until the defect progressed to the limit of operation, and the data after defect occurrence was used for evaluation. The time required for the defect to reach a specific size was taken as the reference point for RUL. The RUL was then determined by subtracting the operating time for each measurement from the reference. Examples of measurement data and defect conditions were shown in Fig. 1 to Fig. 3 in Section 2.

One measurement data of vibration acceleration of one direction $X^{(M)} = [x_1, x_2, \dots, x_j, \dots, x_N]$, $M \in \{\text{Vertical, Horizontal}\}$

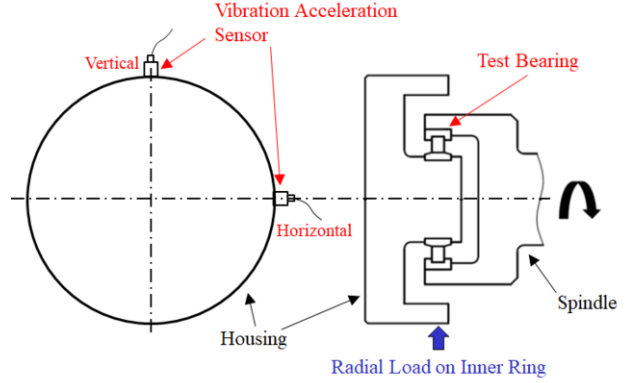


Figure 5: Test equipment.

was measured at a sampling frequency of 50 kHz and a sampling time of 20 seconds. Here, index j indicates the time series order, and x_j indicates the instantaneous value of vibration acceleration amplitude at index j .

Table 1: Operating condition.

Bearing	Cylindrical Roller Bearing (Type: NU224)
Rotation Speed	500 min ⁻¹
Radial Load	90 kN
Measurement	Vibration Acceleration (Vertical and Horizontal)
Number of Bearing Samples	33
Snapshot for each Sample	51 - 129

5.2 Feature Vector

From data measured in Section 5.1, we obtain the feature vector by band-pass filtering and calculating statistical features. For each measurement data $X^{(M)}$, amplitude data obtained by filtering in each frequency band (shown in Table 2) was used as time domain data $X^{(M, TIME)}$. Frequency domain amplitude data obtained by performing envelope[11] and FFT processing on the time domain data $X^{(M, TIME)}$ was used as frequency domain data $X^{(M, FREQ)}$. In addition, the amplitude data obtained by performing FFT processing again on the frequency domain data $X^{(M, FREQ)}$ was used as que-frency domain data $X^{(M, CEPS)}$. From $X^{(M, D)}$, $D \in \{\text{TIME, SPEC, CEPS}\}$, we calculated RMS, Max value (MAX), Crest Factor (CF), Kurtosis (KS), Skewness (SKN), which are often used for diagnostics of rolling bearings[3].

RMS, MAX, CF, KS, SKN are calculated as follows;

$$RMS^{(M, D)} = \sqrt{\frac{1}{n} \sum_{k=1}^n (X_k^{(M, D)})^2} \quad (9)$$

$$MAX^{(M, D)} = \max_{1 \leq k \leq n} X_k^{(M, D)} \quad (10)$$

$$CF^{(M, D)} = MAX^{(M, D)} / RMS^{(M, D)} \quad (11)$$

$$KS^{(M, D)} = \frac{1}{n} \sum_{k=1}^n \frac{(X_k^{(M, D)} - \bar{X}^{(M, D)})^4}{(\sigma^{(M, D)})^4} \quad (12)$$

$$SKN^{(M, D)} = \frac{1}{n} \sum_{k=1}^n \frac{(X_k^{(M, D)} - \bar{X}^{(M, D)})^3}{(\sigma^{(M, D)})^3} \quad (13)$$

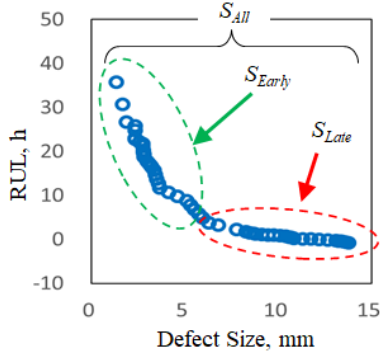


Figure 6: Stages for evaluating accuracy.

Here, M indicates the sensor measurement direction, D indicates domains, $X_k^{(M,D)}$ means index k of $\mathbf{X}^{(M,D)}$, $\bar{X}^{(M,D)}$ means average value of $\mathbf{X}^{(M,D)}$, $\sigma^{(M,D)}$ means standard deviation of $\mathbf{X}^{(M,D)}$. In addition to these features, we also calculated eRMS. eRMS is defined as RMS of enveloped signal of $\mathbf{X}^{(M,D)}$. Therefore, the feature vector consists of 252 features considering domains (time, frequency, quefrequency), filters (6 types of band-pass filter and w/o filter), sensor directions (vertical, horizontal), and statistics (6 types).

Table 2: Kind of band-pass filter.

Filter	Frequency Range(Hz)
Raw	None
Low1	20-200
Low2	20-1000
Mid1	200-2000
Mid2	1000-5000
High1	2000-20000
High2	5000-20000

5.3 Evaluation Method

The coefficient of determination (R^2) was used as an evaluation index of prediction accuracy. R^2 was calculated for each bearing sample, and average and variation of R^2 for all bearing samples were evaluated by leave-one (bearing)-out cross-validation. So we can assume that higher value and less variation of R^2 , the better prediction result. Figure 6 shows a schematic diagram of the evaluation stage of prediction accuracy. The evaluation of defect progression is divided into entire stage (S_{All}), early stage (S_{Early}), and late stage (S_{Late}). The early and late stages of defect progression were defined based on the change point of the defect progression speed in each bearing sample.

5.4 Preliminary experiment: Estimating ds_{ref} and y_{ref}

Figure 7 shows R^2 of ds_{ref} by various regression methods of Kernel Ridge (KR), Random Forest (RF), Support Vector Regression (SVR), Neural Network consists of four hidden layers (DNN) when using feature vectors as input. And Figure 8 shows R^2 of y_{ref} by same methods as Fig. 7. Hyperparameters for each method were selected with the smallest mean-square-error of prediction result by five-fold cross-validation in training. In Fig. 7, R^2 at the early stage of RF and SVR are higher than KR and DNN. Average of R^2 at the late stage

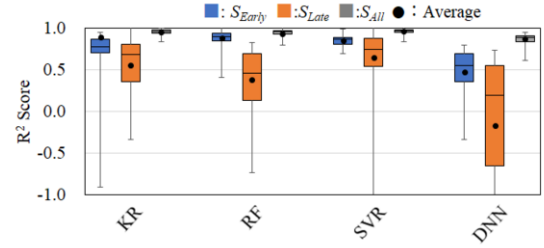


Figure 7: Prediction accuracy for ds (pre-regression model 1 at flow A of Fig. 4).

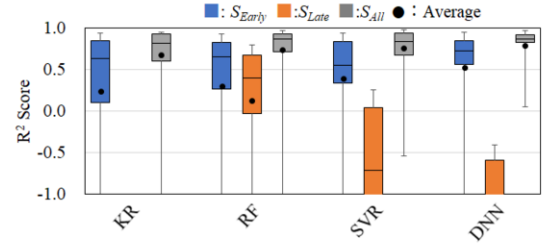


Figure 8: Prediction accuracy for RUL(pre-regression model 2 at flow A of Fig. 4).

of SVR is higher than that of RF, but the variation of SVR is larger than that of RF. In Fig. 8, each method shows large variations in R^2 at the early stage, and R^2 at the late stage was less than 0 in almost all methods. Since only R^2 average of RF exceeded 0 at late stage of RUL prediction, we adopted RF for the pre-regression model of RUL for the proposed method. We also adopted RF for the pre-regression model of defect size in the following section because we obtained the most stable result when using RF for pre-regression models of both defect size and RUL in the proposed method.

5.5 Evaluation of Proposed Method

5.5.1 Relationship between Defect Progress and RUL Curve

Figure 9 shows the relationship between defect progression and the RUL curve predicted by the proposed method. In Fig. 9, the blue plot indicates the measured value and the red plot indicates the true value. Black line indicates the mean curve of the prediction result, dark gray and light gray areas are 50 % and 95 % credible intervals. As the measured data increased depending on the defect progression of the test sample, the mean curve approaches the true value, and the credible interval becomes narrow. By using HBR, the RUL can be expressed as a probability distribution, and as the measurement proceeds, the prediction accuracy of the RUL can be increased along with a reduction of interval.

5.5.2 Distribution of individual parameter δ_i

Figure 10 shows an example of the estimated results of the distribution of the individual parameters δ_i of each bearing sample using

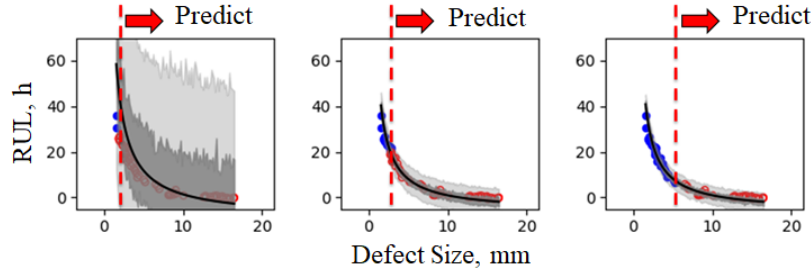


Figure 9: Relation between damage progression and RUL posterior distribution.

Blue plot is the measured value and red plot is the true value.

Black line is the mean curve of prediction result, dark gray and light gray areas are 50% and 95% credible intervals.

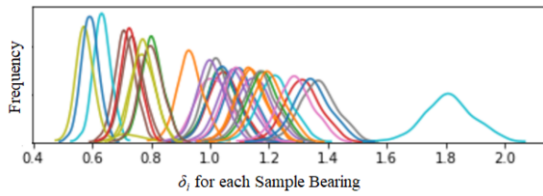


Figure 10: Distribution (KDE plot) of δ_i .

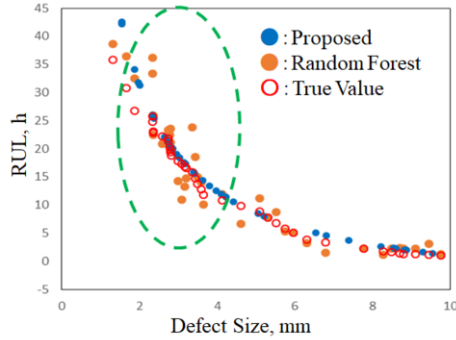


Figure 11: Comparison of regression result.

Kernel Density Estimation (KDE). It can be confirmed that the difference in the RUL of each bearing is expressed as a difference of the distribution. In addition, the mode value of each distribution has a maximum difference of about 3 times, which is almost same as the difference of RUL for each bearing in Fig. 3.

5.5.3 Prediction Accuracy of Proposed Method

Figure 11 shows the comparison of RUL predictions between the proposed method and results of pre-regression with RF. The proposed method can obtain monotonous prediction results and can reduce the possibility of misdiagnoses especially in the early stage (green circled area in Fig. 11).

Figure 12 shows a box plot of the R^2 score of the proposed method. Table 3 shows the average, median, min, max and standard deviation of R^2 . The result of the proposed method was compared with Single Layer BR instead of HBR. The result of RF (same as Fig. 8, result of flow A in Fig. 4) is also compared. In Table 3, the

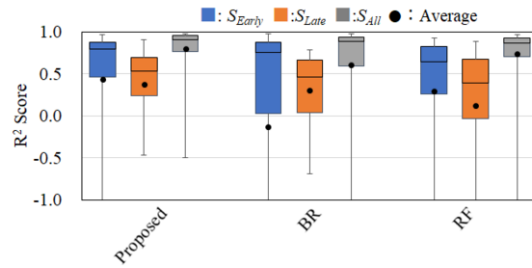


Figure 12: Prediction accuracy for RUL (R^2 score).

Table 3: R^2 score of each method.

		RF	BR	Proposed
Average	S_{Early}	0.300	-0.129	0.436
	S_{Late}	0.122	0.312	0.381
	S_{All}	0.739	0.610	0.800
Median	S_{Early}	0.650	0.755	0.800
	S_{Late}	0.393	0.465	0.539
	S_{All}	0.866	0.886	0.912
Min	S_{Early}	-5.618	-11.582	-3.852
	S_{Late}	-3.385	-0.692	-0.467
	S_{All}	-1.089	-2.957	-0.500
Max	S_{Early}	0.910	0.979	0.966
	S_{Late}	0.789	0.875	0.960
	S_{All}	0.959	0.989	0.983
Standard Deviation	S_{Early}	1.174	2.289	0.934
	S_{Late}	0.917	0.473	0.431
	S_{All}	0.373	0.724	0.293

highest score for average, median, min, max and the lowest score for standard deviation of each stage is indicated in bold. In BR, we use almost the same regression model as the HBR described in section 4.1. The difference of BR instead of HBR is that we assume the common parameter (δ), which has only one distribution for all bearing samples instead of the individual parameter (δ_i), and we do not assume hyperprior distribution for BR model. By using the proposed method, the prediction accuracy in S_{All} and S_{Early} was increased, and the prediction accuracy in the S_{Late} was improved compared to RF. In addition, because standard deviation of the proposed method was decreased in all stages compared to BR, the proposed method can increase the prediction accuracy over different bearing samples by hierarchizing BR model. However, the variation of prediction accuracy of R^2 at the S_{Early} is still large and needs to be improved.

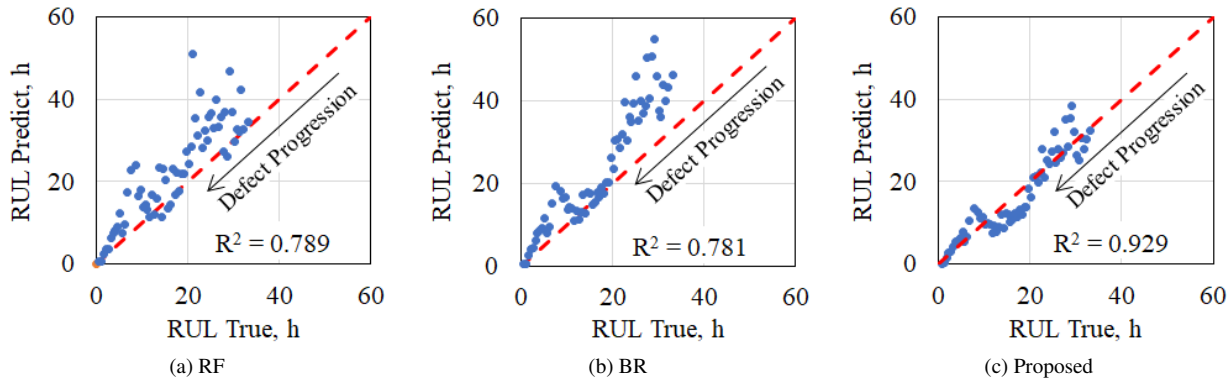


Figure 13: Correlation between predicted value and true value of RUL.

Figures 13 (a) to Figure 13 (c) show examples of the comparison of the predicted value and the true value of RUL by RF, BR, and the proposed method for the same bearing sample. The result of BR has less variation than the result of RF, but the predicted values are larger than the true value for the entire stage, because the individual difference cannot be considered in BR method. In comparison, the proposed method reduces the difference between predicted value and true value by considering individual differences, and as a result, R^2 of the proposed method improved.

6 FUTURE WORK

In this study, by using HBR as a RUL prediction method and using defect size as an intermediate variable, the prediction accuracy for early stage defect progression of the proposed method was improved. However, for some bearing samples, variation of prediction accuracy in the early stage is still large in the proposed method, which may cause misdiagnoses. This is because the reference values of the defect size and the RUL are used as input for HBR, and it is assumed that the estimation accuracy of these reference values is poor. We are currently taking countermeasures.

7 CONCLUSION

For predicting the RUL of rolling bearings under defect progression, we tried to improve prediction accuracy by introducing Hierarchical Bayesian Regression and using defect size as an intermediate variable. The findings obtained in this study are shown below.

- By using Bayesian method as a regression method, the RUL can be evaluated with credible intervals, and prediction accuracy can be increased with each measurement.
- The accuracy of the proposed method is improved especially for the early stage of defect progression in some cases.
- The proposed method can obtain monotonous prediction results and reduce the possibility of misdiagnoses especially in the early stage.
- Because variation of prediction accuracy in the early stage of defect progression is still large in the proposed method, it is necessary to improve in the future.

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