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Towards an Imprecise Probability Approach for Abstract Argumentation[1]

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Abstract. In some abstract argumentation framework (AAF), arguments have a degree of uncertainty, which impacts on the degree of uncertainty of the extensions obtained under a semantics. In these approaches, both the uncertainty of the arguments and of the extensions are modeled by means of precise probability values. However, in many real life situations the exact probabilities values are unknown and sometimes there is a need for aggregating the probability values of different sources. In this paper, we tackle the problem of calculating the degree of uncertainty of the extensions considering that the probability values of the arguments are imprecise.

1 Introduction

The AAF approach that was introduced in the seminal paper of Dung [1] is one of the most significant developments in the computational modelling of argumentation in recent years. The AAF is composed of a set of arguments and a binary relation encoding attacks between arguments. Some recent approaches on abstract argumentation assign uncertainty to the elements of the AAF to represent the degree of believe on arguments or attacks (e.g., [2][5][6]). These works use precise probability approaches to model the uncertainty values. However, precise probability approaches have some limitations to quantify epistemic uncertainty, for example, to represent group disagreeing opinions. These can be better represented by means of imprecise probabilities, which use lower and upper bounds instead of exact values to model the uncertainty values.

Consider a discussion between a group of medicine students (agents). The discussion is about the diagnosis of a patient. In this context, arguments represent the student's opinions and the attacks represent the disagreements between such opinions. Figure 1 shows the argumentation graph where nodes represent arguments and edges the attacks between arguments.

To the best of our knowledge, there is no work that models the uncertainty values of arguments by using an imprecise probability approach. Therefore, we aim to propose an approach for abstract argumentation in which the uncertainty of the arguments is modeled by an imprecise probability value. Thus, the research questions that are addressed in this paper are:

1. How to model the imprecise uncertainty values of arguments?
2. In abstract argumentation, several semantics have been proposed, which return sets of arguments – called extensions – whose basic characteristic is that these arguments do not attack each other, i.e. they are consistent. The fact that the arguments that belong to an extension are uncertain, causes that such extension also has a degree of uncertainty. How to calculate the lower and upper bounds of extensions?

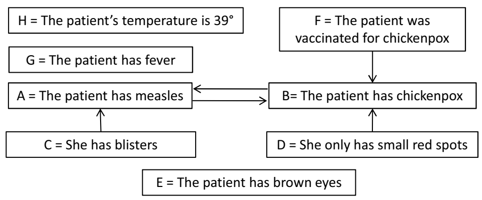

Figure 1. Argumentation graph for the discussion about a diagnose.

In addressing the first question, we use credal sets to model the uncertainty values of arguments. Let $\mathbb{E} = \{E_1, ..., E_n\}$ be a finite set of events and p a probability distribution on $\mathbb{E}$, where p is a mapping $p : \mathbb{E} \to [0, 1]$. A closed convex set of probability distributions p is called a credal set [3]. A credal set for an event E is denoted $K(E)$ and $\mathbb{K} = \{K(E_1), ..., K(E_n)\}$ denotes a set of all credal sets. We assume that the cardinality of every credal sets is the same (denoted by m), we also assume that $p_i(E)$ denotes the suggested probability of agent i w.r.t the event E such that $1 \leq i \leq m$ and $E \in \mathbb{E}$.

Regarding the second question, we base on the credal sets of the arguments to calculate the uncertainty values of extensions obtained under a given semantics. These values are represented by lower and upper bounds. The way to aggregate the credal sets depends on a causal relation between the arguments. For the calculations, we take into account the following equations:

(1) Given a credal set $K(E)$, the lower and upper bounds for event E are $\underline{P}(E) = inf\{p(E) : p(E) \in K(E)\}$ and $\overline{P}(E) = sup\{p(E) : p(E) \in K(E)\}$, respectively.

(2) Given l events $\{E_1, ..., E_l\} \subseteq \mathbb{E}$ and their respective credal sets $K(E_1) = \{p_1(E_1), ..., p_m(E_1)\}, ..., K(E_l) = \{p_1(E_l), ..., p_m(E_l)\}$. If $\{E_1, ..., E_l\}$ are independent events, the lower and upper probabilities are $\underline{P}(\{E_1, ..., E_l\}) = min_{1\leq j\leq m}\{\prod_{i=1}^{i\leq l} p_j(E_i)\}$, respectively, where $p_j \in K(E_i)$ $\overline{P}(\{E_1, ..., E_l\}) = max_{1\leq j\leq m}\{\prod_{i=1}^{i\leq l} p_j(E_i)\}$.

(3) On the other hand, when the independence relation is not assumed, the first step is to calculate a credal set for $\{E_1, ..., E_l\}$: $K(\{E_1, ..., E_l\}) = \{p_E | p_E = min_{1\leq j\leq m}\{p_j(E_1), ...p_j(E_l)\}\}$ where $p_j(E_i) \in K(E_i)$.

(4) Based on $K(\{E_1, ..., E_l\})$, we obtain the lower and upper bounds: $\underline{P}(\{E_1, ..., E_l\}) = min(K(\{E_1, ..., E_l\}))$ and $\overline{P}(\{E_1, ..., E_l\}) = max(K(\{E_1, ..., E_l\}))$.

[1] This is an abstract version of the full article originally presented at the 15th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU'19) [4].

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2 Proposal

Each argument in an AAF has associated a credal set, which contains probability distributions that represent the opinions of the agents about it.

Definition 1. An **AAF based on credal sets** is a tuple $\mathcal{AF}_{\mathcal{CS}} = \langle \texttt{ARG}, \mathcal{R}, \mathbb{K}, f_{\mathcal{CS}} \rangle$ where (i) $\texttt{ARG}$ is a set of arguments, (ii) $\mathcal{R}$ is a binary relation $\mathcal{R} \subseteq \texttt{ARG} \times \texttt{ARG}$ that represents the attack between two arguments of $\texttt{ARG}$, so that $(A, B) \in \mathcal{R}$ denotes that the argument A attacks the argument B, (iii) $\mathbb{K}$ is a set of credal sets, and (iv) $f_{\mathcal{CS}} : \texttt{ARG} \rightarrow \mathbb{K}$ maps a credal set for each argument in $\texttt{ARG}$.

Recall that the cardinality of every credal set depends on the number of agents, which have an opinion about each event.

Definition 2. Let $\mathcal{AF}_{\mathcal{CS}} = \langle \texttt{ARG}, \mathcal{R}, \mathbb{K}, f_{\mathcal{CS}} \rangle$ be a Credal AAF and $\mathbb{AGT} = \{ag_1, ..., ag_m\}$ a set of agents. The **opinion** p_i **of an agent** ag_i (for $1 \leq i \leq m$) is ruled as follows:

1. If $A \in \texttt{ARG}$, there is $p_i(A) \in K(A)$ where $K(A) \in \mathbb{K}$.
2. $\forall A \in \texttt{ARG}$, $0 \leq p_i(A) \leq 1$.

Example 1. Consider that $\mathbb{AGT} = \{ag_1, ag_2, ag_3, ag_4\}$. The Credal AAF for the example given in Introduction is $\mathcal{AF}_{\mathcal{CS}} = \langle \texttt{ARG}, \mathcal{R}, \mathbb{K}, f_{\mathcal{CS}} \rangle$ where: $\texttt{ARG} = \{A, B, C, D, E, F, G.H\}$, $\mathcal{R} = \{(A, B), (B, A), (F, B), (D, B), (C, A)\}$, $\mathbb{K} = \{K(A), ..., K(H)\}$, and $f_{\mathcal{CS}}(A) = K(A)$, $f_{\mathcal{CS}}(B) = K(B)$, ..., $f_{\mathcal{CS}}(H) = K(H)$. The table below shows the credal set of each argument:

	K(A)	K(B)	K(C)	K(D)	K(E)	K(F)	K(G)	K(H)
p_1	0.2	0.8	0.2	0.75	0.8	0.75	0.7	0.8
p_2	0.7	0.25	0.75	0.15	0.65	0.2	0.8	0.9
p_3	0.55	0.45	0.4	0.5	0.8	0.55	1	1
p_4	0.75	0.1	0.2	0.8	0.7	0.8	0.9	0.9

In a Credal AAF, besides the attack relation between the arguments, there may be a causality relation between them.

Definition 3. Let $\mathcal{AF}_{\mathcal{CS}} = \langle \texttt{ARG}, \mathcal{R}, \mathbb{K}, f_{\mathcal{CS}} \rangle$ be a Credal AAF, **a causality graph** $\mathbb{C}$ is a tuple $\mathbb{C} = \langle \texttt{ARG}, \mathcal{R}_{\texttt{CAU}} \rangle$ such that:
(i) $\texttt{ARG} = \texttt{ARG}_{\leftarrow} \cup \texttt{ARG}_{\rightarrow} \cup \texttt{ARG}_{\circ}$ is a set of arguments,
(ii) $\mathcal{R}_{\texttt{CAU}} \subseteq \texttt{ARG} \times \texttt{ARG}$ represents a causal relation between two arguments of $\texttt{ARG}$ (the existence of this relation depends on the domain knowledge), such that $(A, B) \in \mathcal{R}_{\texttt{CAU}}$ denotes that argument A causes argument B. It holds that if $(A, B) \in \mathcal{R}$, then $(A, B) \notin \mathcal{R}_{\texttt{CAU}}$ and $(B, A) \notin \mathcal{R}_{\texttt{CAU}}$,
(iii) $\texttt{ARG}_{\leftarrow} = \{B | (A, B) \in \mathcal{R}_{\texttt{CAU}}\}$, $\texttt{ARG}_{\rightarrow} = \{A | (A, B) \in \mathcal{R}_{\texttt{CAU}}\}$, and $\texttt{ARG}_{\circ} = \{C | C \in \texttt{ARG} - (\texttt{ARG}_{\leftarrow} \cup \texttt{ARG}_{\rightarrow})\}$,
(iv) $\texttt{ARG}_{\leftarrow}$ and $\texttt{ARG}_{\rightarrow}$ are not necessarily pairwise disjoint; however, $(\texttt{ARG}_{\leftarrow} \cup \texttt{ARG}_{\rightarrow}) \cap \texttt{ARG}_{\circ} = \emptyset$.

Example 2. A causality graph for the Credal AAF of Example 1 is $\mathbb{C} = \langle \{A, B, C, D, E, F, G, H\}, \{(D, A), (F, A), (H, A), (G, A), (H, G), (G, B), (C, B)\} \rangle$ (see Figure 2), where $\texttt{ARG}_{\leftarrow} = \{A, B, G\}$, $\texttt{ARG}_{\rightarrow} = \{D, F, H, G, C\}$, and $\texttt{ARG}_{\circ} = \{E\}$.

Considering the causality graph, the arguments of an extension $\mathcal{E}$ may belong to $\texttt{ARG}_{\rightarrow}$, $\texttt{ARG}_{\leftarrow}$, or $\texttt{ARG}_{\circ}$. Depending on it, the calculation of the probabilistic lower and upper bounds of each extension is different. Thus, we can distinguish the following cases: (i) the extension is empty, (ii) the extension has only one argument, and (iii) the extension includes more than one argument.

Definition 4. (Upper and Lower Bounds of Extensions) Let $\mathcal{AF}_{\mathcal{CS}} = \langle \texttt{ARG}, \mathcal{R}, \mathbb{K}, f_{\mathcal{CS}} \rangle$ be a Credal AAF, $\mathbb{C} = \langle \texttt{ARG}, \mathcal{R}_{\texttt{CAU}} \rangle$ a causality graph, and $\mathcal{E} \subseteq \texttt{ARG}$ an extension under any semantics. The lower and uppers bounds of $\mathcal{E}$ are obtained as follows:
1. If $\mathcal{E} = \{\}$, then $\underline{P}(\mathcal{E}) = 0$ and $\overline{P}(\mathcal{E}) = 1$, which denotes ignorance.
2. If $|\mathcal{E}| = 1$, then $\underline{P}(\mathcal{E}) = \underline{P}(A)$ and $\overline{P}(\mathcal{E}) = \overline{P}(A)$ s.t. $A \in \mathcal{E}$, where $\underline{P}(A)$ and $\overline{P}(A)$ are obtained by applying Equation (1).
3. If $|\mathcal{E}| > 1$, we apply an algorithm[4] for calculating $(\underline{P}(\mathcal{E}), \overline{P}(\mathcal{E}))$. This algorithm takes as input the extension $\mathcal{E}$ and the causality graph and returns the upper and lower bounds. The main idea is to find subsets of arguments that are dependent and apply Equation (3) to calculate a unique credal set for each subset. If there is only one subset then it is applied Equation (4) for obtaining the upper and lower bounds; otherwise, it is applied Equation (2).

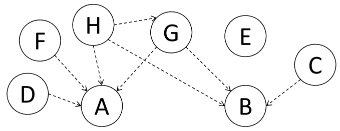

Figure 2. Causality graph. Traced edges represent the causality relation.

3 Conclusions and future work

This work presents an approach for abstract argumentation under imprecise probability. We defined a credal AAF, in which credal sets are used to model the the uncertainty values of the arguments, which correspond to opinions of a set of agents about their degree of believe about each argument. We have considered that – besides the attack relation – there also exists a causality relation between the arguments of a credal AAF. Based on the credal sets and the causality relation, the lower and upper bounds of the extensions – obtained from a semantics – are calculated.

So far, we have calculated the lower and upper bounds of extensions obtained under a given semantics. The next step is to compare these bounds in order to determine an ordering over the extensions, which can be used to choose an extension that resolves the problem.

Some properties were studied in the full version of the article; however, it is necessary a more complete analysis and study of the properties of the approach. We also plan to further study the causality relations, more specifically in the context of credal networks. Finally, we want to study the relation of this approach with bipolar argumentation frameworks.

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[4] Due to space restriction, the algorithm will not be presented. However, the reader can find it in [4].