

# On the Relation Between Claim-Augmented Argumentation Frameworks and Collective Attacks

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**Abstract.** Dung’s abstract argumentation frameworks (AFs) are a popular conceptual tool to define semantics for advanced argumentation formalisms. Hereby, arguments representing a possible inference of a claim are constructed and an attack relation between arguments indicates certain conflicts between the claim of one argument and the inference of another. Based on this abstract model, sets of jointly acceptable arguments are then gathered and finally interpreted in terms of their claims. Argumentation formalisms following this type of instantiating Dung AFs naturally produce several arguments with the same claim. This causes several issues and challenges for argumentation systems: on the one hand, the relation between claims remains implicit and, on the other hand, determining the acceptance of claims requires additional computations on top of argument acceptance. An instantiation that avoids this situation could provide additional insights and advantages, thus complementing the standard instantiation process via Dung AFs. Consequently, the research question we tackle is as follows: Can one combine different arguments sharing the same claim to a single abstract argument without affecting the overall results (and which abstract formalisms can serve such a purpose)? As a main result we show that a certain class of frameworks, where arguments with the same claim have the same outgoing attacks, can be equivalently (for all standard semantics) represented as argumentation frameworks with collective attacks where each claim occurs in exactly one argument. We further identify a class of frameworks where one even obtains an equivalent Dung AF with just one argument per claim.

## 1 Introduction

The formal analysis of human reasoning facing uncertain information and conflicting beliefs is an important research area within AI. Abstract argumentation, as introduced by Dung [16], has been established as an important tool to analyze and evaluate the structure of argumentation systems by treating arguments as abstract entities. Depending on the particular task, various instantiation processes are used to model discourses, medical and legal cases [4], but also logic programs and non-monotonic reasoning formalisms [16, 13].

The general schema is often referred to as the argumentation pipeline and involves: (1) instantiation of a problem (which is given in terms of a knowledge base) into an abstract argumentation framework (AF); (2) application of semantics yielding sets of collectively acceptable arguments (the extensions of the AF); (3) re-interpretation of the extensions in terms of the original problem. Different instantiation processes have been established, see e.g. [23, 27, 13]. They all have in common that each generated argument stands for a statement

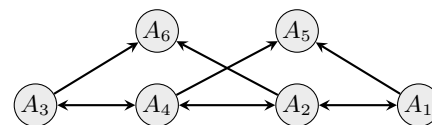


Figure 1: Resulting AF from Example 1.

(claim) that follows from elements of the initial knowledge base. Step (3) hence usually consists of inspecting the claims of the arguments occurring in the extensions; for instance, one might ask whether a particular claim is justified in each extension of the AF which is the case if each extension contains an argument featuring that claim.

**Example 1.** We consider an instantiation procedure using  $ASPIC^+$  [27]. Let  $\mathcal{K}_p = \{b, \bar{b}, c, \bar{c}\}$  be the set of premises,  $\mathcal{K}_s = \{\bar{b} \rightarrow a, \bar{c} \rightarrow a\}$  be the set of strict rules and let the pairs  $(b, \bar{b})$ ,  $(c, \bar{c})$ ,  $(\bar{b}, \bar{c})$  be contradictory. We instantiate this knowledge base into a Dung AF which consists of abstract arguments and an attack from argument  $A$  to argument  $B$  is present if the claim of  $A$  is contradictory to the support of  $B$ . The arguments are listed in the table below and the resulting Dung AF is depicted in Figure 1.

Argument	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$
Structure	$b$	$\bar{b}$	$c$	$\bar{c}$	$A_2 \rightarrow a$	$A_4 \rightarrow a$
Claim	$b$	$\bar{b}$	$c$	$\bar{c}$	$a$	$a$

The evaluation of this AF under stable semantics<sup>2</sup> yields the sets  $\{A_1, A_3\}$ ,  $\{A_2, A_3, A_5\}$  and  $\{A_1, A_4, A_6\}$ . The re-interpretation in terms of claims gives us the sets  $\{b, c\}$ ,  $\{a, \bar{b}, c\}$  and  $\{a, b, \bar{c}\}$ . The arguments  $A_5$  and  $A_6$  both refer to the same claim  $a$  and while the second and third extensions are disjoint on their arguments they are not on their claims. That is, multiple arguments with the same claim can lead to subtle differences in reasoning about claim acceptance. It is thus a natural question whether or not we can avoid multiple occurrences of claims in the abstract arguments.

In this paper we are interested under which conditions an instantiation procedure can be modified such that it results in an abstract representation that requires only one argument per claim and that is equivalent to the standard attack-model. Some clarifications are in order: (1) When we talk about *abstract representations*, we mean any formalism that abstracts away from the contents of the arguments, and allows for an evaluation that is solely depending on the relations between arguments. We have already seen the simple Dung AFs at work. A generalization which we use in this paper are SETAFs (AFs with collective attacks [26]), where arguments can be jointly attacked

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<sup>2</sup> A set  $S$  of arguments is stable if it attacks exactly those arguments which do not belong to  $S$ .

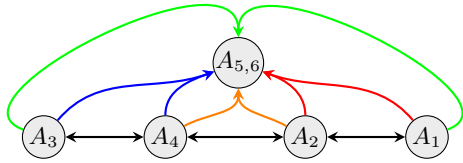


Figure 2: Example 1 expressed as SETAF.

by other arguments. (2) The term *equivalent* refers to the fact that two abstractions deliver the same result when their extensions are interpreted in terms of their claims.

While undoubtedly there are good reasons to keep several arguments with the same claim in the abstract representation (in particular, to preserve the different lines of support for a specific claim), there are also advantages of having only one argument per claim on the abstract level:

- In a representation with only one argument per claim we have an explicit representation of the conflict relation between the claims, which remains implicit otherwise.
- In abstract representations with several arguments per claim we need to add an additional layer in argumentation systems to obtain the acceptance status of the claims from the acceptance status of the arguments. This increases not only the complexity of argumentation systems in practice; recent results also show higher computational complexity of certain tasks [21] compared to the standard acceptance problems on the argument level.
- When allowing for several arguments with same claim, the number of arguments might be exponentially larger than in abstract representations with only one argument per claim, e.g. in the instantiations for LPs discussed below. Again, this might have significant impact on computational aspects.

To exemplify the research question, we show in Figure 2 a SETAF that fulfills above criteria for Example 1. Hereby, arguments  $A_1$  to  $A_4$  have the same claims as the Dung AF; argument  $A_{5,6}$  combines the two arguments  $A_5$  and  $A_6$  with claim  $a$ . A collective attack  $(\{A_1, A_2\}, A_{5,6})$  is interpreted as neither  $A_1$  nor  $A_2$  attacks  $A_{5,6}$  but when both are considered together they jointly attack  $A_{5,6}$ . It can be shown that the stable extensions of this SETAF are equivalent to the stable extensions of the Dung AF in Example 1 when interpreted in terms of claims but duplicate claims are avoided. Notice, that this cannot be accomplished with Dung AFs as one would need an AF such that the stable extensions are given by the sets  $\{b, c\}$ ,  $\{a, \bar{b}, c\}$  and  $\{a, b, \bar{c}\}$ . However, such an AF does not exist as follows from results on expressiveness of AF semantics by Dunne *et al.* [17]. Notice, that the attack  $(\{A_1, A_3\}, A_{5,6})$  in Figure 2 reflects the fact that the three statements  $\{a, b, c\}$  are in conflict although there is no pairwise conflict. Also observe that this relation remains implicit in Figure 1.

The right choice of the abstract representation used in an instantiation procedure of an advanced argumentation formalism is indeed crucial. For instance, instantiations from logic programs (LPs) are a prominent example where instantiations to different abstract representations have been considered in the literature. Several instantiations from LPs into Dung AFs have been proposed, e.g. the instantiation from [13]. The instantiation typically constructs an AF with multiple arguments supporting the same claim. This causes obstacles when instantiating LPs with L-stable semantics into Dung AFs, i.e. there is no abstract argumentation semantics that mirrors L-stable semantics. Moreover, also the skeptical reasoning (a.k.a. cautious reasoning) of LPs does not map to the skeptical reasoning of AFs [21]. In order to overcome the obstacles with L-stable semantics, Alcan-

tara *et al.* [1] recently proposed an alternative instantiation procedure that results in a specific class of ADFs (which in the light of [25] can also be interpreted as SETAFs) that maintains the existing equivalences between LP and Dung AF semantics and avoids duplicates of claims. This one-to-one correspondence between arguments and claims, i.e. the atoms of the LP, resolves the above mentioned obstacle of instantiating LPs with L-stable semantics when using Dung AFs. Moreover, we further have that skeptical acceptance of an atom in a LP now directly corresponds to the skeptical acceptance of an argument in the corresponding SETAF. In our paper we investigate such matters independent from concrete advanced argumentation formalisms.

In order to base our studies independently from concrete instantiations, we utilize the concept of *claim-augmented argumentation frameworks*, CAFs for short, as introduced by Dvořák and Woltran [21]. CAFs are AFs where arguments are associated with claims and semantics deliver those sets of claims attached to the arguments in the extensions under standard AF semantics. CAFs provide a natural intermediate layer between structured and purely abstract argumentation formalisms, since they carry the necessary information to compute the extensions and at the same time re-interpret them in terms of the instantiated problem. This is exactly the level of detail we require for our investigations. Our research question now amounts to the question of translating CAFs into purely abstract formalisms such that duplicate claims are avoided, hence allowing for a one-to-one correspondence between argument names and their claims. We will focus on the most common semantics, namely preferred, stable, complete, admissible and grounded semantics [16]. A crucial notion for our purpose relies on the following restriction on the attack relation: A CAF is *well-formed* if arguments with the same claim attack the same arguments (the AF from Example 1 is indeed well-formed). This constraint naturally occurs in instantiation processes: attacks are usually generated under consideration of the claim of the attacker.<sup>3</sup>

Translating CAFs into an abstract representation without an additional claim layer allows to investigate the capabilities of abstract argumentation formalisms from different viewpoints. (1) Several approaches to extend Dung AFs have been discussed in the literature. However, the question on their advantages in the instantiation process has often been neglected. Our research targets this question by investigating whether such generalizations are capable of representing scenarios not directly expressible via Dung AFs. Our main focus here will be on SETAFs, which have received significant interest in the last years [22, 32, 18]. (2) From a theoretical perspective, we study under which circumstances different arguments  $S_1 : c, \dots, S_n : c$  (with the same claim but different support) and their relation to other arguments can be combined into a single argument  $\bigvee_i S_i : c$  such that this amalgamation can still be represented on the abstract level.

The main results of our paper are:

- We show that well-formed CAFs can be equivalently expressed via SETAFs where the argument names in the SETAF refer to the claims in the CAF, thus providing a single argument for each claim (in fact, we show that well-formed CAFs and SETAFs are equally powerful w.r.t. the semantics under consideration). This result demonstrates that *all* instantiation procedures that result in Dung structures where the arguments' claims imply well-formedness can be defined in terms of SETAFs where each argument stands for a different claim.

<sup>3</sup> Exceptions are instantiation procedures which allow rule and claim preferences (cf. ASPIC<sup>+</sup>).

- We complement the above results by showing that each SETAF can be equivalently represented by a well-formed CAF. It thus follows that SETAFs and well-formed CAFs have the same expressiveness.
- We strengthen the result for well-formed CAFs by characterizing a class of CAFs, so-called attack-unitary CAFs, that, under admissibility-based semantics, can be transformed to an equivalent standard Dung AF.
- We give a rewriting technique for CAFs in order to reduce the number of arguments with the same claim, even in case when a full translation to SETAFs or AFs is not possible.
- Finally, we show that for the classes of well-formed CAFs and attack-unitary CAFs only one attack violating the respective property can lead to a situation where a translation to SETAFs (resp. AFs) is not possible anymore.

Omitted technical details are available in a technical report [20].

## 2 Preliminaries

In this section, we first introduce standard abstract argumentation frameworks [16] and recall the semantics we study (for a comprehensive introduction, see [5]). We then introduce their generalization to collective attacks.

**Dung AFs.** We start with some basic definitions.

**Definition 1.** A Dung argumentation framework (AF) is a pair  $F = (A, R)$  where  $A$  is a finite set of arguments and  $R \subseteq A \times A$  is the attack relation. The pair  $(a, b) \in R$  means that  $a$  attacks  $b$ . Given an argument  $a$ , we use  $a_R^+ = \{b \mid (a, b) \in R\}$  and  $a_R^- = \{b \mid (b, a) \in R\}$ ; we extend both notions to sets  $S$  as expected:  $S_R^+ = \bigcup_{a \in S} a_R^+$ ,  $S_R^- = \bigcup_{a \in S} a_R^-$ . We say that an argument  $a \in A$  is defended (in  $F$ ) by  $S \subseteq A$  if  $a_R^- \subseteq S_R^+$ .

Semantics for AFs are defined as functions  $\sigma$  which assign to each AF  $F = (A, R)$  a set  $\sigma(F) \subseteq 2^A$  of extensions. We consider for  $\sigma$  the functions  $cf$ ,  $adm$ ,  $com$ ,  $grd$ ,  $stb$ , and  $prf$ , which stand for conflict-free, admissible, complete, grounded, stable, and preferred extensions, respectively.

**Definition 2.** Let  $F = (A, R)$  be an AF. A set  $S \subseteq A$  is conflict-free (in  $F$ ), if there are no  $a, b \in S$ , such that  $(a, b) \in R$ .  $cf(F)$  denotes the collection of sets being conflict-free in  $F$ . For a conflict-free set  $S \in cf(F)$ , it holds that

- $S \in adm(F)$ , if each  $a \in S$  is defended by  $S$  in  $F$ ;
- $S \in com(F)$ , if  $S \in adm(F)$  and each  $a \in A$  defended by  $S$  in  $F$  is contained in  $S$ ;
- $S \in grd(F)$ , if  $S = \bigcap_{T \in com(F)} T$ ;
- $S \in stb(F)$ , if each  $a \in A \setminus S$  is attacked by  $S$  in  $F$ ;
- $S \in prf(F)$ , if  $S \in adm(F)$  and there is no  $T \supset S$  such that  $T \in adm(F)$ .

We recall that for each AF  $F$ ,  $stb(F) \subseteq prf(F) \subseteq com(F) \subseteq adm(F)$ , and  $grd(F)$  yields a unique extension. Moreover, semantics  $\sigma \in \{stb, prf\}$  deliver incomparable sets, i.e. for all  $S, T \in \sigma(F)$ ,  $S \subseteq T$  implies  $S = T$ .

**SETAFs and Collective Attacks.** SETAFs, as introduced by Nielsen and Parsons [26], generalize the binary attack-relation in AFs to collective attacks of arguments. The formalism captures situations in which a single argument might be too weak to attack more powerful statements.

**Definition 3.** A SETAF is a pair  $SF = (A, R)$  where  $A$  finite, and  $R \subseteq (2^A \setminus \{\emptyset\}) \times A$  is the attack relation.

Given a SETAF  $SF = (A, R)$ , then  $S \subseteq A$  attacks  $a$  if there is a set  $S' \subseteq S$  with  $(S', a) \in R$ .  $S$  is conflicting in  $SF$  if  $S$  attacks some  $a \in S$ ;  $S$  is conflict-free in  $SF$ , if  $S$  is not conflicting in  $SF$ , i.e.  $S' \cup \{a\} \not\subseteq S$  for each  $(S', a) \in R$ . Finally,  $a \in A$  is defended by  $S$  if for each set  $B \subseteq A$  with  $(B, a) \in R$ , there is some  $b \in B$  such that  $S$  attacks  $b$ . With these extended notions of conflict and defense at hand, the semantics of AFs generalize to SETAFs as follows.

**Definition 4.** Given a SETAF  $SF = (A, R)$ , we denote the set of all conflict-free sets in  $SF$  as  $cf_s(SF)$ . For  $S \in cf_s(SF)$ , it holds that

- $S \in adm_s(SF)$  if each  $a \in S$  is defended by  $S$  in  $SF$ ;
- $S \in com_s(SF)$ , if  $S \in adm_s(SF)$  and  $a \in S$  for all  $a \in A$  defended by  $S$  in  $SF$ ;
- $S \in grd_s(SF)$ , if  $S = \bigcap_{T \in com_s(SF)} T$ ;
- $S \in stb_s(SF)$ , if each  $a \in A \setminus S$  is attacked by  $S$  in  $SF$ ;
- $S \in prf_s(SF)$ , if  $S \in adm_s(SF)$  and there is no  $T \supset S$  such that  $T \in adm_s(SF)$ .

We introduce attack formulas as an alternative formalisation of the attack-structure in SETAFs.

**Definition 5.** For any SETAF  $SF = (A, R)$  and  $a \in A$ , let

$$\mathcal{D}_a^{SF} = \bigvee_{B \subseteq A, (B, a) \in R} \bigwedge_{b \in B} b$$

denote the attack-formula of  $a$  in  $SF$ .

For each  $s \in A$ , the models of the attack-formula  $\mathcal{D}_s^{SF}$  coincide with the sets  $S \subseteq A$  such that  $S$  attacks  $s$  in  $SF$ . Using this identity, the semantics for SETAFs can be rephrased in terms of attack-formulas. For example, a set  $S$  is stable in  $SF$ , if for each  $s \in A$ , we have that  $s \in S$  if and only if  $\delta \not\subseteq S$  for all  $\delta \in \mathcal{D}_s^{SF}$  (following standard conventions, we will occasionally identify formulas in CNF or DNF as a collection of sets of literals; in our case, atoms).

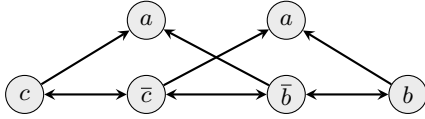
## 3 Argumentation Frameworks with Claims

In order to abstract from a concrete instantiation we use an augmentation of Dung AFs that allows for a uniform representation of the outcome of a wide variety of instantiation procedures. That is we consider claim-augmented argumentation frameworks (CAF) as recently introduced by Dvořák and Woltran [21]. The idea is to assign each argument in an AF a claim, i.e. an element from a countable infinite domain of claims  $C$ . Hence, different arguments can have the same claim, but no further information about claims is available. Notice that CAFs provide exactly the necessary information to compute the extensions and re-interpret them in terms of the instantiated problem.

**Definition 6.** A Claim-augmented Argumentation Framework (CAF) is a triple  $(A, R, claim)$  where  $(A, R)$  is an AF and  $claim: A \rightarrow C$  assigns a claim to each argument of  $A$ .

Figure 3 shows the CAF for Example 1. Notice that CAFs cannot only model the outcome of ASPIC style instantiations but is also applicable to e.g. instantiations from logic programming [21] and assumption-based argumentation (ABA).

Semantics of CAFs are defined from the standard semantics of the underlying AF, but interpret the extensions in terms of the claims of their arguments. To this end, we extend the claim function to sets, i.e.  $claim(S) = \{claim(s) \mid s \in S\}$ .



**Figure 3:** The AF  $F$  from Example 1 as CAF; here, claims are depicted instead of argument names.

**Definition 7.** For a semantics  $\sigma$ , we define its claim-based variant  $\sigma_c$  as follows. For any CAF  $CF = (A, R, \text{claim})$ ,  $\sigma_c(CF) = \{\text{claim}(S) \mid S \in \sigma((A, R))\}$ . Given  $S \in \sigma_c(CF)$ , we say that  $E \subseteq A$  is a  $\sigma$ -realization of  $S$  in  $CF$  if  $\text{claim}(E) = S$  and  $E \in \sigma((A, R))$ .

**Example 2.** Let  $CF = (A, R, \text{claim})$  be given with  $(A, R)$  as depicted in Figure 4, and  $\text{claim}(x_i) = x$ ,  $\text{claim}(y_i) = y$  for  $i = 1, 2$ . We have  $\text{com}_c(CF) = \text{adm}_c(CF) = \{\emptyset, \{x\}, \{x, y\}\}$ ,  $\text{grd}_c(CF) = \{\emptyset\}$ , and  $\text{stb}_c(CF) = \text{prf}_c(CF) = \{\{x\}, \{x, y\}\}$ . Note that  $\{x\} \in \text{adm}_c(CF)$  has two adm-realizations, namely  $E_1 = \{x_1\}$  and  $E_2 = \{x_2\}$ .



**Figure 4:** The AF from Example 2.

Some basic relations between different semantics carry over from standard AFs. In fact, we have for any CAF  $CF$

$$\text{stb}_c(CF) \subseteq \text{prf}_c(CF) \subseteq \text{com}_c(CF) \subseteq \text{adm}_c(CF) \quad (1)$$

and  $\text{grd}_c(CF)$  is unique and contained in  $\text{com}_c(CF)$ . Moreover, the claim-based grounded extension  $S \in \text{grd}_c(CF)$  is still the unique minimal claim-based complete extension.

As Example 2 shows, claim-based semantics lose some basic properties, for instance incomparability of stable and preferred extensions. However, this can be circumvented with a particular subclass called *well-formed CAFs* which has been defined in [21]; a similar property has been studied in [23, 3].

**Definition 8.** A CAF  $(A, R, \text{claim})$  is called well-formed if  $a_R^+ = b_R^+$  for any  $a, b \in A$  with  $\text{claim}(a) = \text{claim}(b)$ , i.e. arguments with the same claim attack the same arguments.

**Proposition 1.** For any well-formed CAF  $CF$ ,  $S \in \text{prf}_c(CF)$  iff  $S \in \text{com}_c(CF)$  and there is no  $T \in \text{com}_c(CF)$  with  $T \supset S$ .

*Proof.* Consider a well-formed CAF  $CF = (A, R, \text{claim})$ . We show that for  $D, E \in \text{com}((A, R))$ ,  $D \subseteq E$  iff  $\text{claim}(D) \subseteq \text{claim}(E)$ . The assertion then follows immediately. As  $\text{claim}(\cdot)$  is monotone we have that  $D \subseteq E$  implies  $\text{claim}(D) \subseteq \text{claim}(E)$ . We next show the converse, i.e. that  $\text{claim}(D) \subseteq \text{claim}(E)$  implies  $D \subseteq E$ . As  $CF$  is well-formed,  $\text{claim}(D) \subseteq \text{claim}(E)$  implies  $D_R^+ \subseteq E_R^+$ . That is all arguments defended by  $D$  in  $(A, R)$  are also defended by  $E$  in  $(A, R)$ . Finally as  $D$  defends all its arguments and  $E$  contains all arguments it defends we have  $D \subseteq E$ .  $\square$

Notice that the above result generalises similar observations for instantiations from logic programming [13] and ABA [12], which both result in well-formed CAFs. The following statement follows from Proposition 1 and (1).

**Proposition 2.** For  $\sigma \in \{\text{stb}, \text{prf}\}$  and every well-formed CAF  $CF = (A, R, \text{claim})$ , we have (a)  $\sigma_c(CF)$  is incomparable, and (b)  $|\sigma((A, R))| = |\sigma_c(CF)|$ .

## 4 Well-formed CAFs and SETAFs

Our aim is to translate CAFs to argumentation frameworks such that each claim that appears in the CAF corresponds to exactly one argument in the argumentation framework. Thus we do not allow additional auxiliary arguments that lack a counterpart in their claims with respect to the original domain. The two main results of this section will show that CAFs and SETAFs can be translated to each other while preserving the outcome in terms of claims. This shows that in the abstract representation of an instantiation procedure one can trade multiple claims for collective attacks and vice versa.

### 4.1 Expressing Well-formed CAFs as SETAFs

We show that each well-formed CAF can be reduced to an equivalent SETAF by identifying claims in well-formed CAFs with arguments in SETAFs. In well-formed CAFs, arguments with the same claim are indistinguishable in terms of their outgoing attacks. Hence, in contrast to general CAFs, one can speak about claims attacking arguments. We will introduce attack formulas for each claim  $c$ . These formulas intuitively capture all possible sets of claims which jointly contradict each occurrence of claim  $c$ .

**Definition 9.** Given a well-formed CAF  $CF = (A, R, \text{claim})$ , then for each claim  $c \in \text{claim}(A)$ , the CNF-attack-formula of  $c$  in  $CF$  is defined as

$$\mathcal{CD}_c^{CF} = \bigwedge_{a \in A, \text{claim}(a)=c} \bigvee_{(x,a) \in R} \text{claim}(x).$$

$\mathcal{D}_c^{CF}$  denotes any equivalent DNF-formula over the same set of variables and is called DNF-attack-formula of  $c$  in  $CF$ .

Note that the attack-formula  $\mathcal{CD}_c^{CF}$  is unsatisfiable iff there exists an argument  $x$  in  $CF$  with  $\text{claim}(x) = c$  such that  $x_R^- = \emptyset$ .

Similarly to SETAFs, attack formulas allow for an exact characterization of well-formed CAFs, i.e. each well-formed CAF  $CF = (A, R, \text{claim})$  is uniquely determined (modulo argument names) via its attack formulas  $\mathcal{CD}_c^{CF}$ .

**Example 3.** Consider the following CNF-attack formulas:

$$\begin{aligned} \mathcal{CD}_a^{CF} &= (c \vee \bar{b}) \wedge (\bar{c} \vee b) \\ \mathcal{CD}_b^{CF} &= \bar{b} & \mathcal{CD}_c^{CF} &= \bar{c} \\ \mathcal{CD}_{\bar{b}}^{CF} &= b \vee \bar{c} & \mathcal{CD}_{\bar{c}}^{CF} &= \bar{b} \vee c \end{aligned}$$

The two conjuncts of  $\mathcal{CD}_a^{CF}$  determine that we have two arguments with claim  $a$ , while the remaining claims appear only once. The disjunctions refer to the attackers of the arguments. In fact, we obtain the CAF as depicted in Figure 3.

We introduce the translation  $T_{cts}$ , which maps each well-formed CAF  $CF$  to a corresponding SETAF  $T_{cts}(CF)$ . Each claim  $c$  in the original framework  $CF$  corresponds to an argument in  $T_{cts}(CF)$ , furthermore we identify each disjunct of the DNF-attack-formula  $\mathcal{D}_c^{CF}$  with a collective attack against  $c$ . Consequently, the formula  $\mathcal{D}_c^{CF}$  coincides with the attack-formula  $\mathcal{D}_{cts}^{T_{cts}(CF)}$  of the resulting SETAF  $T_{cts}(CF)$ . Therefore, the SETAF  $T_{cts}(CF)$  solely depends on the DNF-attack-formulas  $\mathcal{D}_c^{CF}$ .

**Translation 1.** For a well-formed CAF  $CF = (A, R, \text{claim})$  we define  $T_{cts}(CF) = (A', R')$  with  $A' = \text{claim}(A)$  and

$$R' = \{(\delta, c) \mid c \in A', \delta \in \mathcal{D}_c^{CF}\}.$$

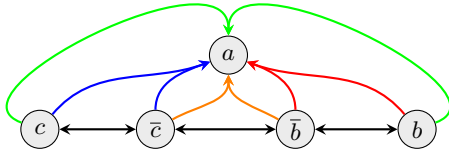


Figure 5: SETAF  $T_{cts}(CF)$  from Example 4.

**Example 4.** In Example 3 we have already provided the attack formulas for the CAF  $CF$  depicted in Figure 3. The attack formula  $CD_a^{CF}$  for claim  $a$  in DNF representation yields  $\mathcal{D}_a^{CF} = (c \wedge \bar{c}) \vee (c \wedge b) \vee (\bar{b} \wedge \bar{c}) \vee (\bar{b} \wedge b)$ . The attack formulas for the remaining claims are readily given in DNF. Applying Translation 1 yields the SETAF given in Figure 5: for  $a$ , we need a collective attack for each disjunct in  $\mathcal{D}_a^{CF}$ ; for the remaining arguments, each disjunct contains one atom, thus the incoming attacks remain binary.

Notice that the translation links multiple occurrences of a claim with collective attacks on a corresponding single argument. Therefore, it interlinks claim-based extensions of CAFs with extensions (on the argument level) of SETAFs. As we show next, this is performed in a faithful way, that is, the reviewed semantics of well-formed CAFs can be reduced to their counterparts in SETAFs.

**Theorem 1.** For each well-formed CAF  $CF$  and  $\sigma \in \{cf, adm, com, grd, prf, stb\}$ , Translation 1 provides an equivalent SETAF  $T_{cts}(CF)$  such that  $\sigma_c(CF) = \sigma_s(T_{cts}(CF))$ .

*Proof* (for  $\sigma \in \{cf, stb\}$ ). First, let  $\sigma = cf$ . We rephrase conflict-freeness in terms of CNF- resp. DNF-attack-formulas. In well-formed CAFs,  $S \in cf_c(CF)$  iff for each  $s \in S$  there is an  $a \in A$  with  $claim(a) = s$  such that  $a$  is not attacked by any argument  $b$  with  $claim(b) \in S$ . Note that, for any  $s \in claim(A)$ ,  $CD_s^{CF}$  identifies each clause with the set of attacking claims for a particular occurrence of  $s$  in  $CF$ . That is,  $S \in cf_c(CF)$  iff for each  $s \in S$ ,

$$\text{there is some } \gamma \in CD_s^{CF} \text{ such that } \gamma \cap S = \emptyset. \quad (C1)$$

In a SETAF  $SF = (A', R')$ , a set  $S \subseteq A'$  is conflict-free iff for all  $S' \subseteq S$  and all  $s \in S$ ,  $(S', s) \notin R'$ . In terms of attack formulas,  $S \in cf_s(SF)$  iff for each  $s \in S$ , it holds that

$$\text{for all } \delta \in \mathcal{D}_s^{SF} \text{ we have } \delta \not\subseteq S. \quad (C2)$$

Now let  $SF = T_{cts}(CF) = (A', R')$ . We have (i)  $\mathcal{D}_s^{SF} = \mathcal{D}_s^{CF}$  for each  $s \in A'$  by construction, and (ii) that no  $\delta \in \mathcal{D}_s^{CF}$  is a subset of  $S$  iff there exists  $\gamma \in CD_c^{CF}$  such that  $\gamma \cap S = \emptyset$ , and thus we obtain that (C1) is equivalent to (C2), hence the statement follows.

Let  $\sigma = stb$ . A set  $S$  is stable on claim-level in  $CF$  if for each  $s \in claim(A)$ , it holds that  $s \in S$  if and only if (C1). Similarly,  $S$  is stable in  $T_{cts}(CF)$  if for each  $s \in A' = claim(A)$ , it holds that  $s \in S$  if and only if (C2). Again, the statement follows by the equivalence of (C2) and (C1).  $\square$

The result shows that multiple occurrences of claims in CAFs can be equivalently treated as collective attacks, if the framework satisfies well-formedness. Indeed, in our running example (cf. Figure 5), the stable extensions  $stbs(T_{cts}(CF))$  are given by the sets  $\{a, \bar{b}, c\}$ ,  $\{a, b, \bar{c}\}$  and  $\{b, c\}$ .

A natural question that arises is about the contents the arguments in the resulting SETAF are representing with respect to an initial instantiation. To this end, let us have one more look on the example from the introduction, where argument  $A_5$  for claim  $a$  builds

on a support consisting of  $\bar{b}$  and rule  $\bar{b} \rightarrow a$  while argument  $A_6$  has support  $\bar{c}$ ;  $\bar{c} \rightarrow a$ . The combined argument  $A_{5,6}$  (see Figure 2) thus can be interpreted as argument for  $a$  with *disjunctive* support  $(\bar{b}; \bar{b} \rightarrow a) \vee (\bar{c}; \bar{c} \rightarrow a)$ . Now, in order to attack this argument we need to find combinations of arguments that are contradictory to each of the disjuncts. Recall that the contradictory relation is given by  $\{(b, \bar{b}), (c, \bar{c}), (\bar{b}, \bar{c})\}$  and arguments  $A_1, A_2, A_3, A_4$  have as their respective claims  $b, \bar{b}, c, \bar{c}$ . We thus need together either  $A_1$  and  $A_3$ ,  $A_1$  and  $A_4$ ,  $A_2$  and  $A_3$ , or  $A_2$  and  $A_4$  to attack  $A_{5,6}$ . This is exactly the collective attack structure in Figure 2 and likewise the result of our translation which delivers this combination by applying the logical rule of distributivity to the CNF-attack-formula  $CD_a^{CF}$  of claim  $a$  in order to obtain the corresponding attack-formula  $\mathcal{D}_a^{SF}$  of argument  $a$  for constructing the SETAF.<sup>4</sup>

## 4.2 Expressing SETAFs as well-formed CAFs

Our next result shows that it is equally possible to map each SETAF to a well-formed CAF while preserving the reviewed semantics. That is, we show that well-formed CAFs and SETAF have the same expressiveness and one can easily translate between them.

We will provide a translation  $T_{stc}$  which maps each SETAF  $SF$  to an equivalent well-formed CAF using attack formulas. Each argument  $a$  will correspond to a claim in the resulting CAF; furthermore, we introduce for each clause  $\gamma$  in the attack formula  $CD_a^{SF}$  an argument  $a_\gamma$  labeled with claim  $a$ . The clause  $\gamma$  also determines the set of attackers of the argument  $a_\gamma$ .

**Translation 2.** For each SETAF  $SF = (A', R')$ , we define  $T_{stc}(SF) = (A, R, claim)$  as follows:

$$\begin{aligned} A &= \{a_\gamma \mid a \in A', \gamma \in CD_a^{SF}\} \cup \{a_\emptyset \mid a \in A', CD_a^{SF} = \emptyset\}, \\ R &= \{(x, a_\gamma) \mid a \in A', \gamma \in CD_a^{SF}, claim(x) \in \gamma\}, \\ claim(a_\gamma) &= a. \end{aligned}$$

**Example 5.** Let  $SF = (A', R')$  be a SETAF as in Figure 5. In order to apply the translation  $T_{stc}$  we first compute the attack formulas of the arguments in  $SF$ . For instance, the attack formula  $\mathcal{D}_a^{SF}$  for the argument  $a \in A'$  is given by  $(c \wedge \bar{c}) \vee (b \wedge \bar{b}) \vee (\bar{b} \wedge \bar{c}) \vee (b \wedge c)$  where every disjunct represents a set which attacks  $a$ . The corresponding CNF attack formula is  $CD_a^{SF} = (c \vee \bar{b}) \wedge (\bar{c} \vee b)$ . For the remaining arguments, the CNF and DNF representation coincides. Once every attack formula is constructed, one can apply Translation 2, where the arguments in the SETAF  $SF$  correspond to the claims in the CAF  $T_{stc}(SF)$ : For  $a$ , we introduce two arguments  $a_{c \vee \bar{b}}, a_{\bar{c} \vee b}$  with  $claim(a_{c \vee \bar{b}}) = claim(a_{\bar{c} \vee b}) = a$ ; the attacks are determined by the claims which appear in the clauses, e.g. every argument  $x$  with  $claim(x) = c$  attacks the argument  $a_{c \vee \bar{b}}$ . The resulting CAF  $T_{stc}(SF)$  corresponds to the CAF depicted in Figure 3.

By applying similar techniques as in the proof of Theorem 1, it can be shown that the reviewed semantics for SETAFs correspond to their counterparts in well-formed CAFs.

**Theorem 2.** For each SETAF  $SF$  and  $\sigma \in \{cf, adm, com, grd, prf, stb\}$ , Translation 2 provides an equivalent CAF  $T_{stc}(SF)$  such that  $\sigma_s(SF) = \sigma_c(T_{stc}(SF))$ .

Thus by the Theorems 1 and 2, we conclude that well-formed CAFs and SETAFs are equally powerful with respect to the semantics under consideration.

<sup>4</sup> Recall that in our translation we use in the SETAF claims anonymously as argument names.

**Corollary 1.** *Let  $\sigma \in \{cf, adm, com, grd, prf, stb\}$ . For any well-formed CAF  $CF$ , there is a SETAF  $SF$  such that  $\sigma_c(CF) = \sigma_s(SF)$ , and vice versa.*

Given the exact characterizations of the expressiveness of semantics of SETAFs in [18] the above translation results immediately provide exact characterizations of the expressiveness of the respective semantics of well-formed CAFs.

## 5 Classes of CAFs Expressible as AFs

In this section we investigate classes of CAFs that can be directly expressed by AFs. Prominently, one can show that for well-formed CAFs  $(A, R, claim)$  where the attack relation  $R$  is symmetric there is an equivalent Dung AF. This type of CAFs naturally occurs in instantiations with only rebutting attacks (see [30] for a discussion of different types of attacks) which is reflected by work on symmetric AFs [15]. However, as we show next, a weaker condition on the attack structure is already sufficient.

**Definition 10.** *A CAF  $(A, R, claim)$  is called attacker-unitary (att-unitary) if, for any  $a, b \in A$  with  $claim(a) = claim(b)$ , it holds that  $a_R^- = b_R^-$ , i.e. arguments with the same claim are attacked by the same arguments.*

The crucial property is that in att-unitary CAFs, a set of arguments  $E$  defends either all or no occurrences of a claim  $c$ . Thus, for admissible-based semantics, each claim-extension  $S$  can be realized by a maximal representative. That is, the maximal representative of a claim-extension  $S$  is given by the set  $E_S^{max}$  containing all arguments with a claim from  $S$ .

**Lemma 1.** *Let  $CF = (A, R, claim)$  be att-unitary and let  $S \in \sigma_c(CF)$  for  $\sigma \in \{adm, com, grd, prf, stb\}$ . Then  $E_S^{max} = \{x \in A \mid claim(x) \in S\} \in \sigma((A, R))$ .*

Next, we present a translation  $T_{cta}$  from att-unitary CAFs to AFs. Given CAF  $CF = (A, R, claim)$ , each claim  $c \in claim(A)$  is mapped to a single argument  $c$  in the resulting AF  $T_{cta}(CF)$ , wherein  $c$  attacks  $d$  if at least one argument with claim  $c$  attacks the arguments with claim  $d$  in  $(A, R)$ .

**Translation 3.** *For an att-unitary CAF  $CF = (A, R, claim)$ , we define  $T_{cta}(CF) = (claim(A), R')$  with*

$$R' = \{(claim(x), claim(y)) \mid (x, y) \in R\}.$$

**Example 6.** *Let  $CF' = (A, R, claim)$  be an att-unitary CAF, with  $A = \{x_1, y_1, x_2\}$ ,  $R = \{(x_1, y_1), (y_1, x_1), (y_1, x_2)\}$  and  $claim(x_1) = claim(x_2) = x$ ,  $claim(y_1) = y$ . The AF  $T_{cta}(CF')$  constructed by Translation 3 is given by  $(\{x, y\}, \{(x, y), (y, x)\})$  and we have  $stb_c(CF') = stb_s(T_{cta}(CF')) = \{\{x\}, \{y\}\}$ .*

We next show that  $CF$  and  $T_{cta}(CF)$  are equivalent for the admissible-based semantics under our considerations.

**Theorem 3.** *For each att-unitary CAF  $CF$  and  $\sigma \in \{adm, com, grd, prf, stb\}$ , Translation 3 provides an equivalent AF  $T_{cta}(CF)$  such that  $\sigma_c(CF) = \sigma_s(T_{cta}(CF))$ .*

*Proof (for  $\sigma = stb$ ).* Let  $CF = (A, R, claim)$ ,  $F = T_{cta}(CF) = (A', R')$  and consider  $S \in stb_c(CF)$ . By Lemma 1,  $E_S^{max} \in stb((A, R))$ . Note that  $S \in cf(F)$  by definition of  $R'$ . For each  $c \in A' \setminus S$  there is an  $x \in A \setminus E_S^{max}$  with  $claim(x) = c$

and there is  $y \in E_S^{max}$  such that  $(y, x) \in R$ . Consequently  $(claim(y), claim(x)) \in R'$ , and  $S$  attacks in  $F$  all arguments  $a \in A' \setminus S$ .

Now, let  $S \in stb(F)$  and let  $c \in A' \setminus S$ . Then there is some argument  $a \in S$  such that  $(a, c) \in R'$ . By definition of  $R'$ , there are arguments  $x, y \in A$ ,  $claim(x) = a$ ,  $claim(y) = c$ , such that  $(x, y) \in R$ . By att-unitarity,  $(x, z) \in R$  for each  $z \in A$  such that  $claim(z) = c$ . Hence each argument  $y \in A \setminus E_S^{max}$  is attacked by  $E_S^{max}$ .  $\square$

Notice that Theorem 3 does not extend to *cf* semantics. For conflict-free semantics well-formed and att-unitary CAFs are inter-translatable, as the orientation of attacks is immaterial. Thus, under conflict-free semantics att-unitary CAFs correspond to SETAFs.

## 6 General CAFs

This section is concerned with CAFs neither satisfying well-formedness nor att-unitarity. We show that in general one cannot avoid multiple occurrences of claims. However, we provide a result that allows to reduce the number of arguments having the same claim. On the one hand we show that there is no translation from general CAFs into SETAFs. Moreover, already a small deviation of the respective properties of being well-formed or att-unitary can prohibit a translation to SETAFs or AFs, respectively. On the other hand, we provide a result to reduce the number of arguments having the same claim, even in the case that a translation to a SETAF or an AF is impossible.

We first investigate whether the introduced translation  $T_{cts}$  from well-formed CAFs to SETAFs can be extended to arbitrary CAFs that only slightly deviate from well-formedness. The following example shows that this is not the case by providing an example where already removing one attack suffices to make a semantics-preserving representation as SETAF impossible.

**Example 7.** *Let  $CF = (A, R, claim)$  be the well-formed CAF from Figure 6 with  $A = \{x_1, x_2, y_1\}$ ,  $R = \{(x_1, y_1), (y_1, x_1), (x_2, y_1)\}$  and  $claim(x_1) = claim(x_2) = x$ ,  $claim(y_1) = y$ . Observe that in the CAF  $CF_1 = (A, R', claim)$  which arises after removing the attack  $a_1 = (x_2, y_1)$  (i.e.  $R' = R \setminus \{a_1\}$ ), we have that  $stb_c(CF_1) = \{\{x\}, \{x, y\}\}$ . However, recent results on the expressiveness of semantics in SETAFs [18] show that there is no SETAF  $SF$  such that  $\sigma(SF) = \sigma_c(CF_1)$ .*

Example 7 shows that already a deviation of well-formedness by a single attack is sufficient to prohibit a faithful translation from CAFs to SETAFs. We provide a similar result for att-unitary CAFs: A semantics-preserving representation for general CAFs that deviate from att-unitarity by only one attack is in general not possible.

**Example 8.** *We consider the att-unitary CAF  $CF' = (A, R, claim)$  from Figure 6 with  $A = \{x_1, x_2, y_1\}$ ,  $R = \{(x_1, y_1), (y_1, x_1), (y_1, x_2)\}$  and  $claim(x_1) = claim(x_2) = x$ ,  $claim(y_1) = y$ . The CAF  $CF'_1 = (A, R', claim)$  which arises when removing the attack  $b_1 = (y_1, x_2)$ , i.e.  $R' = R \setminus \{b_1\}$ , possesses the stable extensions  $stb_c(CF'_1) = \{\{x\}, \{x, y\}\}$ . Thus*



Figure 6: CAFs  $CF$ ,  $CF'$  from Examples 7 and 8.

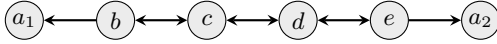


Figure 7: The AF from Example 9.

by similar arguments as in Example 7, we can conclude that there is no AF  $F$  such that  $\sigma(F) = \sigma_c(CF'_1)$ .

Although CAFs cannot be fully translated to SETAFs or AFs with unique claims in general, there is still potential to reduce the number of arguments having the same claim. Towards such a simplification procedure we introduce the concept of redundant arguments.

**Definition 11.** Let  $CF = (A, R, \text{claim})$ . An argument  $a \in A$  is called *redundant* (in  $CF$ ) w.r.t. argument  $b \in A$  if  $a \neq b$ ,  $\text{claim}(a) = \text{claim}(b)$ ,  $a_R^+ = b_R^+$ , and  $a_R^- \supseteq b_R^-$ .

In the CAF  $CF$  from Example 7, the argument  $x_1$  is redundant w.r.t.  $x_2$ : Indeed, we have that  $x_1^+ = x_2^+ = \{y_1\}$  and  $x_1^- = \{y_1\} \supseteq \emptyset = x_2^-$ . Observe that the framework  $CF' = (\{x_2, y_1\}, \{(x_2, y_1)\}, \text{claim})$  where the redundant argument  $x_1$  is deleted yields the same stable extension  $\{x\}$ ; in fact,  $\sigma_c(CF) = \sigma_c(CF')$  for every semantics  $\sigma$  under consideration. The following proposition states that, for an arbitrary CAF, redundant arguments do not influence the outcome of the reviewed semantics.

**Proposition 3.** Let  $CF = (A, R, \text{claim})$ ,  $a \in A$  be redundant in  $CF$  w.r.t. some  $b \in A$ , and let  $CF' = (A', R', \text{claim})$  with  $A' = A \setminus \{a\}$ , and  $R' = R \cap (A' \times A')$ . Then, for  $\sigma \in \{cf, adm, com, grd, stb, prf\}$ ,  $\sigma_c(CF) = \sigma_c(CF')$ .

It follows that redundant arguments can be safely removed without changing the semantics. One can apply the above proposition iteratively in order to obtain a CAF without redundant arguments. If the resulting CAF can be translated into a SETAF or an AF then also the original CAF can be translated into such a framework. Another consequence of Proposition 3 is that each well-formed CAF which violates att-unitariness by only one attack can be translated to an equivalent AF. This shows that Translation 3 is indeed robust against minimal deviations if well-formedness is guaranteed. However, well-formed CAFs violating att-unitariness to a slightly higher extent cannot, in general, be expressed as AFs, as illustrated next.

**Example 9.** Let  $CF = (A, R, \text{claim})$  with  $(A, R)$  as depicted in Figure 7,  $\text{claim}(a_1) = \text{claim}(a_2) = a$ , and  $\text{claim}(x) = x$  for  $x \in \{b, c, d, e\}$ .  $CF$  is well-formed and  $\text{stb}_c(CF) = \{\{a, b, d\}, \{b, e\}, \{a, c, e\}\}$ . Results on signatures for AFs [17] imply that there is no AF  $F$  such that  $\text{stb}(F) = \text{stb}_c(CF)$ . Observe that retaining att-unitariness requires the deletion (or addition) of at least two attacks, e.g. removing  $(b, a_1)$  and  $(e, a_2)$  yields an att-unitary CAF.

## 7 Discussion

**Related Work.** The work by Amgoud et al. [3] is probably closest to ours. They investigate equivalence in logic-based argumentation systems and study conditions under which arguments can be removed from a system without affecting its semantics. Their setting is more limiting as they require arguments to have both equivalent support and equivalent claims in order to remove one of them. The notion of CAFs is borrowed from Dvořák and Woltran [21] who introduce CAFs for a different purpose, namely to analyze the complexity of acceptance problems in terms of CAFs. Also related are semantics-preserving translations to standard AFs that have been investigated for several generalizations of AFs, e.g. [14, 8, 6]. In contrast to our main results where claims are mapped to arguments, all

of these translations concern the argument level only. An exception is the work by Strass [29] on expressiveness of AFs compared to the expressiveness of logic programs and propositional logic, where arguments are mapped to propositional atoms. Furthermore, we mention studies [9, 23, 2] that analyze whether particular properties on the level of claims (rationality and consistency postulates) are fulfilled by AF extensions in certain instantiation scenarios. Moreover, Caminada and Oren [11] classified arguments that can be neglected when computing the grounded extension of infinitary AFs. Their classification is similar to our notion of redundant arguments but their results are limited to grounded semantics.

Finally, the idea of merging arguments with the same claim has been considered in a different context. Beirlaen et al. [7] extend ASPIC+ by a reasoning by cases inference scheme that allows to generate additional arguments; in [24], Heynick and Strasser model case-based reasoning by constructing independent assumption sets of the given arguments (i.e. disjunctive supports) in their argumentative approach on dynamic proof theories for defeasible reasoning. In contrast to our approach both studies do not consider collective attacks; moreover, the newly generated arguments add to the number of arguments featuring the same claim, that is, multiple arguments with the same claim are not avoided in those works.

**Summary and Outlook.** In this work we tackled the research question under which circumstances instantiation-based argumentation can be achieved in a way where the abstract representation does not require multiple arguments for the same claim. This alternative approach provides an orthogonal view (compared to the standard way of instantiation) that is centered on the relation between claims rather than on the relation between single arguments.

We based our investigations on claim-augmented AFs (CAFs), which reflect the actual structure of the standard instantiation into Dung AFs but carry information about the claims associated to the arguments. Our research question then translates into the question whether (certain sub-classes of) CAFs can be equivalently expressed in a purely abstract setting (where claims equal argument names). We showed that well-formed CAFs are equally powerful to SETAFs with respect to the reviewed semantics and that attacker-unitary CAFs can be represented as Dung AFs with respect to admissible-based semantics. We also argued that these classes are adequately defined in the sense that minimal deviations make these translations invalid.

In terms of our research question, we thus have shown that (i) *all* instantiation procedures that result in Dung structures where arguments that implicitly share the same claim have the same outgoing attacks can be equivalently mapped to SETAFs where each argument stands for a different claim; (ii) *all* instantiation procedures that result in Dung structures where arguments that implicitly share the same claim have the same incoming attacks can be equivalently mapped to Dung AFs where each argument stands for a different claim. These findings thus give rise to alternative instantiation methods which (a) provide a single argument for each claim and (b) thus reduce the number of arguments on the abstract level. A concrete formalization of such direct instantiation procedures tailored to advanced argumentation formalisms (like ASPIC+ without preferences or logic programs) that result in SETAFs is subject of ongoing work. Investigations of this kind are also relevant from a practical side as nowadays abstract argumentation is not only understood as conceptual tool, but also the base formalism in implementations of argumentation systems, e.g. in the TOAST system [28]. Moreover, first implementations for SETAFs entered the stage [19] and can be employed for direct instantiations into SETAFs.

In this initial study we have focused on two classes of CAFs, well-formed and attacker-unitary. To extend our investigations to further instantiations, for instance full ASPIC<sup>+</sup> with preferences, further such sub-classes need to be investigated and corresponding abstract formalisms have to be found which allow for similar translations as we have shown here. We anticipate that AFs with some form of recursive attacks might be the adequate formalism for this purpose. Another direction for future research is to extend our studies to further prominent argumentation semantics, e.g. naïve, stage and semi-stable semantics [31], and labelling-based semantics [10].

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## REFERENCES

- [1] João F. L. Alcântara, Samy Sá, and Juan Carlos Acosta Guadarrama, ‘On the equivalence between abstract dialectical frameworks and logic programs’, *TPLP*, **19**(5-6), 941–956, (2019).
- [2] Leila Amgoud and Philippe Besnard, ‘Logical limits of abstract argumentation frameworks’, *Journal of Applied Non-Classical Logics*, **23**(3), 229–267, (2013).
- [3] Leila Amgoud, Philippe Besnard, and Srdjan Vesic, ‘Equivalence in logic-based argumentation’, *Journal of Applied Non-Classical Logics*, **24**(3), 181–208, (2014).
- [4] Katie Atkinson, Pietro Baroni, Massimiliano Giacomin, Anthony Hunter, Henry Prakken, Chris Reed, Guillermo Ricardo Simari, Matthias Thimm, and Serena Villata, ‘Towards artificial argumentation’, *AI Magazine*, **38**(3), 25–36, (2017).
- [5] Pietro Baroni, Martin Caminada, and Massimiliano Giacomin, ‘An introduction to argumentation semantics’, *Knowledge Eng. Review*, **26**(4), 365–410, (2011).
- [6] Pietro Baroni, Federico Cerutti, Massimiliano Giacomin, and Giovanni Guida, ‘AFRA: Argumentation framework with recursive attacks’, *Int. J. Approx. Reasoning*, **52**(1), 19–37, (2011).
- [7] Mathieu Beirlaen, Jesse Heyninck, and Christian Straßer, ‘Reasoning by cases in structured argumentation’, in *Proceedings of the Symposium on Applied Computing, SAC 2017, Marrakech, Morocco, April 3-7, 2017*, eds., Ahmed Seffah, Birgit Penzenstadler, Carina Alves, and Xin Peng, pp. 989–994. ACM, (2017).
- [8] Gerhard Brewka, Paul E. Dunne, and Stefan Woltran, ‘Relating the semantics of abstract dialectical frameworks and standard AFs’, in *Proceedings of the 22nd International Joint Conference on Artificial Intelligence, IJCAI 2011*, ed., Toby Walsh, pp. 780–785. AAAI Press, (2011).
- [9] Martin Caminada and Leila Amgoud, ‘On the evaluation of argumentation formalisms’, *Artif. Intell.*, **171**(5–6), 286–310, (2007).
- [10] Martin Caminada and Dov M. Gabbay, ‘A logical account of formal argumentation’, *Studia Logica*, **93**(2), 109–145, (2009).
- [11] Martin Caminada and Nir Oren, ‘Grounded semantics and infinitary argumentation frameworks’, in *Proceedings of the 26th Benelux Conference on Artificial Intelligence (BNAIC 2014)*, pp. 25–32, (2014).
- [12] Martin Caminada, Samy Sá, João Alcântara, and Wolfgang Dvořák, ‘On the difference between assumption-based argumentation and abstract argumentation’, *IfCoLog Journal of Logic and its Applications*, **2**(1), 15–34, (2015).
- [13] Martin Caminada, Samy Sá, João Alcântara, and Wolfgang Dvořák, ‘On the equivalence between logic programming semantics and argumentation semantics’, *Int. J. Approx. Reasoning*, **58**, 87–111, (2015).
- [14] Claudette Cayrol and Marie-Christine Lagasque-Schieux, ‘Bipolar abstract argumentation systems’, in *Argumentation in Artificial Intelligence*, eds., I. Rahwan and G. Simari, 65–84, Springer, (2009).
- [15] Sylvie Coste-Marquis, Caroline Devred, and Pierre Marquis, ‘Symmetric argumentation frameworks’, in *Proceedings of the 8th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU 2005)*, ed., Lluís Godo, volume 3571 of *Lecture Notes in Computer Science*, pp. 317–328. Springer, (2005).
- [16] Phan Minh Dung, ‘On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games’, *Artif. Intell.*, **77**(2), 321–358, (1995).
- [17] Paul E. Dunne, Wolfgang Dvořák, Thomas Linsbichler, and Stefan Woltran, ‘Characteristics of multiple viewpoints in abstract argumentation’, *Artif. Intell.*, **228**, 153–178, (2015).
- [18] Wolfgang Dvořák, Jorge Fandinno, and Stefan Woltran, ‘On the expressive power of collective attacks’, *Argument & Computation*, **10**(2), 191–230, (2019).
- [19] Wolfgang Dvořák, Alexander Grebler, and Stefan Woltran, ‘Evaluating setafs via answer-set programming’, in *Proceedings of the Second International Workshop on Systems and Algorithms for Formal Argumentation (SAFA 2018)*, eds., Matthias Thimm, Federico Cerutti, and Mauro Vallati, volume 2171 of *CEUR Workshop Proceedings*, pp. 10–21. CEUR-WS.org, (2018).
- [20] Wolfgang Dvořák, Anna Rapberger, and Stefan Woltran, ‘On the relation between claim-augmented argumentation frameworks and collective attacks’, Technical Report DBAI-TR-2020-118, Technische Universität Wien, (2020). available at <https://www.dbai.tuwien.ac.at/research/report/dbai-tr-2020-118.pdf>.
- [21] Wolfgang Dvořák and Stefan Woltran, ‘Complexity of abstract argumentation under a claim-centric view’, in *Proceedings of the Thirty-Third AAAI Conference on Artificial Intelligence (AAAI 2019)*, pp. 2801–2808. AAAI Press, (2019).
- [22] Giorgos Flouris and Antonis Bikakis, ‘A comprehensive study of argumentation frameworks with sets of attacking arguments’, *Int. J. Approx. Reasoning*, **109**, 55–86, (2019).
- [23] Nikos Gorogiannis and Anthony Hunter, ‘Instantiating abstract argumentation with classical logic arguments: Postulates and properties’, *Artif. Intell.*, **175**(9-10), 1479–1497, (2011).
- [24] Jesse Heyninck and Christian Straßer, ‘Dynamic proof theories for structured argumentation’, Technical report, academia.edu, (2015). available at [https://www.academia.edu/19401613/Dynamic\\_Proof\\_Theories\\_for\\_Structured\\_Argumentation](https://www.academia.edu/19401613/Dynamic_Proof_Theories_for_Structured_Argumentation).
- [25] Thomas Linsbichler, Jörg Pührer, and Hannes Strass, ‘A uniform account of realizability in abstract argumentation’, in *Proceedings of the 22nd European Conference on Artificial Intelligence (ECAI 2016)*, eds., Gal A. Kaminka, Maria Fox, Paolo Bouquet, Eyke Hüllermeier, Virginia Dignum, Frank Dignum, and Frank van Harmelen, volume 285 of *Frontiers in Artificial Intelligence and Applications*, pp. 252–260. IOS Press, (2016).
- [26] Søren Holbech Nielsen and Simon Parsons, ‘A generalization of Dung’s abstract framework for argumentation: Arguing with sets of attacking arguments’, in *Argumentation in Multi-Agent Systems, Third International Workshop (ArgMAS 2006)*, volume 4766 of *Lecture Notes in Computer Science*, pp. 54–73. Springer, (2006).
- [27] Henry Prakken, ‘An abstract framework for argumentation with structured arguments’, *Argument and Computation*, **1**(2), 93–124, (2010).
- [28] Mark Snaith and Chris Reed, ‘TOAST: online ASPIC<sup>+</sup> implementation’, in *Computational Models of Argument - Proceedings of COMMA 2012*, eds., Bart Verheij, Stefan Szeider, and Stefan Woltran, volume 245 of *Frontiers in Artificial Intelligence and Applications*, pp. 509–510. IOS Press, (2012).
- [29] Hannes Strass, ‘The relative expressiveness of abstract argumentation and logic programming’, in *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence (AAAI’15)*, eds., Blai Bonet and Sven Koenig, pp. 1625–1631. AAAI Press, (2015).
- [30] Frans H. van Eemeren and Bart Verheij, ‘Argumentation theory in formal and computational perspective’, in *Handbook of Formal Argumentation*, eds., Pietro Baroni, Dov Gabbay, Massimiliano Giacomin, and Leendert van der Torre, chapter 1, 3–73, College Publications, (February 2018).
- [31] Bart Verheij, ‘Two approaches to dialectical argumentation: admissible sets and argumentation stages’, in *Proceedings of the 8th Dutch Conference on Artificial Intelligence (NAIC’96)*, eds., John Jules C. Meyer and Linda C. van der Gaag, pp. 357–368, (1996).
- [32] Bruno Yun, Madalina Croitoru, Srdjan Vesic, and Pierre Bisquert, ‘Naked: N-ary graphs from knowledge bases expressed in datalog<sup>±</sup>’, in *Proceedings of the 18th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS ’19)*, eds., Edith Elkind, Manuela Veloso, Noa Agmon, and Matthew E. Taylor, pp. 2390–2392. International Foundation for Autonomous Agents and Multiagent Systems, (2019).