

# False-Name-Proof Facility Location on Discrete Structures

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**Abstract.** We consider the problem of locating a single facility on a vertex in a given graph based on agents' preferences, where the domain of the preferences is either single-peaked or single-dipped, depending on whether they want to access the facility (a public good) or be far from it (a public bad). Our main interest is the existence of deterministic social choice functions that are Pareto efficient and *false-name-proof*, i.e., resistant to fake votes. We show that regardless of whether preferences are single-peaked or single-dipped, such a social choice function exists (i) for any tree graph, and (ii) for a cycle graph if and only if its length is less than six. We also show that when the preferences are single-peaked, such a social choice function exists for any ladder (i.e.,  $2 \times m$  grid) graph, and does not exist for any larger (hyper)grid.

## 1 INTRODUCTION

Social choice theory analyzes how collective decisions are made based on the preferences of individual agents. Its typical application field is voting, where each agent reports a preference ordering over a set of alternatives and a social choice function selects one. One of the most well-studied criteria for social choice functions is robustness to agents' manipulations. An SCF is said to be *truthful* if no agent can benefit by telling a lie, and *false-name-proof* if no agent can benefit by casting multiple fake votes. The Gibbard-Satterthwaite theorem implies that any deterministic, truthful, and Pareto efficient social choice function must be dictatorial. Also, it is known that any false-name-proof social choice function must be randomized [7].

Overcoming such negative results has been a crucial research direction, and there have been a bunch of research directions that overcome those negative results. One of the most popular approaches is to restrict agents' preferences. For example, when their preferences are restricted to being single-peaked, the well-known median voter schemes are truthful, Pareto efficient, and anonymous [17], and their strict subclass, called *target rule*, is also false-name-proof [27]. The model where agents' preferences are single-peaked has also been called the *facility location problem*, where each agent has an ideal point on an interval, e.g., on a street, and a social choice function locates a public good, e.g., a train station, to which agents want to stay close.

Moulin [17], as well as many other works on facility location problems, considered an interval as the set of possible alternatives, where any point in the interval can be chosen by a social choice function. In several practical situations, however, the set of possible alternatives is discrete and has slightly more complex underlying network structure, which the agents' preferences also respect. For example, when we are choosing the server to store the corrected data among a cluster network, whose nodes usually have an acyclic structure, i.e., a *tree graph*. To avoid the risk of data loss, each client machine in the network prefers that the data is stored in a server close from it; in each hop, an error occurs with a certain probability, and thus the farther the data server is, the less happy the client is. In multi-criteria voting with two criteria, each of which has only three options, the underlying network is a three-by-three *grid*. When we need to choose a time-slot to organize a joint meeting, the problem resembles choosing a point on a *discrete cycle*; people living in different countries are setting up a 1h meeting, each person has the ideal (or the worst) time slot, and the potential time slots/outcomes are, say, 0am, 1am, ..., 11pm in GMT, which form a discrete cycle with 24 vertices. Dokow et al. [8] studied truthful social choice functions on discrete lines and cycles. Ono et al. [20] considered false-name-proof social choice functions on a discrete line. However, there has been very few works on false-name-proof social choice functions on more complex structures (see Section 2).

In this paper we tackle the following question: for which graph structures does a false-name-proof and Pareto efficient social choice function exist? When the mechanism designer can arbitrarily modify the network structure of the set of possible alternatives, the problem is simplified. The network structure, however, is a metaphor of a common feature among agents' preferences, where modifying the network structure equals changing the domain of agents' preferences. This is almost impossible in practice because agents' preferences are their own private information. The mechanism designer, therefore, first faces the problem of verifying whether, under a *given* network structure (or equally, a given preference domain), a desirable social choice function exists.

Locating a bad is another possible extension of the facility location problem, where a social choice function is required to locate a public bad, e.g., a nuclear plant or a disposal station, which each agent wants to avoid. Agents' preferences are therefore assumed to be single-dipped, which is sometimes called obnoxious. Actually, some existing works have studied truthful facility location with single-dipped preferences [5]. Nevertheless, to the best of our knowledge, no work has dealt with both false-name-proofness and more complex structures than a path, such as cycles.

Table 1 summarizes our contribution. Regardless of whether the preferences are single-peaked or single-dipped, there is a false-name-

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**Table 1.** Summary of our contributions. ✓ indicates a false-name-proof and Pareto efficient social choice function, and ✗ indicates that no such social choice function exists. It remains open to clarify whether such a social choice function exists for general hypergrid graphs when agents' preferences are single-dipped.

	Tree	Cycle $C_k$	Hypergrid
Single-Peaked	Any ✓ [18]	$k \leq 5$ ✓ (Thm. 3) $k \geq 6$ ✗ (Thm. 4)	Ladder ✓ [18] <b>Other</b> ✗ (Thm. 5 and 6)
Single-Dipped	Any ✓ (Thm. 7)	$k \leq 5$ ✓ (Thm. 8) $k \geq 6$ ✗ (Thm. 9)	Open

proof and Pareto efficient social choice function for any tree graph and any cycle graph of length less than six, and there is no such social choice function for any larger cycle graph. For hypergrid graphs, when preferences are single-peaked, such a social choice function exists if and only if the given hypergrid graph is a ladder, i.e., of dimension two and at least one of which has at most two vertices.

## 2 RELATED WORKS

In the literature of facility location (and social choice with single-peaked preferences), one of the most popular direction is to design and analyze truthful social choice functions. Moulin [17] proposed generalized median voter schemes, which are the only deterministic, truthful, Pareto efficient, and anonymous social choice functions. Procaccia and Tennenholtz [21] proposed a general framework of approximate mechanism design, which evaluates the worst case performance of truthful social choice functions from the perspective of competitive ratio. Recently, some models for locating multiple heterogeneous facilities have also been studied [23, 11, 3]. Wada et al. [29] considered the agents who dynamically arrive and depart. Some research also considered facility location on grids [25, 9] and cycles [1, 2, 8]. Melo et al. [16] overviewed applications in practical decision making.

Over the last decade, false-name-proofness has also been scrutinized in various mechanism design problems [30, 4, 26, 31, 28], as a refinement of truthfulness for such open and anonymous environments, as the internet. Bu [6] clarified a connection between false-name-proofness and population monotonicity in general social choice problems. Todo et al. [27] provided a complete characterization of false-name-proof and Pareto efficient social choice functions for the facility location problem with single-peaked preferences on a continuous line. Lesca et al. [14] also addressed false-name-proof social choice functions that are associated with monetary compensation. Sonoda et al. [24] considered the case of locating two homogeneous facilities on a continuous line and a circle. Ono et al. [20] studied some discrete structures, but focused on randomized social choice functions and clarifies the relation between false-name-proofness and population monotonicity.

One of the most similar works to this paper is Nehama et al. [18], which also clarified the network structures under which false-name-proof and Pareto efficient social choice functions exist for single-peaked preferences. One clear difference from ours is that, in their paper they proposed a new class of graphs, called ZV-line, as a generalization of path graphs. In our paper, on the other hand, we investigate well-known existing structures, namely tree, hypergrid, and cycle graphs. ZV-line graphs contain any tree and ladder (i.e.,  $2 \times m$ -grid for arbitrary  $m \geq 2$ ), but do not cover any other graphs considered in this paper, such as larger (hyper-)grid graphs and cycle graphs of lengths not equal to four.

Locating a public bad has also been widely studied in both economics and computer science fields, which essentially equals to consider single-dipped preferences. Manjunath [15] characterized truthful social choice functions on an interval. Lahiri et al. [13] studied the model for locating two public bads. Feigenbaum and Sethuraman [10] considered the cases where single-peaked and single-dipped preferences coexist. Nevertheless, all of these works just focused on truthful social choice functions. To the best of our knowledge, this paper is the very first work that considers false-name-proof facility location when agents' preferences are single-dipped.

## 3 PRELIMINARIES

In this section, we describe the formal model of the facility location problem considered in this paper. Let  $\Gamma := (V, E)$  be an undirected and connected graph, defined by the set  $V$  of vertices and the set  $E$  of edges. The distance function  $d : V^2 \rightarrow \mathbb{N}_{\geq 0}$  is such that for any  $v, w \in V$ ,  $d(v, w) := \#\{e \in E \mid e \in s(v, w)\}$ , where  $s(v, w)$  is the shortest path between  $v$  and  $w$ . We say that a graph  $\Gamma' := (V', E')$  has another graph  $\Gamma := (V, E)$  as a *distance-preserving induced subgraph* if  $\Gamma'$  has  $\Gamma$  as an induced subgraph, where the corresponding pair of two vertices are represented as  $v \equiv v'$ , and for any pair  $v, w \in V$  and their corresponding vertices  $v', w' \in V'$ , i.e.,  $v \equiv v'$  and  $w \equiv w'$ , it holds that  $d(v', w') = d(v, w)$ . For these  $\Gamma$  and  $\Gamma'$ , let us also denote  $V'_{|\Gamma} := \{v' \in V' \mid \exists v \in V, v' \equiv v\}$ , and  $U \equiv U'$  for  $U \subseteq V$  and  $U' \subseteq V'_{|\Gamma}$  if  $\forall u \in U, \exists u' \in U'$  such that  $u \equiv u'$ .

In this paper we focus on three classes of graphs, namely tree, cycle, and hypergrid. A *tree* graph is an undirected, connected and acyclic graph. A special case of tree graphs is called as a *path* graph, in which only two vertices have a degree of one and all the others have a degree of two. Indeed, tree graphs are a simplest generalization of path graphs, so that most of the properties of path graphs, such as the uniqueness of the shortest path between two vertices, carries over to tree graphs. A *cycle* graph is an undirected and connected graph that only consists of a single cycle. When a cycle graph has  $k$  vertices, we refer to it as  $C_k$ , and its vertices are labeled, in a counter-clockwise order, from  $v_1$  to  $v_k$ .

A *hypergrid* graph is a Cartesian product of more than one path graphs. When a hypergrid  $\Gamma$  is a Cartesian product of  $k$  path graphs, we call it a  $k$ -dimensional ( $k$ -D, in short) grid. In this paper, a 2-D grid is sometimes represented by the number of vertices on each path, as  $l \times m$ -grid. In a given  $k$ -D grid graph, each vertex  $v$  is represented as a  $k$ -tuple  $(v_{k'})_{1 \leq k' \leq k}$ . Note that the 2-D  $2 \times 2$ -grid is a cycle graph  $C_4$ .

Let  $\mathcal{N}$  be the set of potential agents, and  $N \subseteq \mathcal{N}$  be a set of agents who actually participate into the problem. Each agent  $i \in N$  has a type  $\theta_i \in V$ . When an agent  $i$  has type  $\theta_i$ , agent  $i$  is said to be *located on* vertex  $\theta_i$ . Let  $\theta := (\theta_i)_{i \in N} \in V^{|N|}$  denote a profile of the agents'

types (or *type profile* in short), and  $\theta_{-i} := (\theta_{i'})_{i' \neq i}$  denote the type profile without  $i$ 's type. Given  $\theta$ , let  $I(\theta) := \bigcup_{i \in N} \theta_i \subseteq V$  be the set of vertices on which at least one agent is located. Also, given  $\theta$  and  $v \in I(\theta)$ , let  $\theta_{-v}$  be the type profile obtained by removing all the agents located on  $v$  from  $\theta$ . By definition,  $I(\theta_{-v}) = I(\theta) \setminus \{v\}$ .

Given  $\Gamma$  and  $v \in V$ , let  $\succsim_v$  indicate the preference of the agent located on  $v$  over the set  $V$  of alternatives, where  $\succ_v$  and  $\sim_v$  indicate the strict and indifferent parts of  $\succsim_v$  respectively, each of which is represented as a binary relation. A preference  $\succsim_v$  is *single-peaked* (resp. *single-dipped*) under  $\Gamma$  if, for any  $w, x \in V$ ,  $w \succ_v x$  if and only if  $d(v, w) < d(v, x)$  (resp.  $d(v, w) > d(v, x)$ ), and  $w \sim_v x$  if and only if  $d(v, w) = d(v, x)$ . That is, an agent located on  $v$  strictly prefers alternative  $w$ , which is strictly closer to (resp. farther from)  $v$ , than alternative  $x$ , and is indifferent between these alternatives when they are in the same distance from  $v$ . By definition, for each type  $\theta_i$ , the single-peaked (resp. single-dipped) preference is unique.

A deterministic social choice function maps a type profile to a vertex. Since each agent may pretend to be multiple agents in our model of facility location, a social choice function must be defined over type profiles of different sizes. To describe this feature, we define a social choice function  $f = (f_N)_{N \subseteq N}$  as a family of functions, where each  $f_N$  is a mapping from  $V^{|N|}$  to  $V$ . When a set  $N$  of agents participates, the social choice function  $f$  uses function  $f_N$  to determine the outcome. The function  $f_N$  takes profile  $\theta$  of types jointly reported by  $N$  as an input, and returns  $f_N(\theta)$  as an outcome. We denote  $f_N$  as  $f$  if it is clear from the context. We further assume that a social choice function  $f$  is anonymous, i.e., for any input  $\theta$  and its permutation  $\theta'$ ,  $f(\theta') = f(\theta)$  holds.

Here we define two desirable properties of social choice functions, *false-name-proofness* and *Pareto efficiency*, which are in our main interest of this paper.

**Definition 1** (False-Name-Proofness). *A social choice function  $f$  is said to be false-name-proof if for any  $N$ , any  $\theta$ , any  $i \in N$ , any  $\theta_i \in V$ , any  $\theta'_i \in V$ , any  $\Phi_i \subseteq N \setminus \{i\}$ , and any  $\theta_{\Phi_i} \in V^{|\Phi_i|}$ , it holds that*

$$f(\theta) \succsim_i f(\theta'_i, \theta_{\Phi_i}, \theta_{-i}).$$

The set  $\Phi_i$  indicates the set of identities added by  $i$  for the manipulation. The property coincides with the canonical *truthfulness* when  $\Phi_i = \emptyset$ , i.e., agent  $i$  only uses one identity.

**Definition 2** (Pareto Efficiency). *An alternative  $v \in V$  is said to Pareto dominate  $w \in V$  under  $\theta$  if both*

- $v \succsim_i w$  for all  $i \in N$ , and
- $v \succ_j w$  for some  $j \in N$

*hold. a social choice function  $f$  is said to be Pareto efficient if for any  $N$  and any  $\theta$ , no alternative  $v \in V$  Pareto dominates  $f(\theta)$ .*

Given  $\theta$ , let  $\text{PE}(\theta) \subseteq V$  indicate the set of all alternatives that are not Pareto dominated by any alternative.

The following theorem on a general property of false-name-proof and Pareto efficient social choice functions has recently been provided by the authors' another paper [19], which justifies to focus on a special class of false-name-proof and Pareto efficient social choice functions. a social choice function  $f$  is said to *ignore duplicate ballots* (or satisfies IDB in short) if for any pair  $\theta, \theta'$ ,  $I(\theta) = I(\theta')$  implies  $f(\theta) = f(\theta')$ . In words, any social choice function that satisfies IDB cares about whether there exists at least one agent on each vertex, but does not care about how many agents are located in each vertex.

**Theorem 1** (Okada et al. [19]). *Assume that the domain of agents' preferences are either single-peaked or single-dipped. If there is a false-name-proof and Pareto efficient social choice function  $f$  that does not satisfy IDB, we then can find another false-name-proof and Pareto efficient social choice function  $f'$  that also satisfies IDB and such that for any  $N$ , any  $i \in N$  and any  $\theta$ ,*

$$f'(\theta) \sim_i f(\theta).$$

That is, for any social choice function  $f$  violating IDB, we can find another social choice function  $f'$  that is indifferent with  $f$ , for any possible input  $\theta$ , from the perspective of any participating agent. Therefore, in what follows, we focus on such false-name-proof and Pareto efficient social choice functions  $f'$  that also satisfies IDB.

## 4 SINGLE-PEAKED PREFERENCES

In this section, we focus on single-peaked preferences, i.e., every agent prefers to have the facility closer to her. It is already known that for any tree graph, and thus for any path graph, a false-name-proof and Pareto efficient social choice function exists.

**Theorem 2** (Nehama et al. [18]). *Assume that agents' preferences are single-peaked. For any tree graph, there is a false-name-proof and Pareto efficient social choice function.*

An example of such a social choice function is the target rule [12], originally proposed for an interval, i.e., a continuous line such as  $[0, 1]$ . It is shown that the target rule is false-name-proof and Pareto efficient for any tree metric [27]. Almost the same proof works for any tree graph.

In the following two subsections, we investigate the existence of such social choice functions for cycle and hypergrid graphs. The two lemmata presented below are useful to prove the impossibility results for single-peaked preferences. Lemma 1 intuitively shows that, when there is no social choice function that is truthful and Pareto efficient simultaneously for a distance-preserving induced subgraph, then the impossibility carries over to the original graph. Lemma 2 shows that, when the current alternative is still Pareto efficient after removing a preference, then the alternative must be still chosen. Notice that both lemmata does not assume any specific structure of the graphs, i.e., does hold not only for grids and cycles but also for general structures.

**Lemma 1.** *Let  $\Gamma = (V, E)$  be an arbitrary graph. Assume that agents' preferences are single-peaked under  $\Gamma$  and there is no truthful and Pareto efficient social choice function for  $\Gamma$ . Then, for any graph  $\Gamma' = (V', E')$  that contains  $\Gamma$  as a distance-preserving induced subgraph, there is no truthful and Pareto efficient social choice function.*

*Proof.* Consider an arbitrarily chosen social choice function  $f'$  for  $\Gamma'$ . Because (i)  $\Gamma'$  has  $\Gamma$  as a distance-preserving induced subgraph and (ii) agents' preferences are single-peaked, for any profile  $\theta'$  on  $V'_{|\Gamma|}$  and corresponding profile  $\theta$  on  $V$  such that  $I(\theta) = I(\theta')$ , it holds that

$$\text{PE}(\theta') \equiv \text{PE}(\theta),$$

that is, the structure of the set of Pareto efficient alternatives are totally the same. Therefore, for the arbitrarily chosen social choice function  $f'$  for  $\Gamma'$ , its behavior for the set of profiles  $\theta'$  such that  $I(\theta') \subseteq V'_{|\Gamma|}$  must be equal to a social choice function  $f$  for  $\Gamma$ , i.e., there exists a social choice function  $f$  for  $\Gamma$  such that

$$\forall \theta' \text{ s.t. } I(\theta') \subseteq V'_{|\Gamma|}, \forall \theta \text{ s.t. } I(\theta) = I(\theta'), f(\theta) \equiv f'(\theta').$$

By the assumption, such a social choice function  $f$  for  $\Gamma$  is not truthful and Pareto efficient simultaneously. Therefore,  $f'$  also violate one of the properties.  $\square$

**Lemma 2.** Let  $\Gamma$  be an arbitrary graph. Assume that agents' preferences are single-peaked under a graph  $\Gamma$ . Then, for any false-name-proof social choice function  $f$ , any  $\theta$  and any  $v \in I(\theta)$ ,

$$[f(\theta) \in I(\theta) \wedge f(\theta) \in I(\theta_{-v})] \Rightarrow [f(\theta_{-v}) = f(\theta)].$$

*Proof.* Assume for the sake of contradiction that the alternative chosen by the social choice function  $f$  changes after the removal of such  $v$ , i.e.,

$$f(\theta_{-v}) \neq f(\theta).$$

Here,

$$f(\theta) \in I(\theta)$$

implies that there is some agent, say  $i$ , located at  $f(\theta)$ , who incurs the cost of zero when  $\theta$  is reported and is still present when  $v$  is removed. Since

$$f(\theta) \neq f(\theta_{-v})$$

and agents' preferences are single-peaked, such an agent  $i$  incurs the cost of more than zero when  $v$  is removed. Thus, the agent  $i$  located at  $f(\theta)$  has an incentive to add identities at  $v$ , so that the situation becomes identical to the case of  $\theta$ , which contradicts the assumption that  $f$  is false-name-proofness.  $\square$

#### 4.1 Single-Peaked Preferences on Cycles

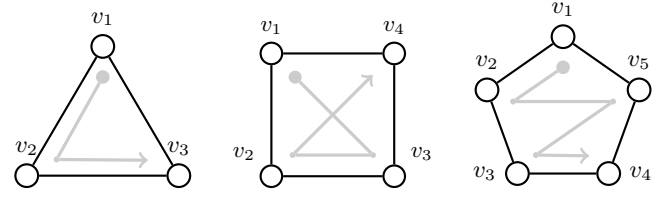
In this section, we show that, under single-peaked preferences, there is a false-name-proof and Pareto efficient social choice function for  $C_k$  if and only if  $k \leq 5$ .

For the if direction, to explain the existence of such social choice functions, we first define a class of social choice functions, called *sequential Pareto* rules. Given cycle  $C_k$ , a sequential Pareto rule has an ordering  $\sigma$  of all the alternatives in  $C_k$ , which is given independently from agents' preferences  $\theta$ . For a given  $\theta$ , it sequentially checks, in the order specified by  $\sigma$ , whether the first (second, third, and so on) alternative is Pareto efficient, and terminates when it finds a Pareto efficient one. By definition, any sequential Pareto rule is automatically Pareto efficient.

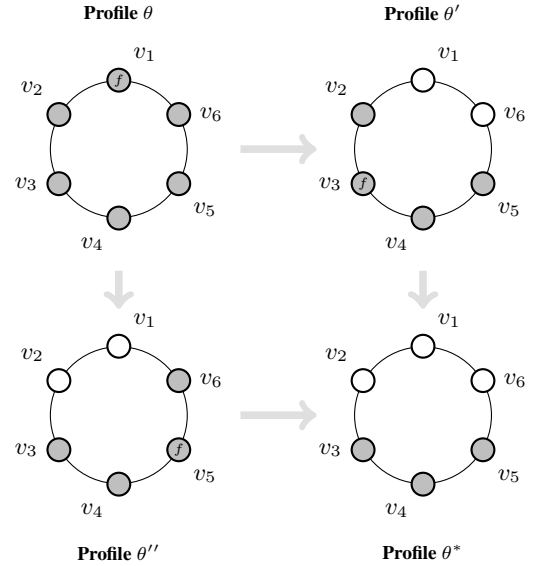
For a continuous circle, any truthful and Pareto efficient social choice function is dictatorial [22]. Since choosing such a dictator in a non-manipulable manner, when there is uncertainty on identities, is quite difficult, false-name-proof and Pareto efficient social choice functions are not likely to exist for a continuous circle. Our results in this section thus demonstrate the power of the discretization of the alternative space; by discretizing the set of alternatives so that at most five alternatives exist along with a cycle, we can avoid falling into the impossibility.

Dokow et al. [8] showed that any truthful and onto social choice function is nearly dictatorial for a cycle  $C_k$  with  $k \geq 22$ . In this paper we clarify a stricter threshold on such an impossibility when agents can pretend to be multiple agents; false-name-proof social choice functions exist for a cycle  $C_k$  if and only if  $k \leq 5$ . Theorem 3 shows the if direction, and Theorem 4 shows the only if direction.

**Theorem 3.** Let  $\Gamma$  be a cycle graph  $C_k$  s.t.  $3 \leq k \leq 5$ . When preferences are single-peaked, there is a false-name-proof and Pareto efficient social choice function.



**Figure 1.** Sequential Pareto rules for  $C_3$ ,  $C_4$ , and  $C_5$ . The arrows indicate the associated ordering  $\sigma$ , according to which the Pareto efficiency condition will be checked one by one.

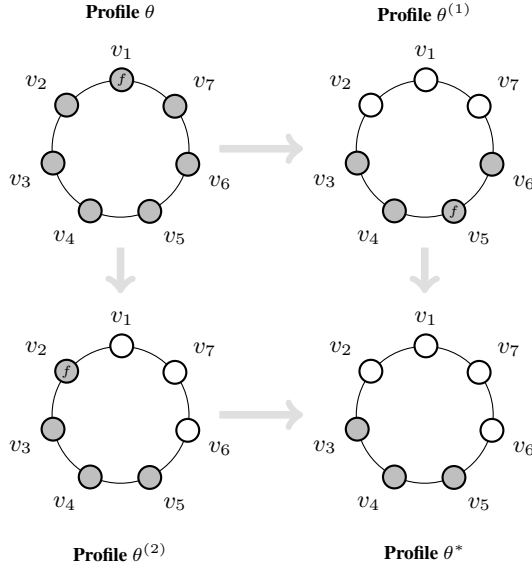


**Figure 2.** Type profiles used for  $C_6$  in the proof of Theorem 4. On each gray vertex there is some agent, and the vertex with label  $f$  must be chosen under the profile. The proof derives a contradiction on  $\theta^*$ .

*Proof.* It is obvious that any sequential Pareto rule is false-name-proof for  $C_3$ . For  $C_4$ , the sequential Pareto rule with the ordering  $v_1 \rightarrow v_3 \rightarrow v_2 \rightarrow v_4$  is false-name-proof, which is actually the same rule mentioned by Nehama et al [18]. Finally, for  $C_5$ , the sequential Pareto rule with the ordering  $v_1 \rightarrow v_2 \rightarrow v_5 \rightarrow v_3 \rightarrow v_4$  is false-name-proof, which was also informally mentioned in [18]. These rules are described in Fig. 1.  $\square$

One might think that a sequential Pareto rule associated with any possible ordering is false-name-proof. However, the following example shows that the ordering must be carefully chosen to guarantee false-name-proofness (and truthfulness as well). Characterizing false-name-proof and Pareto efficient social choice functions for a given cycle graph remains open.

**Example 1.** Consider  $C_5$  and a sequential Pareto rule  $f$  associated with ordering  $v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5$ . Assume that there are three agents, whose types are  $\theta = (v_3, v_4, v_5)$ . Since  $PE(\theta) = \{v_3, v_4, v_5\}$ ,  $v_3$  is chosen as an outcome when all the agents reports truthfully, where the agent located at  $v_5$  incurs the cost of 3. However, she can benefit by reporting  $v_1$  as her type, since  $PE(v_1, v_3, v_4) = V$ , and thus  $f(v_1, v_3, v_4) = v_1$  reduces her cost to 1.



**Figure 3.** Type profiles used for  $C_7$  in the proof of Theorem 4. On each gray vertex there is some agent, and the vertex with label  $f$  must be chosen under the profile. The proof derives a contradiction on  $\theta^*$ .

**Theorem 4.** Let  $\Gamma$  be a cycle graph  $C_k$  s.t.  $k \geq 6$ . When preferences are single-peaked, there is no false-name-proof and Pareto efficient social choice function.

The impossibility for  $C_6$  is important because we will use it in the proof of Theorem 6 in the next subsection. Due to space limitations, we show the proof for  $C_6$  and  $C_7$ .

*Proof.* Assume for the sake of contradiction that a false-name-proof and Pareto efficient social choice function  $f$  exists for  $C_6$ , and assume w.l.o.g. that  $f(\theta) = v_1$  for any profile  $\theta$  s.t.  $I(\theta) = V$  (see the top-left cycle of Fig. 2).

**For  $C_6$ :** Consider a type profile  $\theta'$  s.t.

$$I(\theta') = \{v_2, v_3, v_4, v_5\}$$

(see the top-right cycle of Fig. 2). From Pareto efficiency, it must be the case that

$$f(\theta') \in \text{PE}(\theta') = \{v_2, v_3, v_4, v_5\}.$$

If  $f(\theta') = v_2$ , then agents at  $v_5$  have an incentive to add fake identities at both  $v_1$  and  $v_6$ . If  $f(\theta') = v_4$  or  $f(\theta') = v_5$ , then agents at  $v_2$  have an incentive to add fake identities at both  $v_1$  and  $v_6$ . Therefore, false-name-proofness implies that  $f(\theta') = v_3$ . Let  $\theta^*$  then be a type profile s.t.  $I(\theta^*) = \{v_3, v_4, v_5\}$ , i.e., the antipodal to the vertex  $v_1 = f(\theta)$  and its two neighbors. Since  $\theta^*$  can be obtained by removing  $v_2$  from  $\theta'$  and  $f(\theta') = v_3 \in I(\theta^*)$ , Lemma 2 implies

$$f(\theta^*) = v_3.$$

We then consider another profile  $\theta''$  s.t.

$$I(\theta'') = \{v_3, v_4, v_5, v_6\}$$

(see the bottom-left cycle of Fig. 2). From symmetry and Lemma 2,

$$f(\theta'') = v_5,$$

which contradicts  $f(\theta^*) = v_3$ . Almost the same argument holds for any larger even  $k$ .

**For  $C_7$ :** Consider a type profile  $\theta^{(1)}$  s.t.

$$I(\theta^{(1)}) = \{v_3, v_4, v_5, v_6\}$$

(see the top-right cycle of Fig. 3). From Pareto efficiency, it must be the case that

$$f(\theta^{(1)}) \in \text{PE}(\theta^{(1)}) = \{v_3, v_4, v_5, v_6\}.$$

If  $f(\theta^{(1)}) = v_6$ , then agents at  $v_3$  have an incentive to add fake identities at  $v_1, v_2$  and  $v_7$ . If  $f(\theta^{(1)}) = v_3$ , then agents at  $v_6$  have an incentive to add fake identities at  $v_1, v_2$  and  $v_7$ . Therefore, false-name-proofness implies that

$$f(\theta^{(1)}) \in \{v_4, v_5\}.$$

From the symmetry between  $v_4$  and  $v_5$  in the profile  $\theta^{(1)}$ , assume w.l.o.g. that  $f(\theta^{(1)}) = v_5$ . From Lemma 2, removing  $v_6$  from  $\theta^{(1)}$  does not change the outcome. That is, for  $\theta^*$  s.t.  $I(\theta^*) = \{v_3, v_4, v_5\}$ , it must be the case that

$$f(\theta^*) = v_5.$$

Then let us consider another profile  $\theta^{(2)}$  s.t.

$$I(\theta^{(2)}) = \{v_2, v_3, v_4, v_5\}$$

(see the bottom-left cycle of Fig. 3). Since  $f$  is false-name-proof and Pareto efficient,  $f(\theta^{(2)}) \notin \{v_4, v_5\}$ ; otherwise agents located on  $v_2$  would add fake identities so that the outcome changes to  $v_1$ . Similarly,  $f(\theta^{(2)}) \neq v_3$ ; otherwise, it must be the case that  $f(\theta^*) = v_3$ , which yields a contradiction. Thus,

$$f(\theta^{(2)}) = v_2.$$

However,  $f(\theta^{(2)}) = v_2$  implies

$$f(\theta^*) \neq v_5;$$

otherwise agents located on  $v_3$  would add fake identities so that the outcome changes to  $v_2$ . This also yields a contradiction. Almost the same argument holds for any larger odd  $k$ .  $\square$

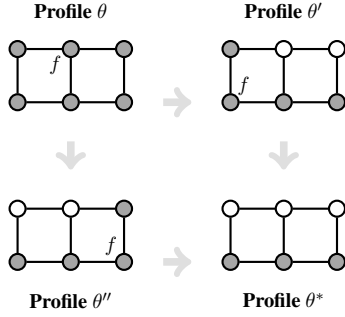
## 4.2 Single-Peaked Preferences on Hypergrids

The facility location on a hypergrid graph is a reasonable simplification of multi-criteria voting [25], where each candidate has a pledge for each criteria, such as taxation and diplomacy, that is embeddable on a hypergrid. Each voter then has the most/least preferred point on the hypergrid.

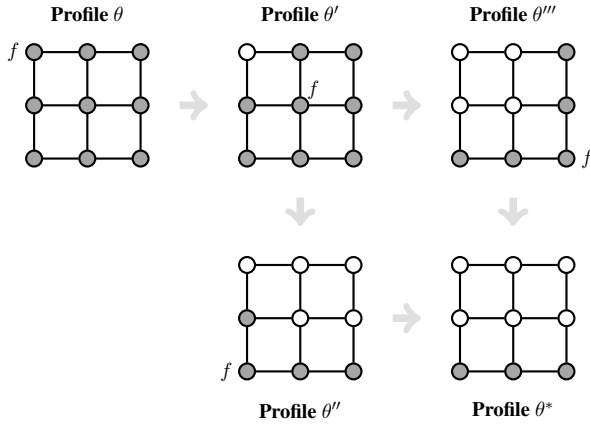
In this section, we completely clarify under which condition on a given hypergrid graph a false-name-proof and Pareto efficient social choice function exists when agents' preferences are single-peaked.

It is already known that, when preferences are single-peaked, a false-name-proof and Pareto efficient social choice function exists for any  $2 \times m$ -grid [18]. Theorem 5 complements their result; no such social choice function exists for any other 2-D grid. Theorem 6 further shows that this impossibility carries over into any  $k$ -D grid with  $k \geq 3$ .

**Lemma 3.** Let  $\Gamma$  be the  $2 \times 3$ -grid, where the set of vertices  $V = \{v_{1,1}, v_{1,2}, v_{1,3}, v_{2,1}, v_{2,2}, v_{2,3}\}$ . Assume that agents' preferences are single-peaked under  $\Gamma$  and there is a false-name-proof and Pareto efficient social choice function  $f$ . Then, for any  $\theta$  s.t.  $I(\theta) = V$ ,  $f(\theta)$  must be one of the four corners of  $\Gamma$ , i.e.,  $f(\theta) \in \{v_{1,1}, v_{1,3}, v_{2,1}, v_{2,3}\}$ .



**Figure 4.** Four type profiles used in the proof of Lemma 3. On each gray vertex there is some agent, and the vertex with label  $f$  must be chosen under the profile. The proof derives a contradiction on  $\theta^*$ .



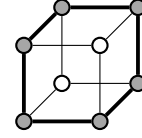
**Figure 5.** Five type profiles used in the proof of Lemma 4. On each gray vertex there is some agent, and the vertex with label  $f$  must be chosen under the profile. The proof derives a contradiction on  $\theta^*$ .

*Proof.* Assume w.l.o.g. that  $f(\theta) = v_{1,2}$  (see the top-left grid in Fig. 4). We construct a type profile  $\theta'$  s.t.  $I(\theta') = \{v_{1,1}, v_{2,1}, v_{2,2}, v_{2,3}\}$ . Since  $f$  is false-name-proof,  $f(\theta') = v_{2,1}$  (see the top-right grid in Fig. 4). We also construct another profile  $\theta''$  s.t.  $I(\theta'') = \{v_{1,3}, v_{2,1}, v_{2,2}, v_{2,3}\}$ . Since  $f$  is false-name-proof,  $f(\theta'') = v_{2,3}$  (see the bottom-left grid in Fig. 4). Finally, let  $\theta^*$  be the profile constructed by removing all the agents located on  $v_{1,1}$ ,  $v_{1,2}$ , and  $v_{1,3}$ . By applying Lemma 2 to those profiles, we obtain both  $f(\theta^*) = v_{2,1}$  and  $f(\theta^*) = v_{2,3}$ , which yield a contradiction.  $\square$

Now we are ready to present the general impossibility results for general 2-D grids (Theorem 5) and hypergrids (Theorem 6), which are our main contribution in this subsection.

**Theorem 5.** *Let  $\Gamma$  be an  $l \times m$ -grid, where  $l, m \geq 3$ . When preferences are single-peaked, there is no false-name-proof and Pareto efficient social choice function.*

*Proof.* Lemma 4 below shows that, for the  $3 \times 3$ -grid, there is no false-name-proof and Pareto efficient social choice function. Since any  $l \times m$ -grid  $\Gamma$ , for arbitrary  $l, m \geq 3$ , contains the  $3 \times 3$ -grid graph as a distance-preserving induced subgraph, the impossibility carries over into  $\Gamma$  according to Lemma 1.  $\square$



**Figure 6.** The three-dimensional  $2 \times 2 \times 2$ -grid, also known as a binary cube, contains  $C_6$  (emphasized by bold edges) as a distance-preserving induced subgraph, which consists of the grayed vertices.

**Lemma 4.** *Let  $\Gamma$  be the 2-D  $3 \times 3$ -grid. When preferences are single-peaked, there is no false-name-proof and Pareto efficient social choice function.*

*Proof.* Assume that a false-name-proof and Pareto efficient social choice function  $f$  exists for the  $3 \times 3$ -grid. From Lemmata 2 and 3, for any  $\theta$  s.t.  $I(\theta) = V$ ,  $f(\theta) \in \{(1,1), (1,3), (3,1), (3,3)\}$  holds. From symmetry, assume w.l.o.g. that  $f(\theta) = (1,1)$  (see the top-left grid in Fig. 5).

We now remove all the agents located at  $(1,1)$  from the above profile  $\theta$ , and refer to the profile as  $\theta'$ . Since  $f$  is false-name-proof and Pareto efficient,  $f(\theta') = (2,2)$ . Here, let  $\theta''$  be the profile that further removes all the agents located at  $(1,2)$ ,  $(1,3)$ ,  $(2,2)$ , and  $(2,3)$  from  $\theta'$ . Note that  $I(\theta'') = \{(2,1), (3,1), (3,2), (3,3)\}$ , and thus  $f(\theta'') = (3,1)$  holds by the same argument. We also consider another profile,  $\theta'''$ , which is obtained by removing all the agents at  $(1,2)$ ,  $(2,1)$ , and  $(2,2)$  from  $\theta'$ . Note that  $I(\theta''') = \{(1,3), (2,3), (3,1), (3,2), (3,3)\}$ , and  $f(\theta''') = (3,3)$  by the same argument.

Then we construct  $\theta^*$  by removing all the agents in the vertices except for  $(3,1)$ ,  $(3,2)$ , and  $(3,3)$  from  $\theta'$ . Since  $\theta^*$  is reachable from both  $\theta''$  and  $\theta'''$ , Lemma 2 implies  $f(\theta^*) = (3,1)$  and  $f(\theta^*) = (3,3)$ , which yields a contradiction.  $\square$

**Theorem 6.** *Let  $\Gamma$  be an arbitrary  $k(> 2)$ -D grid. When preferences are single-peaked, there is no false-name-proof and Pareto efficient social choice function.*

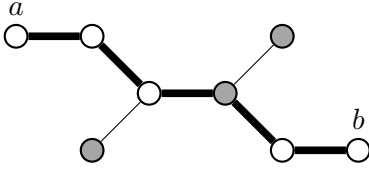
*Proof.* We can easily observe that the three-dimensional  $2 \times 2 \times 2$ -grid, a.k.a. the binary cube, contains  $C_6$  as a distance-preserving induced subgraph, e.g., the subgraph induced from the grayed vertices in Fig. 6. As we showed in Theorem 4 in the previous subsection, there is no false-name-proof and Pareto efficient social choice function for  $C_6$ . Therefore, by Lemma 1, no such social choice function exists for the  $2 \times 2 \times 2$ -grid. Any other larger grid (possibly of more than three dimensions) contains the three-dimensional  $2 \times 2 \times 2$ -grid, and thus the impossibility carries over by Lemma 1.  $\square$

## 5 SINGLE-DIPPED PREFERENCES

As we already mentioned in Section 2, this paper is the very first work that considers false-name-proof social choice function for discrete facility location problem when agents' preferences are single-dipped. We therefore begin with the discussion on tree graphs.

### 5.1 Single-Dipped Preferences on Trees

For the case of a public bad, where agents' preferences are single-dipped, we can find a false-name-proof and Pareto efficient social choice function.



**Figure 7.** The bold edges construct a longest path  $\pi^*$ . For the profile where some agent exists on each of the gray vertices, the social choice function described in the proof of Theorem 2 returns  $a$  as an outcome.

**Theorem 7.** Let  $\Gamma$  be an arbitrary tree graph. When preferences are single-dipped, there is a false-name-proof and Pareto efficient social choice function.

*Proof.* Consider the social choice function described as follows. First, choose an arbitrary longest path  $\pi^*$  of a given tree, whose extremes are called  $a$  and  $b$ . Then, return  $a$  as an outcome if at least one agent strictly prefers  $a$  to  $b$ ; otherwise return  $b$  as an outcome.

For each agent  $i$ , either  $a$  or  $b$  is one of the most preferred alternative; otherwise, the path from the most preferred point of  $i$  to one of the two extremes is strictly longer than  $\pi^*$ , which violates the assumption that  $\pi^*$  is a longest path. In Fig. 7, the agents at the bottom left gray vertex most prefer  $b$ , while agents at the middle or top-right gray vertices most prefer  $a$ . It is therefore obvious that the above social choice function is Pareto efficient, since either  $a$  or  $b$  is the most preferred alternative for each agent, and the choice between  $a$  and  $b$  is made by a unanimous voting, guaranteeing that the outcome is the most preferred for at least one agent. Furthermore, such a unanimous voting over two alternatives is obviously false-name-proof.  $\square$

## 5.2 Single-Dipped Preferences on Cycles

We next consider locating a public bad on a cycle. Single-dipped preferences quite resemble single-peaked preferences for cycle graphs, especially for sufficiently large ones. Actually, in this subsection we provide almost the same results with the case of single-peaked preferences.

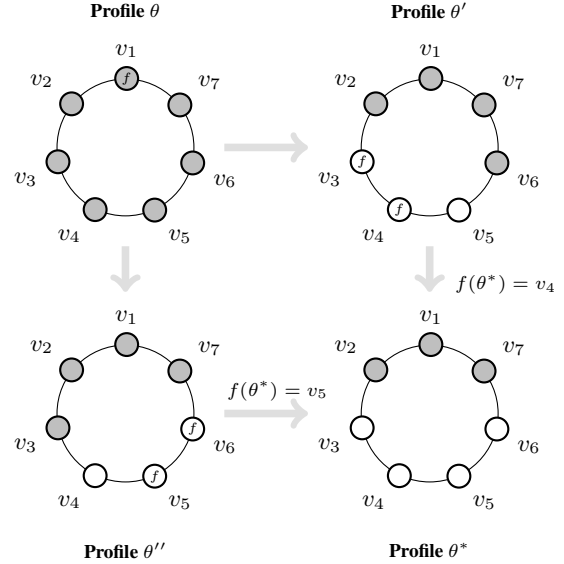
**Theorem 8.** Let  $\Gamma$  be a cycle graph  $C_k$  s.t.  $3 \leq k \leq 5$ . When preferences are single-dipped, there is a false-name-proof and Pareto efficient social choice function.

*Proof.* For  $C_3$ , it is easy to see that any sequential Pareto rule is false-name-proof. For  $C_4$ , the domain of single-dipped preferences coincides with the domain of single-peaked preferences, since the point diagonal from a dip point can be considered as a peak point. Therefore, the sequential Pareto rule with ordering  $v_1 \rightarrow v_3 \rightarrow v_2 \rightarrow v_4$  is false-name-proof, as shown in Theorem 3. Finally, for  $C_5$ , the sequential Pareto rule with ordering  $v_1 \rightarrow v_2 \rightarrow v_5 \rightarrow v_3 \rightarrow v_4$  is false-name-proof.  $\square$

**Theorem 9.** Let  $\Gamma$  be a cycle graph  $C_k$  s.t.  $k \geq 6$ . When preferences are single-dipped, there is no false-name-proof and Pareto efficient social choice function.

*Proof.* The identical proof of Theorem 4 applies for any even  $k \geq 6$ , since a single-dipped preference over a cycle of even length, with a dip point  $v$ , coincides with the single-peaked one with the peak point that is antipodal to  $v$ .

We therefore focus on odd  $k \geq 7$ . Due to space limitations, we assume for the sake of contradiction that a false-name-proof and



**Figure 8.** Type profiles used for  $C_7$  in the proof of Theorem 9. On each gray vertex there is some agent, and one of the vertices with label  $f$  must be chosen under the profile. The proof derives a contradiction on  $\theta^*$ .

Pareto efficient social choice function  $f$  exists for  $C_7$ , and w.l.o.g. that  $f(\theta) = v_1$  for any  $\theta$  s.t.  $I(\theta) = V$ .

Consider a type profile  $\theta'$  s.t.  $I(\theta') = \{v_1, v_2, v_6, v_7\}$  (see the top-right cycle in Fig. 8). Since  $f$  is false-name-proof and Pareto efficient,  $f(\theta')$  must be either  $v_3$  or  $v_4$ ; otherwise some agent has incentive to add fake identities. Furthermore, for the profile  $\theta^*$  s.t.  $I(\theta^*) = \{v_1, v_2, v_7\}$ ,  $\text{PE}(\theta^*) = \{v_4, v_5\}$  holds. Therefore,  $f(\theta^*) = v_4$  holds; otherwise the agent located at  $v_7$  has incentive to add fake identity on  $v_6$ , which moves the facility to either  $v_3$  or  $v_4$ .

On the other hand, for another profile  $\theta''$  s.t.  $I(\theta'') = \{v_1, v_2, v_3, v_7\}$  (see the bottom-left cycle in Fig. 8),  $f(\theta'')$  must be either  $v_5$  or  $v_6$  due to symmetry. Therefore, for the above  $\theta^*$ ,  $f(\theta^*) = v_5$  must hold, which contradicts the condition of  $f(\theta^*) = v_4$ . Almost the same argument holds for any larger odd  $k$ .  $\square$

## 6 CONCLUSIONS

We tackled whether there exists a false-name-proof and Pareto efficient social choice function for the facility location problem under a given graph. We gave complete answers for path, tree, and cycle graphs, regardless whether the preferences are single-peaked or single-dipped. For hypergrid graphs, an open problem remains for single-dipped preferences. When such social choice functions exist, completely characterizing their class of such social choice functions is crucial future work, as many other works did for continuous structure [17, 22, 27]. Investigating randomized social choice functions is another interesting direction.

## ACKNOWLEDGEMENTS

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