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An Analysis of Two Methods for Reflection Imaging and Decisive Inequalities of Interference Images by Thin Lenses Resulting in Real Images

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Abstract. The measurement of the focal length of a bi-convex lens using auto-collimation imaging is a fundamental experiment in college. However, some teachers and students are confused by unwanted reflection images unaffected by the plane mirror when tracking the accurate image reflected by that mirror. By delving into the formulas of the reflection imaging of thin lenses through two different reflection imaging methods and introducing the notion of focal power, this paper obtained the inequalities determining the formation of the reflection images. These inequalities can explain the existence of the reflection images, analyze the general cases of reflection imaging using a thin lens, and do a quantitative calculation of a thin lens with specific parameters. This work can help teachers and students understand the reflection imaging of thin lenses, and it can be designed as an experimental procedure to foster greater scientific interest among students in this subject by combining different teaching methods.

Keywords. Thin lenses; Reflection imaging methods; Inequalities; Interference images.

1. Introduction

Lenses play a fundamental role in physics, mechanical engineering, materials science, clinical medicine, and other fields, and their focal lengths have been extensively researched [1-4]. Using the auto-collimation method to measure the focal length of the bi-convex is a basic experiment in university labs. In this experiment, some teachers and students are confused about the reflection image shown in Figure 1, which is not unaffected by moving or rotating the plane mirror [5].

To deeply understand, researchers have conducted massive research on the reflection imaging of thin lenses. Generally, they analyze the reflection phenomena using the step-by-step imaging method under paraxial approximation. Among them, Yousheng Shu establishes the object-image relationship of thin lenses through refraction-reflection-refraction (R-L-R) imaging, while Guijuan Yang through thin lenses' first-sphere imaging [6, 7]. In addition, they primarily discuss and apply the relationships to measure the radii and refractive index of the thin lenses [8].

However, the reason why the reflection images of thin lenses exist there in different surroundings is unknown. To get that reason, we did an in-depth analysis of the reflection

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imaging formulas of thin lenses in this paper and obtained the inequalities that determine whether an image is produced through the reflection imaging of a thin lens. Based on these inequalities, various scenarios for reflection imaging of thin lenses are explored and discussed. This paper provides a deep understanding of these phenomena for teachers and students.

Currently, college physics experimental teaching places great emphasis on developing students' abilities and improving classroom teaching methods [9-12]. Various teaching models such as micro-lectures, heuristic teaching, flipped classrooms, graphical programming environments, and project-driven approaches are employed [13,14]. Through these methods, teachers can design experiments on reflection imaging with thin lenses to help students develop a scientific mindset for problem discovery and solving [15,16]. Exploring problem-based learning methods, adopting a combination of qualitative and quantitative approaches, and exploring improvements and evaluations in teaching methods are all effective measures and approaches in the teaching process [17].

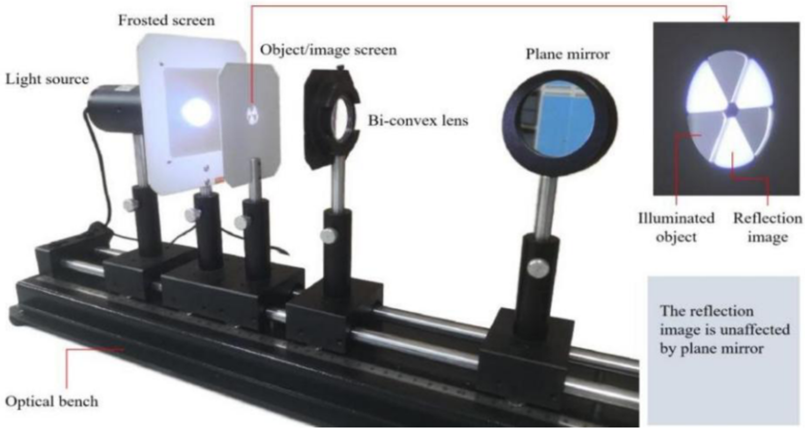

Figure 1. The reflection image of bi-convex

2. Two Reflection Imaging Methods of Thin lenses

A thin lens is a coaxial spherical system consisting of two refractive spheres. Through this lens, light undergoes refraction at its first sphere, reflection at the second sphere, and then refraction again through the first sphere to form an image. This imaging method is known as R-L-R imaging. Furthermore, when the first sphere of the thin lens is a concave spherical surface, light can also be imaged solely through the first sphere of the thin lens. Hence, there are two approaches to imaging in reflection utilizing a thin lens.

To analyze the reflection image of a thin lens, we agreed that the light rays propagate from left to right and follow the sign rule: The length of the line starts from the vertex of the sphere, and when the intersection point of the ray and principal axis is on the right of the vertex, the length of line is positive, and vice versa, if it's on the left side, it's negative; The distance from object point or image point to principle axis is positive above and negative below; Use positive values for each length quantity in figure.

Assume the thin lens is displayed in an environment with a specific refractive index n and has a refractive index n'. The first sphere of the lens is marked as "S_1", while the second sphere is "S_2". The schematic diagrams of *R-L-R* imaging and first-sphere reflection imaging can be expressed as Figures 2(a) and 2(b), respectively. The corresponding lens formulas for these maps are equation (1) [6, 8] and equation (2), respectively.

$$\frac{n}{l'}+\frac{n}{l}=-2\left(\frac{n'-n}{r_1}-\frac{n'}{r_2}\right) \tag{1}$$

$$\frac{1}{l'}+\frac{1}{l}=\frac{2}{r_1} \tag{2}$$

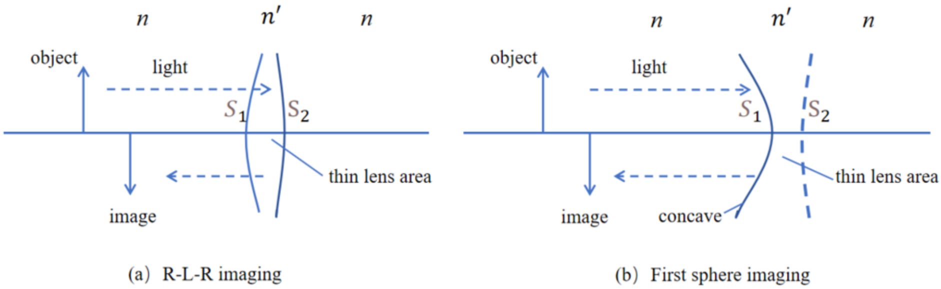

Figure 2. Schematic diagrams of reflection imaging using a thin lens

3. Decisive Inequalities for the Formation of the Reflection Images of Thin Lens Corresponding to the Two Reflection Imaging Methods

According to the sign rule, if a thin lens forms a real image, both the object distance and the image distance must be less than 0, indicating that the object and the image are on the same side. Since the absolute refractive index of the thin lens is greater than 1, we can derive from equations (1) and (2) to obtain the following two inequalities.

$$\frac{n'-n}{r_1}>\frac{n'}{r_2} \tag{3}$$

$$r_1<0 \tag{4}$$

Inequality (3) indicates that the ability of a thin lens to form a real image through the process of R-L-R imaging depends on various factors, including the radii of the two spheres, the refractive index of the thin lens, and the refractive index of its surrounding medium. Inequality (4) states that for the first sphere of a thin lens to produce a real image through reflection, it must have a concave shape, implying that its radius should be negative.

If we use focal power instead, the refraction focal power for S_1 sphere of the thin lens is $\Phi_{1refr}=\frac{n'-n}{r_1}$, the reflection focal power is $\Phi_{1refl}=-\frac{2n}{r_1}$, and for S_2 sphere, the reflection focal power is $\Phi_{2refl}=-\frac{2n'}{r_2}$ in the direction of the light. Consequently, inequalities (3) and (4) can be modified as follows.

$$\Phi_{refl}=2\Phi_{1refr}+\Phi_{2refl}>0 \tag{5}$$

$$\Phi_{1refl}>0 \tag{6}$$

It is evident that the total focal power of light passing through a thin lens must be greater than 0 to form a real image. In the case of thin-lens reflection imaging, inequalities (3) and (4), or (5) and (6), serve as decisive formulas for R-L-R imaging and first-sphere reflection imaging respectively.

4. Application

4.1. The General Cases of Reflection Imaging Using Thin Lenses

Table 1. The general cases of *R-L-R* imaging for thin lenses lactated in different media

Thin lens	Radius	$n'>n$			$n'<n$		
		Φ_{refl}	Image	condition	Φ_{refl}	Image	condition
Bi-convex	$r_1>0,\ r_2<0$	>0	√	-	>0	√	$\frac{n'-n}{r_1}>\frac{n'}{r_2}$
Plano-convex	$r_1=\infty,\ r_2<0$	>0	√	-	>0	√	-
	$r_1>0,\ r_2=\infty$	>0	√	-	<0	-	-
Concavo-convex	$r_1<r_2<0$	>0	√	-	>0	√	-
	$r_2>r_1>0$	>0	√	$\frac{n'-n}{r_1}>\frac{n'}{r_2}$	<0	-	-
Bi-concave	$r_1<0,\ r_2>0$	<0	-	-	>0	√	$\frac{n'-n}{r_1}>\frac{n'}{r_2}$
Plano-concave	$r_1<0,\ r_2=\infty$	<0	-	-	>0	√	-
	$r_1=\infty,\ r_2>0$	<0	-	-	<0	-	-
Convexo-concave	$r_2<r_1<0$	>0	√	$\frac{n'-n}{r_1}>\frac{n'}{r_2}$	>0	√	-
	$r_1>r_2>0$	<0	-	-	<0	-	-

Φ_{refl} : the total focal power of reflection imaging using a thin lens; n’: the refractive index of thin lens; n: the refractive index of surrounding medium

The first-sphere reflection imaging is simple and will not be discussed here. By analyzing inequalities (3) and (5), the general cases of R-L-R imaging can be determined, as shown in Table 1.

Table 1 presents two primary findings. First, for bi-convex, plano-convex, concavo-convex, or convexo-concave in a place where the refractive index of the thin lens is higher than that of its surrounding environment, they can obtain a real image. However, in this scenario, the bi-concave or plano-concave fails to achieve an image. Second, for bi-convex, plano-convex, concavo-convex, bi-concave, plano-concave, or convexo-concave in a place where the refractive index of the thin lens is lower than that of its surrounding medium, they can produce a real image.

These general cases can be designed to help understand the reflection images when measuring the focal length of a thin lens using auto-collimation imaging. First, measure the focal length of a thin lens using the auto-collimation method and finish it. Second, show the phenomena of the reflection images of the thin lens and raise the question of how they happen. Third, students experiment and discuss it. Fourth, send the materials that can help understand these phenomena, let the students learn, and do the calculation. Fifth, discuss and conclude. Six, show it together on the experiment report. Besides, other teaching methods can also be designed into the procedure.

4.2. Analysis of the Interference Images

Take the experiment of determining the focal length of a H-K9L uncoated bi-convex lens using the auto-collimation method as an example. The experimental apparatus is displayed in the air, and Figure 3 displays the main parameters of the bi-convex lens. During the experimental procedure, when positioning the bi-convex in a specific location, a clear interference image appears on the object screen. This image remains unaffected even when moving or rotating the plane mirror. Thus, we can conclude that this interference image is a reflection image. However, how exactly does the bi-convex reflect the light to form a real image still remains unknown.

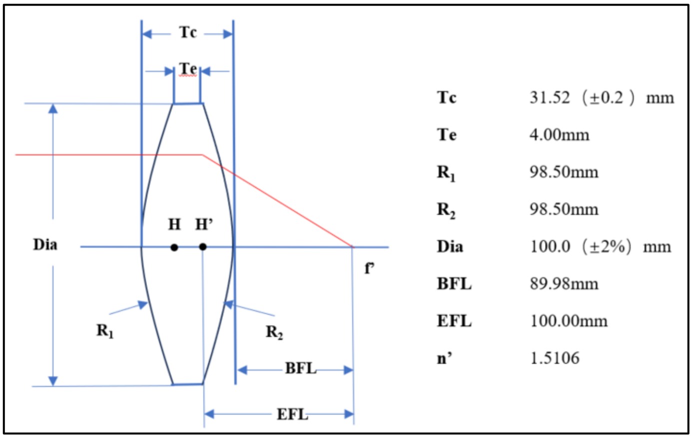

Figure 3. The bi-convex used to determine the focal length and its basic parameters

To get the reason, the following quantitative calculations will be conducted utilizing the given convex parameters. For S_1 sphere of the convex, the refraction focal power is

$$\Phi_{1refr} = \frac{n' - n}{r_1} = \frac{1.5106 - 1}{0.09850} \approx 5.1838/\mathrm{m},$$

the reflection focal power is

$$\Phi_{1refl} = -\frac{2n}{r_1} = -\frac{2}{0.09850} \approx -20.3046/\mathrm{m}$$

For S_2 sphere, the reflection focal power is

$$\Phi_{2refl} = -\frac{2n'}{r_2} = -\frac{2\times 1.5106}{-0.09850} \approx 30.6721/\mathrm{m}$$

Assume the interference image is introduce by R-L-R imaging, the total focal power of bi-convex is

$$\Phi_{refl} = 2\Phi_{1refr} + \Phi_{2refl} \approx 41.0396/\mathrm{m}$$

By inequality (5), we confirm that this image can be introduced by R-L-R imaging of the bi-convex lens.

Since the first sphere has a reflection focal power lower than 0, according to inequality (6), it can be concluded that obtaining the interference image through the first-sphere imaging is impossible.

Based on the above quantitative analysis, we can conclude that the interference image observed in the experiment to determine the focal length of a bi-convex lens using auto-collimation imaging is caused by R-L-R reflection imaging.

5. Conclusion

By analyzing the two methods of reflection imaging with thin lenses and introducing the concept of focal power, We have reached the following conclusions.

1) The imaging of thin lenses through reflection includes two methods: R-L-R imaging and first-sphere imaging. R-L-R imaging depends on the radii of the two spheres, the refractive index of the thin lens, and its surrounding medium, while first-sphere imaging on the radius of its first sphere.

2) To form a real reflection image, the parameters of thin lenses must adhere to the inequalities (3)~(6). For first-sphere imaging, inequality (4) indicates that the first sphere of the thin lens must be concave. In addition, analyzed from a focal power perspective, inequalities (5) and (6) show that the total focal power of the sub-interface through which the light passes in the thin lens must be greater than 0.

3) The inequalities of reflection imaging in thin lenses can be utilized to analyze the general situations of thin lens reflection imaging and qualitatively analyze the interference images that appear in the experiment of determining the thin lens' focal length through the auto-collimation method.

Furthermore, although this paper provides a basic understanding of the reflection imaging of thin lenses, other aspects are still worth studying. In terms of education, measuring the focal length of a thin lens using the auto-collimation method can be combined with the experiment on thin lens reflection imaging, which can help address students' confusion, enhance their knowledge, and foster their interest in scientific exploration. Regarding applications, measuring the focal length of optical lenses can be further studied. Additionally, in terms of theory, there is an expectation for additional research on the reflection imaging of thick lenses or multiple lenses.

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