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From Chaos to Control: Leveraging Multi-Expert Strategies for Predictive Accuracy in Autonomous Systems

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Abstract. This paper addresses the challenge of predicting chaotic behavior in automatic control systems and robotics, especially in unstable environments. Chaotic elements in observational data diminish the effectiveness of traditional statistical methods, necessitating novel predictive approaches. We propose a predictive framework based on a multi-expert data analysis model for control applications. Preliminary predictions are generated by software experts as weak classifiers, while a supervising expert consolidates these into a final decision. This approach resembles stacking algorithms used in ensemble decision-making. Our methodology enhances predictive accuracy in chaotic environments, leveraging the structural redundancy of multi-expert systems for improved robustness. Empirical results indicate that it strengthens decision-making in unpredictable scenarios, paving the way for future research on managing chaotic dynamics in automatic control and robotics.

Keywords. Predicting, chaotic processes, automatic control systems, robotics, multi-expert data analysis, instability.

1. Introduction

Contemporary methods for managing complex dynamic systems increasingly rely on predictive technologies that utilize advanced algorithms. A major challenge is predicting the behavior of chaotic systems in unstable environments, where changes in the state vector are hard to anticipate. These systems encompass various phenomena, from turbulent gas-dynamic processes to the volatility of capital markets. The main difficulty lies in chaotic components represented by complex nonlinear differential equations and incomplete disturbance data, necessitating robust predictive techniques for automatic control and robotics [1-3].

Traditional statistical extrapolation methods are often inadequate for these chaotic systems due to their unpredictability. This has created a demand for novel computational approaches [4, 5]. Leveraging high-performance computing and advancements in data

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analytics and AI, these methods enhance prediction accuracy and reliability, crucial for effective decision-making in dynamic environments.

One promising solution is the Multi-Expert System (MES) framework, which coordinates the efforts of multiple independent Software Experts (SEs) to improve predictive outcomes [6-8]. Unlike decentralized multi-agent systems, MES employs a collaborative approach where each SE generates preliminary predictions that a supervisory expert aggregates into a final decision[9-11]. This resembles ensemble learning, particularly stacking algorithms [12-14]. By utilizing the unique strengths of each SE, MES enhances prediction reliability and robustness, outperforming traditional methods.

The two-tiered structure of MES—integrating findings from various SEs—offers a flexible framework well-suited for navigating the complexities and uncertainties of chaotic systems, ultimately enabling more effective and resilient automated control strategies in robotics.

2. Data Model

To formally describe this time series, we employ a conventional additive model. This model posits that the observed time series can be decomposed into several distinct components, each contributing to its overall structure. These components typically include a systematic trend, seasonal variations, and random noise. By representing the time series in this manner, we can analyze and isolate the individual effects of each component, thus enhancing our ability to predict future values with greater precision.

Mathematically, an additive model can be expressed as:

$$z_k = x_k + v_k, \quad k = 1, \dots, N \tag{1}$$

where the systemic component $x_k, \quad k = 1, \dots, N$ represents the regular dynamics of the process being predicted, which is utilized in the formation of control decisions, and $v_k, \quad k = 1, \dots, N$ is the random component reflecting observational noise that needs to be filtered.

Traditional additive models, as per Wold's decomposition [15], assume the systemic component x_k is a smooth process, yet under unstable environmental conditions, chaotic dynamics emerge-characterized by high sensitivity to initial conditions which complicate long-term predictions and require advanced frameworks like nonlinear dynamics and bifurcation theory [16-20].

Typically, the noise component $v_k, \quad k = 1, \dots, N$ is modeled as white noise. However, in practice, this is often an inadequate representation, particularly for systems in unstable environments. In these cases, the noise is a non-stationary random process, better represented by a Gaussian model of the Huber type [21] $v \in (1 - \varepsilon)N(0, \ \sigma_1^2) + \varepsilon N(0, \ \sigma_2^2), \quad \varepsilon \in (0, \ 1), \quad \sigma_2 >> \sigma_1$, where the contamination coefficient ε is usually significantly less than 1. For heteroscedastic non-stationary processes, the variance may change over time $\sigma = \sigma(t)$.

In robotics and automatic control systems, traditional computational methods often struggle with chaotic conditions, as they are slow to adapt to fluctuations, whereas Multi-Expert Systems (MES), which leverage a group of Software Experts (SEs) to provide tailored strengths, enhance adaptability and resilience to unpredictable changes, ultimately supporting more reliable predictions and decision-making, despite challenges in combining conflicting outputs and varying predictor accuracies.

3. Problem Statement

From a mathematical standpoint, each software expert (SE) engaged in the predicting function embodies a transformation, which can be articulated as follows:

$$Exp(S,p): \quad [Y_S,\ y_k] \rightarrow \tilde{x}_{k+\tau}, \quad k = 1,\ldots,\ N_p \tag{2}$$

where S is the structure of the predicting algorithm, parameterized by the set p, τ is the predicting horizon, and N_p is the total number of predicts generated. The input data for generating the prediction is a set of retrospective data

$$Y_S = \{y_i, \quad i = 1,\ldots,\ N_S\} \tag{3}$$

used for training the SE, along with the current observation y_k.

The ensemble of m simultaneously operating SEs $Exp_i(S,p)$, $i = 1,\ldots,\ m$ allows the generation of a set of possible predictions at each predicting step $k = 1,\ldots,\ N_p$:

$$\tilde{X}_{k+\tau} = \{\tilde{x}_{k+\tau}(i), \quad i = 1,\ldots,m\}. \tag{4}$$

The objective of the proposed methodology centers around the development of a System of Software Experts (SEs), denoted as SEs (2). This system is tasked with the intricate process of aggregating and jointly analyzing the outputs of individual predictive models, each contributing partial predictions, symbolically represented as PF (4). The fundamental goal is to synthesize these partial predicts into a cohesive, comprehensive prediction that optimally aligns with a designated efficiency metric $Q(\tilde{X}_{k+\tau},\ x_{k+\tau})$

$$\tilde{x}^*_{k+\tau} = extr(Q(\tilde{X}_{k+\tau}, \quad x_{k+\tau})), \quad k = 1,\ldots,N_p. \tag{5}$$

In the context of linear computational frameworks within robotics and automatic control systems, we assert that the average value of this prediction quality indicator, when evaluated across N_p discrete time steps, will reach an optimal state. This underscores the importance of employing robust statistical metrics to assess the predictive efficacy of models operating in dynamic environments. Among these metrics, Mean Squared Error (MSE) is a prominent criterion, capturing the average of the squared differences between predicted and actual values. Mathematically, MSE is expressed as:

$$MSE = \left(\frac{1}{N_p}\sum_{k=1}^{N_p}(\tilde{x}_{k+\tau} - x_{k+\tau})^2\right)^{\frac{1}{2}} \tag{6}$$

or the Mean Absolute Deviation (MAD):

$$MAD = \frac{1}{N_p}\sum_{k=1}^{N_p}|\tilde{x}_{k+\tau} - x_{k+\tau}|. \tag{7}$$

Prediction is a vital tool for enhancing decision-making in complex systems like robotics and automated control. Metrics assessing predictive accuracy should be seen as preliminary indicators guiding system improvements, not final outcomes. The true value of predictions lies in their ability to enhance control system efficiency; accurate predicts of environmental variables can significantly improve robotic responses to changing conditions.

For instance, in navigation, the accuracy of predicting obstacles is important, but success should be measured by improvements in operational efficiency, such as faster travel times and better obstacle avoidance. Thus, evaluating predictive quality requires consideration of broader system performance metrics linked to informed decision-making. Ultimately, the impact of predictions on system effectiveness is what determines their true value.

4. Methodology

Numerous methodologies exist for creating predictive algorithms that predict non-stationary stochastic processes, particularly in robotics and automated control. We focus on a method using polynomial approximation with a moving window, followed by extrapolation over a defined future time, τ.

According to the Weierstrass approximation theorem, any continuous process can be uniformly approximated by a sequence of polynomials $P_c(t), \quad c = 1,2,\ldots$, where c is the order of the approximating polynomial [22]. Due to the rapid fluctuations of the observed process, the polynomial model uses a moving observation window that matches the prediction interval. This is crucial in robotics and automated control, where conditions can change unpredictably. In applications like autonomous navigation, adapting to variable environments is essential. The model captures local trends and quickly responds to changes by continuously integrating the latest data for accurate predictions. This approach enhances prediction accuracy and provides the robotic system with the capability to make timely, informed decisions based on current conditions:

$$Y_{k-L+1,k} = (y_{k-L+1,k},\ldots,y_k)\,, \quad k = 1,\ldots,\ N_p. \tag{8}$$

For each SE, the structure $S_c(i), \quad i = 1,\ldots,\ M$, is fixed, determined by the chosen order of polynomial approximation m. The parameters $p(i), \quad i = 1,\ldots,\ M$ of each approximating model are determined using the traditional least squares fitting method, i.e., such that:

$$p^{*}\colon\ \sum_{j=1}^{L}(y_j - P_j(S_c,\ p))^2 = min. \tag{9}$$

The predict is then performed by directly substituting the corresponding prediction time into the value of the approximating polynomial, i.e.,

$$\tilde{x}^{*}_{k+\tau} = P_m(k+\tau), \quad k = 1,\ldots,N_p. \tag{10}$$

A fundamental method for collaborative decision-making among independent SEs is simple averaging, but weighted averaging using Bayesian risk evaluations offers a more effective approach [23]. This is crucial in robotics and automated control, where uncertainty and environmental variability are significant. The Bayesian framework considers the reliability of different SEs by applying weights based on prior confidence derived from performance metrics. This results in a more nuanced amalgamation of predictions, enhancing decision-making, situational awareness, and adaptability in complex scenarios, ultimately improving operational efficiency in applications like autonomous vehicles and automated processes. Using the same base of retrospective data Y_S, the predicting task is sequentially solved for each SE, and the average predicting error $\delta\tilde{y}^i_{k,\tau}$ and the corresponding Bayesian risks $r^{\tau}_{k,i}$ are assessed. The final decision is based on the weighted prediction:

$$\tilde{y}_{k+\tau} = \sum_{i=1}^{m} w_{k,i}\tilde{y}^i_{k+\tau} \,/ \sum_{i=1}^{m} w_{k,i},\ k = 1,\dots,\ N-\tau. \quad (11)$$

That is, it is formed as a linear combination of partial predicts $\tilde{y}^i_{k+\tau}$ with weights inversely proportional to the Bayesian risks $w_{k,i} = (r^{\tau}_{k,i})^{-1}$.

5. Experiments

We present a MES for predicting robotic control systems using a low-order statistical extrapolator, benchmarked against a linear extrapolator on a chaotic model's time series. As shown in table 1, our analysis evaluates prediction accuracy through Mean Squared Error (MSE) and Mean Absolute Deviation (MAD), focusing on the impact of observation window size and prediction horizon on accurate prediction for real-time decision-making in robotics.

Table 1. Dependence of Predicting Quality Indicators on Model Parameters

τ=3, L:	**60**	**120**	**180**
MSE	8.99	12.61	14.56
MAD	5.80	7.41	9.17
L=120, τ:	**5**	**15**	**30**
MSE	8.99	12.61	14.56
MAD	5.80	7.41	9.17

Figure 1 illustrates how varying observation window sizes (L=60, 120, 180) and prediction horizons (τ=5, 15, 30) affect prediction quality. Errors increase with larger horizons, significant in chaotic robotic systems due to sensitivity to initial conditions. While increasing the window size enhances smoothing, it doesn't reduce bias from chaotic behavior, posing challenges for precise real-time applications. To overcome the limitations of a single linear extrapolator, we introduce a MES featuring three Software Experts (SEs): a linear extrapolator (SE1), a quadratic extrapolator (SE2), and a cubic extrapolator (SE3).

As shown in figure 2, all three models are initially trained with a dataset of size L = 120 and a prediction interval of τ = 30 time steps. In particular, the reliance on continuity and smoothness assumptions in polynomial models poses a limitation, as chaotic systems often exhibit nonlinear interactions and sensitivity to initial conditions, which violate these assumptions. Consequently, polynomial models tend to produce misleading predictions, as they are unable to capture the erratic shifts that characterize chaotic behavior.

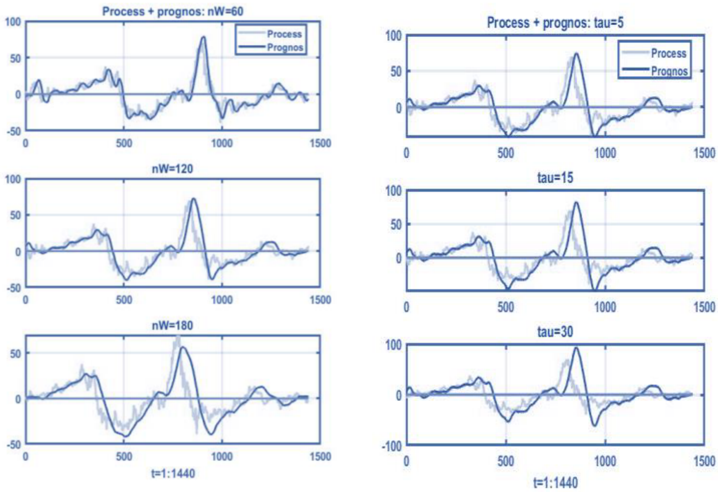

Figure 1. Impact of the learning window size L (Left Graphs) and prediction depth τ (right graphs) on predict quality

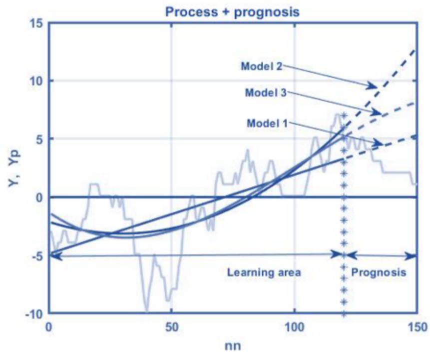

Figure 2. Illustration of the low effectiveness of SEs based on statistical extrapolation of observation series

Increasing the order of polynomial models may seem to improve predictive accuracy, but it often causes overfitting, reducing generalizability and reliability in robotics, where accurate predictions of dynamic environments are crucial. As robotics increasingly interacts with chaotic environments, the development of robust predictive methodologies becomes essential for operational reliability, with future efforts aimed at enhancing adaptability and performance in unpredictable settings.

6. Results

Implementing adaptive Software Experts in a Multi-Expert System (MES) enhances predictive capabilities in robotics by reducing errors in dynamic system predictions.

$$\tilde{y}_{k+\tau} = \tilde{y}_{k+\tau}(i^*_{k-1}), \quad (12)$$

where $i^*_{k-1}\ satisfies\ MSE_{i^*} = min\ \ [MSE_1, \ldots, MSE_m]_{k-1}$.

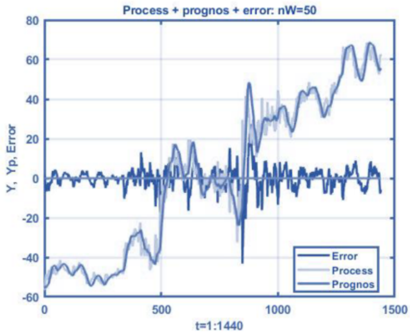

Figure 3. Monitored Process, 3rd-Order Polynomial Prediction, and Prediction Error

To illustrate, we evaluate the average Mean Squared Error (MSE) of daily predictions from SEs 1-3 and the MES strategy, following the algorithm described in equation (11). The accuracy metrics from the numerical experiments for SEs 1-3 and MES (as the fourth component in vector $\hat{S}$) are presented as average MSE values over a 10-day period: $\hat{S}$ = [13.5, 13.1, 12.8, 12.6].

The data show that implementing MES improves prediction quality and solution stability, as illustrated by methodologies in equations (6) and (7). Evaluating prediction quality necessitates a thorough analysis of task efficacy, particularly in robotics and automated control, where adaptability and accuracy are essential for enhancing decision-making. This MES approach demonstrates how adaptive strategies lead to more resilient predictions in chaotic systems, resulting in better outcomes in dynamic, unpredictable environments.

7. Conclusion

This article examines how informational redundancy can improve decision-making in robotics and control systems, drawing inspiration from John von Neumann's concept of reliable systems using unreliable automata. We introduce the Method of Extrapolation Strategies, which leverages an ensemble machine learning approach to generate predictive decisions in chaotic environments, overcoming limitations of traditional probabilistic methods.

Our research demonstrates that MES can enhance predictive accuracy and stability, key factors for autonomous systems. However, empirical findings lack a solid theoretical foundation, highlighting the need for further research. Future directions include:

- Developing advanced state estimation models for adaptability in unpredictable settings.
- Creating a multi-expert system to address MES limitations in chaotic environments.
- Establishing a theoretical framework for control theory in indeterministic conditions using entropy measures.

These efforts aim to enhance the effectiveness and robustness of robotics and automated control systems in real-world applications.

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