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The Role of the Mathematical Modelling in the Analysis of Market Price Conditions

Nataliya MUTOVKINA[1]
Tver State Technical University, Tver (170023), Russia

Abstract. The article describes the results of a study of the influence of various factors on pricing in modern market conditions. Effective pricing policy plays a crucial role in ensuring the competitiveness and profitability of business entities. Maximizing income is one of the main goals of business entities. However, earning income depends on the effectiveness of the company's pricing policy. If the price of a product is too high, consumers will purchase it from competing companies. Underestimating the price of a product in comparison with competing products can cause potential buyers to distrust the product of this company. The psychological factor “it doesn’t happen cheap and good at the same time” is triggered here. Therefore, the management of business entities must constantly remember to balance price and quality, the price of their goods, and the prices of competitors’ goods. The main factors influencing the price level are supply and demand. The higher the price, the lower the demand, and the higher the supply. Therefore, one of the main tasks that the management of commercial enterprises solves is determining a fair price for a product. The article defines a fair price and considers the methods of its establishment. The most effective method is mathematical modelling. The article shows characteristic mathematical models describing the behaviour of market participants. Special attention is to the cobweb model, which gives a complete picture of the tendency of the price of a product to a state of market equilibrium. Of course, the system of factors influencing the pricing strategy is much broader, so the author recommends including other methods in the pricing methodology, for example, demand-oriented pricing, cross-elasticity, and an econometric approach to price forecasting. These and other techniques, taking into account the specifics of the market of goods (or services) under consideration, can be used to build mathematical models describing changes in the result (price) when influencing factors change. Mathematical modelling allows the management of business entities to make informed decisions regarding the prices of goods.

Keywords. Market; market price; pricing; modeling; market equilibrium; decision-making.

1. Introduction

The issue of setting optimal prices for goods (services) does not lose relevance and will be the most important economic issue as long as there is a market and competition. The pricing methodology includes the choice of the method of setting and pricing and the sequence of actions for its application. The main techniques include market, cost, parametric, and administrative methods [1-3]. Pricing tactics depend on pricing

[1] Correspondence author, Nataliya MUTOVKINA, Tver State Technical University, Tver (170023), Russia; Email: letter-boxNM@yandex.ru

methodology, a system of specific practical measures to manage prices for goods and services [4]. Pricing tactics are a logical continuation of the strategy and depend on it. The techniques of pricing tactics allow you to implement a policy in daily activities in the market of goods and services. All pricing tactics are in two groups. The economic techniques included in the first group imply a practice change in prices (increase or decrease) to achieve the set goals. Psychological techniques in the second group assume an impact on the buyer without significant price changes. Both of these groups of techniques are interrelated since a change in the price of a product entails a defined reaction of the buyer and, conversely, seeing the response, the entrepreneur can adjust the purchasing behavior by varying the prices of the product.

However, changing the prices of goods (services) without preliminary calculations in actual market circumstances is a bad practice, leading, in most cases, to the collapse of companies. Conducting experiments with a real object can be expensive for the owner. Therefore, mathematical modeling comes to the rescue [5, 6]. Mathematical models allow you to predict the development of economic systems, and mathematical models of supply and demand give you to calculate the equilibrium market price. The knowledge of which allows companies operating in the market to optimize their activities [7].

In this regard, the study aims to analyze the effectiveness of mathematical pricing models in modern market conditions. The study considers a demand-driven pricing model and a cobweb pricing model.

The study's novelty lies in comparing the effectiveness of the applied models and recommendations for their inclusion in the pricing methodology of an economic entity. The entity in the study is a farm poultry meat and sells through its retail store.

The practical task solved in the study is to achieve the optimal price for poultry meat in this company.

2. Mathematical models in making price decisions

The pricing strategy of an economic entity is of great importance for its effective operation. The activity is as effective, as a result of which the company receives a net profit sufficient for further development, expansion of production, creation of a reserve fund, payment of dividends (if it is a joint-stock company), and the like. The company needs to conduct a competent pricing policy based on optimal prices for goods and services to make a profit. The optimal price can be considered the one at which consumers will willingly buy the product, and the company will be able to recoup production costs and pay off mandatory payments from the sales revenue.

This section discusses the most relevant mathematical models for deciding the price of a product, as well as methods for obtaining initial information for modeling.

2.1. Literature review

Many publications are currently devoted to the theoretical description of mathematical models in economics and the specifics of their application. Many of them are textbooks, for example [1, 4, 8], workshops that consider typical models using numerical examples [9], and articles devoted to solving specific problems, for example [5-7, 10].

The textbook [11] describes the features of economic and mathematical modeling and the stages of model construction. The authors did not ignore the issues of mathematical modeling of equilibrium. For the equilibrium price, you can set various mathematical models. Some researchers use a cobweb model for this [10, 12, 13], other scientists use econometric forecasting methods [14-16].

Thus, currently, mathematical modeling is used very widely to solve a wide variety of economic problems. In particular, to solve optimization problems in pricing [17-19]. A whole system of mathematical models and methods is for this purpose. The choice of the model remains the researcher's right, and the selection criteria are most often the purpose of modeling, the initial data, as the methodological and software tools.

2.2. *Types of pricing models*

Different models can be used to determine the optimal price for a product, depending on the initial data and economic conditions. In market theory, the conditions are maximum revenue and maximum profit. The condition for maximum revenue is by the expression:

$$\frac{\Delta Q}{Q} = -\frac{\Delta P}{P} \tag{1}$$

The maximum revenue is achieved at such a price when the percentage change in sales volume is the percentage change in price (with the opposite sign). Accordingly, if the elasticity is less than one at the current price, then it is advantageous to increase the price to increase the revenue. Conversely, if the elasticity is more than one, then it is profitable to reduce the cost to increase revenue [20].

Although revenue is one of the most significant indicators of a company's performance, it is more important to determine the prices at which maximum profit. The condition for achieving maximum profit looks like (2) or (3):

$$E_{P-AVC} = \frac{P - AVC}{P} \cdot E = 1 \tag{2}$$

$$\frac{\Delta Q}{Q} = -\frac{\Delta P}{P} \cdot \frac{P}{P - AVC} \tag{3}$$

where AVC are variable costs per unit of production.

Thus, the maximum profit is achieved at such a price when the percentage change in sales volume is the percentage change in price multiplied by the coefficient $\frac{P}{P-AVC}$. If the elasticity is less than $\frac{P}{P-AVC}$ at the current price, then it is advantageous to increase the price so that revenue increases. Conversely, if the elasticity is more than $\frac{P}{P-AVC}$, then it is profitable to reduce the price so that revenue increases.

It is advisable to summarize the findings in a table (Table 1) for the prices in the store.

Table 1. Recommendations on price changes to maximize revenue and profit

	$E_P^D < 1$	$E_P^D = 1$	$1 < E_P^D < \frac{P}{P - AVC}$	$E_P^D = \frac{P}{P - AVC}$	$E_P^D > \frac{P}{P - AVC}$
To maximize revenue	raise	save	reduce	reduce	reduce
To maximize profits	raise	raise	raise	save	reduce

The price elasticity of demand coefficient (E_P^D) shows how much percentage of the sales volume (*Q*) will change when the price (*P*) changes by 1%, i.e. it is calculated using the formula:

$$E_P^D = \frac{\Delta Q / Q_1}{\Delta P / P_1} = \frac{(Q_2 - Q_1)/Q_1}{(P_2 - P_1)/P_1} \tag{4}$$

where Q_1 and Q_2 are the value of demand at the same and the new price; P_1 and P_2 are the old and new prices, respectively.

There is a maximum profit and a maximum revenue at different price values. The maximum profit is always at a price higher than the price at which there is a maximum revenue.

The process of approaching normal equilibrium can done using a sequence of discrete steps, analyzing the supply and demand curves. The visualization of such a sequence is a cobweb model since the trajectory of price changes with the supply and demand lines resemble a web [11]. The construction of such a model is on the assumption that the volumes of supply and demand are functions of price:

$$y_t^D = a_0 + a_1 p_t \tag{5}$$

$$y_t^S = b_0 + b_1 p_{t-1} \tag{6}$$

where y_t^D is the volume of demand at time *t*, y_t^S is the volume of supply at time *t*, p_t is the price at time *t*, and p_{t-1} is the price at the previous time. There is a delay in the reaction to price changes. Since demand usually decreases with an increase in price and supply increases, then $a_1 < 0$, $b_1 > 0$.

The equality of the volume of demand and the volume of supply at each moment (7) completes the description of the cobweb model:

$$y_t^D = y_t^S \tag{8}$$

From equality (7), a model for the price is the form of a first-order difference equation:

$$p_t = \frac{b_1}{a_1} \cdot p_{t-1} + \frac{(b_0 - a_0)}{a_1} \tag{9}$$

The price at which equality (7) holds is as r. That is the price for which, in a state of equilibrium of supply and demand, the ratio is fair:

$$p^* = \frac{b_1}{a_1} \cdot p^* + \frac{(b_0 - a_0)}{a_1} \tag{10}$$

Accordingly, the value of the equilibrium price is by the formula:

$$p^* = \frac{c}{1 - b_1/a_1} \tag{11}$$

where $c = \frac{b_0 - a_0}{a_1}$.

The study of the process described by the cobweb model for convergence gives reason to assert:

1. If $\alpha = \left|\frac{b_1}{a_1}\right| < 1$, then for $t \to \infty$ $p_t \to p^*$.

2. If $\alpha = 1$, then at $t \to \infty$, p_t fluctuates near the equilibrium value.

3. If $\alpha > 1$, then at $t \to \infty$, the price will deviate from its equilibrium value more and more.

2.3. Methods of obtaining initial information for modeling

The information needed to build mathematical models differs depending on the specifics of the models. Different sources are also.

So, to build a demand-oriented pricing model, the initial data are the results of a survey of potential product consumers. You can interview a relatively small number of potential consumers among the regular customers of this store; if the situation is favorable, first-time customers will come to the store.

The respondent is asked: “What is the maximum price you are willing to pay for this product?” To eliminate difficulties in answering, we must give the respondent additional information regarding hypothetical prices for new goods or fact prices for similar goods identified during the previous trading day (week, month). This approach was applied to obtain selective demand functions for individual product items in one of the meat retail stores in Tver.

In a cobweb model’s building, it is necessary to know the prices of goods over the past period (price changes) and the corresponding volumes of demand (sales) and supply (production). These data are recorded by the company’s manager and formatted in the table’s form.

3. Modeling the search for an equilibrium price

Since it is enough to make a small statistical sample for modeling and the models are simple, any software environment with sufficient computing capabilities and built-in visualization tools is for their construction.

The software environment in this work is MS Excel.

3.1. Demand-Driven Pricing Model

To build the first model, we are randomly 100 buyers. The suggested price range is from 300 to 440 rubles in increments of 20 rubles. The analytic results of the information received are in the first two columns of Table 2.

Table 2. Calculations to determine the selective demand function for chicken drumstick

Price per 1 kg, rub.	Frequency	Inverse cumulative frequency	Cash receipts, rub.	Variable costs, rub.	Earnings, rub.	Elasticity coefficients	Coefficients *P*/(*P*-*AVC*)
300	4	100	30 000	24 600	5 400	-	5.56
320	5	96	30 720	23 616	7 104	0.6	4.32
340	14	91	30 940	22 386	8 554	0.8	3.62
360	18	77	27 720	18 942	8 778	2.6	3.16
380	33	59	22 420	14 514	7 906	4.2	2.84
400	18	26	10 400	6 396	4 004	10.6	2.60
420	5	8	3 360	1 968	1 392	13.8	2.41
440	3	3	1 320	738	582	13.1	2.27

The first column shows the possible options for the cost of 1 kg. The second column shows the number of respondents who indicated the appropriate price in the answer. The third column calculates the number of people ready to buy the product at that price. At the same time, it is, that those buyers who agree to pay 440 rubles for the product will buy the same product at 300 rubles. Therefore, these buyers are counted in all rows, starting from 300 rubles and above. In the fourth column, revenue is calculated as the product of the product price by the value in the "Inverse cumulative frequency" column. The highest income corresponds to the price of 340 rubles. However, revenue optimization is not always an effective result. The most important thing is to optimize for profit. If the variable cost of 1 kg of goods is 246 rubles, then you can calculate the profit from their sale (excluding fixed prices since fixed costs do not depend on the number of products sold). These calculations are in the sixth column of Table 2. The optimal price for profit turned out to be the price of 360 rubles, which corresponds to 8 778 rubles of profit.

In the price ranges of 300-320 rubles and 320-340 rubles, the elasticity coefficient is less than one, so the price must increase. If the price increases in this range, revenue and profit both grow (Figure 1).

Figure 1. The prices of achieving maximum revenue and maximum profit

In the range from 340 to 360 rubles, the average elasticity is more than one (modulo) but less than the coefficient $\frac{P}{P-AVC}$; so when the price increases in this range, revenue begins to decrease, but profit continues to grow. From 360-380 rubles, the elasticity coefficient is noticeably higher than the coefficient $\frac{P}{P-AVC}$, so both revenue and profit are reduced. Accordingly, we can recommend a retail price of 360 rubles per kilogram of goods.

3.2. Cobweb pricing model

The finding of an equilibrium price using a cobweb model is by the example of the same product. The initial data are in Table 3.

Table 3. Initial data for building a cobweb model

No. of change	Price per 1 kg, rub.	The volume of demand, kg	The volume of supply, kg
1	300	512	223
2	320	425	284
3	340	383	313
4	360	362	328
5	380	328	346
6	400	313	362
7	420	284	383
8	440	223	425

Using the Regression tool in MS Excel, we found the parameters of equations (5) and (6):

$$y^D = 1007.6 - 1.8p \quad (12)$$

$$y^S = -123.8 + 1.2p \quad (13)$$

The calculations required to visualize the model are in Table 4.

Table 4. Calculations for modelling

Price	The trajectory of the demand	The trajectory of the supply	The trajectory of movement toward the equilibrium price
300.00	477.272	246.583	477.272
300.00	477.272	246.583	246.583
430.51	246.583	407.701	246.583
430.51	246.583	407.701	407.701
339.36	407.701	295.173	407.701

339.36	407.701	295.173	295.173
403.02	295.173	373.765	295.173
403.02	295.173	373.765	373.765
358.56	373.765	318.875	373.765
358.56	373.765	318.875	318.875
389.61	318.875	357.211	318.875
389.61	318.875	357.211	357.211
367.92	357.211	330.436	357.211
367.92	357.211	330.436	330.436
383.07	330.436	349.136	330.436
383.07	330.436	349.136	349.136

Figure 2 shows a graph of the price movement to an equilibrium state.

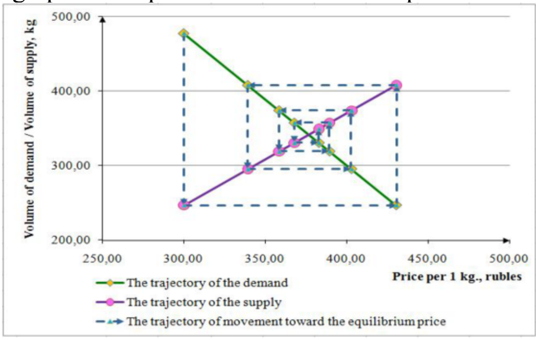

Figure 2. The cobweb model

The equilibrium price was 376.84 rubles, which is higher than the 16.84 rubles than the result obtained using the demand-oriented model.

3.3. Features and disadvantages of the models

Each model has its features and disadvantages. The advantages of both models are their simplicity, visibility, and modeling capabilities in almost any computing software environment.

The disadvantage of the demand-driven model is the subjectivity of respondents' responses. Respondents' opinions may change over time. In addition, there is no guarantee that each respondent answered honestly during the survey. Another disadvantage is the initial assumption that each respondent will buy one kilogram of goods.

The disadvantage of the cobweb model is that it considers the influence of only two factors and does not take other.

Nevertheless, it should be recognized that each of the considered models is effective under certain conditions. The demand-driven pricing model is effective when sales are of a new product; there is still no information about the volume of supply and demand. The cobweb model shows, on the contrary, good results if this information is known. Both models are based on statistical data but on different indicators.

The models proposed in the work allow us to analyze the process of establishing an equilibrium price for goods and services in a competitive market. With the help of the developed models, entrepreneurs can make informed decisions about the prices of goods and services and create an effective pricing policy for their enterprises.

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