Advances in Machinery, Materials Science and Engineering Application X
M. Giorgetti et al. (Eds.)
© 2024 The Authors.
This article is published online with Open Access by IOS Press and distributed under the terms of the Creative Commons Attribution Non-Commercial License 4.0 (CC BY-NC 4.0).
doi:10.3233/ATDE240687

Design of a Gain Scheduling Algorithm for a Three Conical Tank Interacting System

Aravind Pitchai VENKATARAMAN [a], Veeramani VEERAPATHRAN [b], Suleyman Malikmyradovich NOKEROV [c,1], Subramanian Pitchai VENKATARAMAN [d] and Faiza Bait Ali SULEIMAN [b]
[a] *Mattu University, Mettu, Oromia, Ethiopia*
[b] *University of Technology and Applied Science, Salalah, Sultanate of Oman.*
[c] *Oguz Han Engineering and Technology University of Turkmenistan, "Oguz Han" Scientific and Technology Center, Ashgabat, Turkmenistan*
[d] *M.A.M School of Engineering, Thiruchirappalli, Tamil Nadu, India*

Abstract. Controlling fluid levels in cone-shaped discharge tanks can be challenging, especially in the chemical industry. This paper presents a novel approach for achieving precise level control in such tanks using a proportional-integral (PI) control algorithm. Our method relies on a mathematical model of three interacting conical tanks arranged in a single-input, single-output (SISO) system. By implementing this algorithm, operators can effortlessly maintain the desired fluid level in all three tanks, as specified by the set point. This translates to direct and practical application of the derived controller parameters in real-time process control. The key to our approach lies in a gain scheduling algorithm, developed by analyzing the mass balance equation in the form of differential equations. We provide a detailed step-by-step derivation to guide researchers in their own controller design endeavors. The obtained PI controller values are tested in MATLAB and results are presented.

Keywords. Chemical fluid discharge tank, differential equations, nonlinear control, pi controller, conical tank, gain scheduling.

1. Introduction

Many industrial processes face control problems due to the dynamic non-linear characteristics of stationary equipment. Designing controllers for non-linear systems has always been a difficult task, especially for applications with uncertain parameters or operating under changing conditions. Because many industrial processes are non-linear and have no common solution, they represent one of the most difficult problems to fully understand and solve. To control key critical parameters such as level, flow, pressure and temperature of the fluid in the tank, the process industry requires efficient controllers [1]. Tanks of various shapes such as cylindrical, spherical and conical are widely used in industries such as gas processing, petrochemical, paper, hydrometallurgy, food processing and waste water treatment [1-7].

[1] Suleyman Malikmyradovich NOKEROV, Corresponding author, Oguz Han Engineering and Technology University of Turkmenistan, Ashgabat, Turkmenistan; E-mail: suleyman.nokerov@etut.edu.tm.

Conical tanks are widely used in the process industries because they have the advantage of not allowing solid materials to accumulate at the bottom. Conical tanks with gravity discharge provides a cost effective and convenient method of supplying both liquids and solid materials. Changing the shape of a conical tank, deadtime, delay time, setpoint changes create non-linearity characteristics and its level response depends on the geometry of its volume. Due to the non-linear nature of the system, it is very difficult to do research on [2, 8, 9].

In the process industry, the control of liquid levels in tanks and the flow between tanks is a complex process. Failure to maintain the correct level of substances in conical tanks can result in many unforeseen situations, such as reaction imbalances, release of hazardous substances, equipment damage as well as adverse consequences for subsequent operations. This is why the control of liquid levels is an important and common task in the process industry [9, 10].

This work embarks on a journey into the intricate realm of process dynamics, specifically aiding young researchers venturing into the domain of controller design. We embark on a meticulously charted path, laying out a step-by-step procedure for extracting the elusive PI controller gain values – P and I. Such quests for controller optimization have captivated the minds of many a renowned researcher [8, 10], a testament to the enduring allure of process control. Through this guided exploration, we equip fellow travelers with the tools to not only conquer the PI realm, but also delve into the broader expanse of PID control algorithms, ultimately mastering the art of manipulating process variables with precision.

2. Process Description

The process is modeled dynamically using ordinary differential equations. These equations are formulated based on mass conservation within each tank. For simplicity, we consider the fluid to have a constant density. Figure 1 is a process sketch of three vertical conical tanks are connected in series. Figure 2 then transports us to the digital arena, showcasing the SISO model to real time simulation and its operating parameters are given in table 1. Water is supplied to the tank from a preceding unit. To control the liquid level within the tank, the outflow rate is manipulated. The liquid level, represented by height h, is employed as the feedback signal for the controller to achieve and sustain the desired height.

This paper develops a linear model of a nonlinear process tank based on its transient response to changes in liquid height (h). A controller tuned using this model operates optimally only at the specific operating point analyzed. To address this limitation, the process' steady-state input-output curve is divided into five distinct linear regions.

2.1. Problem Formulation

Table 1. Operating parameters for conical tanks.

Parameter	**Description**	**Value**
R	Top radius of the cone	19.25 cm
H	Maximum total height of the tank	73 cm
β	Valve coefficient	55 cm^2/s
Fin	Maximum inflow rate of the tank	111.11 cm^3/s

2.2. Mathematical Model

According to the Law of conservation of mass,

Input – Output=Accumulation

F_{in} - F_{out}=A(dh/dt)

Where,
(dh/dt) is the rate of change of height;
A is the area of cross section, [($\pi R^2 h^2$)/H^2];
h is the liquid level height;
H is the total height of tank.

The below equations are the mathematical model of tank 1, tank 2, and tank 3 respectively.

Tank 1

$$\frac{dh1}{dt} = \left(Fin1 - \beta 12\sqrt{h1 - h2}\right) * \frac{1}{A(h1)} \tag{1}$$

Tank 2

$$\frac{dh2}{dt} = \left(\beta 12\sqrt{h1 - h2} - \beta 23\sqrt{h2 - h3}\right) * \frac{1}{A(h2)} \tag{2}$$

Tank 3

$$\frac{dh3}{dt} = \left(\beta 23\sqrt{h2 - h3} - \beta 3\sqrt{h3}\right) * \frac{1}{A(h3)} \tag{3}$$

Where,
h_1, h_2 and h_3 represents the tank 1, tank 2, and tank 3 parameters;
β is valve coefficient.

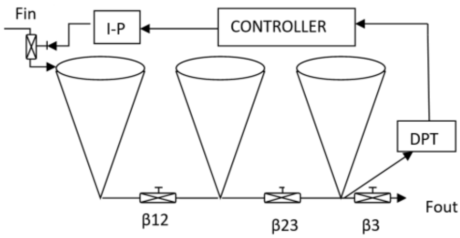

Figure 1. Three tank conical interacting system

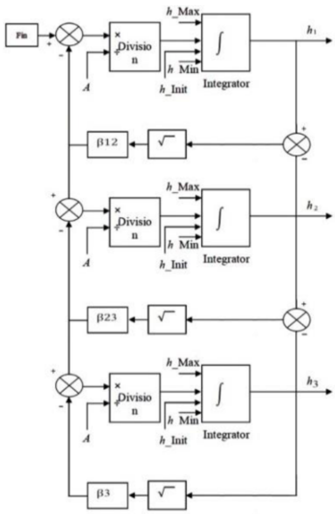

Figure 2. Simulink Model of Three Tank Conical Interacting System.

3. Adaptive Control

3.1. Gain Scheduling Method

A popular approach to nonlinear control design is a divide-and-conquer strategy that breaks down the complex problem into simpler linear sub-problems such technique is gain scheduling algorithm is a one type of adaptive control method. This allows for the application of linear control methods to nonlinear systems. While often categorized as open-loop adaptive control, gain scheduling is a more accurate description. This technique effectively handles systems with varying dynamics by adjusting controller parameters at different operating points based on corresponding process conditions. Block diagram of a system with gain scheduling method is depicted in figure 3.

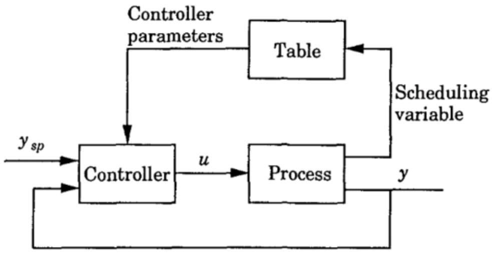

Figure 3. Block diagram of a system with gain scheduling.

Solving the SISO three tank interacting mathematical model equations 1, 2, and 3 to obtain PI controller gain values, as follows

Tank 1,

$$A(h_1)\frac{d}{dt}\partial(h_1) = \partial F_I - \frac{\beta_{12}}{2\sqrt{h_{10}-h_{20}}}\partial h_1 + \frac{\beta_{12}}{2\sqrt{h_{10}-h_{20}}}\partial h_2$$

$$A(h_1)S\partial(h_1) = \partial F_I - C_1\partial h_1 + C_1\partial h_2$$
$$(A(h_1)S + C_1)\partial(h_1) = \partial F_I + C_1\partial h_2$$

$$\partial(h_1) = \frac{\partial F_I + C_1\partial h_2}{(A(h_1)S + C_1)} \tag{4}$$

Tank 2,

$$A(h_2)\frac{d}{dt}\partial(h_2) = \frac{\beta_{12}}{2\sqrt{h_{10}-h_{20}}}\partial h_1 - \frac{\beta_{12}}{2\sqrt{h_{10}-h_{20}}}\partial h_2 - \frac{\beta_{23}}{2\sqrt{h_{20}-h_{30}}}\partial h_2 + \frac{\beta_{23}}{2\sqrt{h_{20}-h_{30}}}\partial h_3$$

$$A(h_2)S\partial(h_2) = C_1\partial h_1 - C_1\partial h_2 - C_2\partial h_2 + C_2\partial h_3$$
$$A(h_2)S\partial(h_2) + C_1\partial(h_2) = C_1\partial h_2 + C_2\partial h_3$$
$$(A(h_2)S + C_1 + C_2)\partial(h_2) = C_1\partial h_I + C_2\partial h_3$$

$$\partial(h_2)=\frac{C_1\partial h_I+C_2\partial h_3}{(A(h_2)S+C_1+C_2)} \tag{5}$$

Tank 3,

$$A(h_3)\frac{d}{dt}\partial(h_3)=\frac{\beta_{23}}{2\sqrt{h_{20}-h_{30}}}\partial h_2-\frac{\beta_{23}}{2\sqrt{h_{20}-h_{30}}}\partial h_3-\frac{\beta_3}{2\sqrt{h_3}}\partial h_3$$

$$A(h_3)S\partial(h_3)=C_2\partial h_2-C_2\partial h_3-C_3\partial h_3$$
$$A(h_3)S\partial(h_3)+C_2\partial h_3+C_3\partial(h_3)=C_2\partial h_2$$

$$\partial(h_3)=\frac{C_2\partial h_2}{(A(h_3)S+C_2+C_3)} \tag{6}$$

Substitute equation 4 in equation 5 to get ∂h_2 ,

$$(A(h_2)S+C_1+C_2)\partial(h_2)=C_1*\left(\frac{\partial F_I+C_1\partial h_2}{(A(h_1)S+C_1)}\right)+C_2\partial h_3$$

Solving the above equation, we get,

$$\partial(h_2)=\frac{C_1\partial F_I+C_2\partial h_3(A(h_1)S+C_1)}{[(A(h_2)S+C_1+C_2)*(A(h_1)S+C_1)]-[C_1C_1]}$$

From the above equation, take $C_1=\beta_{12}C_A$; $C_2=\beta_{23}C_B$; $C_3=\beta_3C_C$, we get,

$$\partial(h_2)=\frac{\beta_{12}C_A\partial F_I+\beta_{23}C_B\partial h_3(A(h_1)S+\beta_{12}C_A)}{[(A(h_2)S+\beta_{12}C_A+\beta_{23}C_B)*(A(h_1)S+\beta_{12}C_A)]-[\beta_{12}C_A*\beta_{12}C_A]} \tag{7}$$

Substitute equation 7 in equation 6,

$$\partial(h_3)*(A(h_3)S+\beta_{23}C_B+\beta_3C_C)=\beta_{23}C_B\left[\frac{\beta_{12}C_A\partial F_I+\beta_{23}C_B\partial h_3(A(h_1)S+\beta_{12}C_A)}{\begin{array}{l}[(A(h_2)S+\beta_{12}C_A+\beta_{23}C_B)*(A(h_1)S+\beta_{12}C_A)]\\-[\beta_{12}C_A*\beta_{12}C_A]\end{array}}\right]$$

$$\partial(h_3)*(A(h_3)S+\beta_{23}C_B+\beta_3C_C)*[(A(h_2)S+\beta_{12}C_A+\beta_{23}C_B)*(A(h_1)S+\beta_{12}C_A)]-[\beta_{12}C_A*\beta_{12}C_A]$$
$$=(\beta_{23}C_B*\beta_{12}C_A\partial F_I)+\beta_{23}C_B*\beta_{23}C_B\partial h_3(A(h_1)S+\beta_{12}C_A)$$

To solve the above equation here consider P and Q as below mentioned to nullifying the size of equation,

$$P=\{[(A(h_3)S+\beta_{23}C_B+\beta_3C_C)]*[(A(h_2)S+\beta_{12}C_A+\beta_{23}C_B)*(A(h_1)S+\beta_{12}C_A)];$$

$$Q=\beta_{23}C_B*\beta_{23}C_B\partial h_3$$

$$\frac{\partial(h_3)}{\partial F_I}*=\left[\frac{\beta_{23}C_B*\beta_{12}C_A}{(P-Q)}\right] \tag{8}$$

In next step we have to solve P and after solving equation P, we have to take out $\beta_{12}C_A * \beta_{23}C_B * \beta_3 C_C$ a terms from denominator of equation 8,

Then we get,

$$\frac{\partial(h_3)}{\partial F_I} * = \left[\frac{\beta_{23}C_B * \beta_{12}C_A}{\beta_{12}C_A * \beta_{23}C_B * \beta_3 C_C (P^l - Q^l)} \right]$$

$P^l - Q^l$ are expressions without $\beta_{12}C_A * \beta_{23}C_B * \beta_3 C_C$

$$\partial(h_3) = \left[\frac{\beta_{23}C_B * \beta_{12}C_A * \partial F_I}{\beta_{12}C_A * \beta_{23}C_B * \beta_3 C_C (P^l - Q^l)} \right]$$

$$\partial(h_3) = \left[\frac{\left(\frac{1}{\beta_3 C_C} \right) * \partial F_I}{(P^l - Q^l)} \right] \tag{9}$$

After solving the equation 8, consider restriction coefficient of valve R_x and time constant τ for three tank system, then

$$R_1 = \frac{1}{\beta_1 C_A}, \; R_2 = \frac{1}{\beta_{23} C_B} \text{ and } R_3 = \frac{1}{\beta_3 C_C}$$

$$\tau_1 = A(h_1)R_1, \; \tau_2 = A(h_2)R_2 \text{ and } \tau_3 = A(h_3)R_3$$

R_1 , R_2, and, R_3 and are τ_1 , τ_2 , τ_3 coefficients tank 1, tank 2, and tank 3 respectively. Coefficients in equation 9 is replaced by respective R and τ . We get,

$$\frac{\partial(h_3)}{\partial F_I} = \left[\frac{R_3}{(\tau_1\,\tau_2\,\tau_3)S^3 + [\tau_3 A(h_1)R_2 + \tau_3\tau_1 + \tau_3\tau_2 + \tau_1 A(h_2)R_3 + \tau_1\tau_2 + \tau_2)S^2 + (\tau_3 + A(h_1)R_3 + A(h_2)R_3 + A(h_1)R_2 + \tau_1 + \tau_2)S + 1} \right] \tag{10}$$

To derive gain scheduling algorithm, consider a P + I controller equation,

$$\text{Closed loop} = \frac{\partial(h_3)}{\partial F_I} * \left(\frac{K_P T_I s + K_P}{T_I s} \right) \tag{11}$$

Equation 10 in equation 11, we get,

$$= \left[\frac{R_3 K_P (1 + T_I S)}{(T_I \tau_1\,\tau_2\,\tau_3)S^4 + T_I(\tau_3 A(h_1)R_2 + \tau_3\tau_1 + \tau_3\tau_2 + \tau_1 A(h_2)R_3 + \tau_1\tau_2 + \tau_2)S^3 + T_I(\tau_3 + A(h_1)R_3 + A(h_2)R_3 + A(h_1)R_2 + \tau_1 + \tau_2)S^2 + T_I S + K_P T_I R_3 S + K_P R_3} \right]$$

The characteristics equation is

$$[(T_I\tau_1\ \tau_2\ \tau_3)S^4 + T_I(\tau_3 A(h_1)R_2 + \tau_3\tau_1 + \tau_3\tau_2 + \tau_1 A(h_2)R_3$$
$$+\tau_1\tau_2 + \tau_2)S^3 + T_I(\tau_3 + A(h_1)R_3 + A(h_2)R_3 + A(h_1)R_2 + \tau_1 + \tau_2)S^2 +$$
$$T_I S + K_P T_I R_3 S + K_P R_3] = 0$$

$$[(T_I\tau_1\ \tau_2\ \tau_3)S^4 + \left[\frac{(\tau_1\tau_3 + \tau_3\tau_2 + \tau_1\tau_2 + \tau_2) + \tau_1(A(h_1)R_3)}{\tau_1\tau_2\tau_3}\right]S^3$$
$$+\left[\frac{\tau_1+\tau_2+\tau_3}{\tau_1\tau_2\tau_3} + \frac{A(h_1)(R_3+R_2) + A(h_2)R_3}{\tau_1\tau_2\tau_3}\right]S^2$$
$$+\left[\frac{1}{\tau_1\tau_2\tau_3} + \frac{K_P R_3}{\tau_1\tau_2\tau_3}\right]S + \left[\frac{K_P R_3}{\tau_1\tau_2\tau_3 T_I}\right] = 0$$

Compare coefficient of S term: $\frac{K_P R_3}{\tau_1\tau_2\tau_3}$ and coefficient of S^2: $\frac{\tau_1+\tau_2+\tau_3}{\tau_1\tau_2\tau_3}$

$$\frac{K_P R_3}{\tau_1\tau_2\tau_3} = \frac{\tau_1+\tau_2+\tau_3}{\tau_1\tau_2\tau_3}$$

we get,

$$K_P = \frac{\tau_1+\tau_2+\tau_3}{R_3}. \tag{12}$$

Compare coefficient of S term: $\frac{1}{\tau_1\tau_2\tau_3}$ and coefficient of constant: $\frac{K_P R_3}{\tau_1\tau_2\tau_3 T_I}$

we get,

$$T_I = K_P R_3. \tag{13}$$

Equations 12 and 13 encapsulate the derived PI controller gains for the interacting three-tank conical system. These expressions empower precise tuning of the control parameters, enabling researchers to meticulously maintain the desired fluid level within the specified operational range.

4. Simulation Results

An adaptive controller was designed using the gain scheduling method. The simulated response of this controller is shown in figure 4. This approach involved establishing a relationship between tank parameters and PI controller parameters given in table 2, followed by simulation to evaluate performance. All simulations were conducted within the MATLAB environment.

Table 2. PI Controller Values.

Region	Kp	Ti
1	0.179746	0.03128
2	0.52907	0.125344
3	0.879969	0.214449
4	1.23541	0.298854
5	1.589793	0.383753

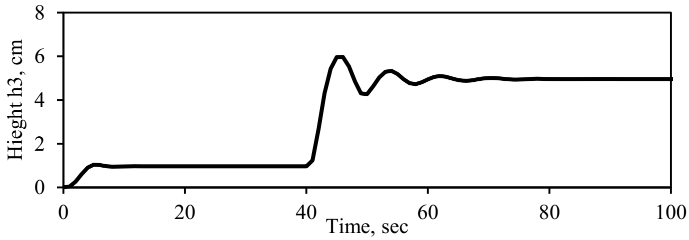

Figure 4. Simulated results of PI controller values in tank system.

5. Conclusion

This study implements an adaptive controller using the gain scheduling technique for a three-tank conical system within the MATLAB Simulink environment. Equations 12 and 13 provide the derived PI controller gains for this interconnected conical tank system. Presented values in table 2 and implemented and results are shown in figure 4. The simulation results shown that these equations facilitate precise adjustment of control parameters, enabling accurate maintenance of the desired fluid level within the operating range.

References

[1] Hamdani CN, Adi WW, Anggriawan DO. Comparative study on fuzzy supervision and gain scheduling for nonlinear level control system. Proceedings of the 2nd Borobudur International Symposium on Science and Technology (BIS-STE 2020), 2021. https://doi.org/10.2991/aer.k.210810.085

[2] Chakravarthi MK, Pannem VK, Venkatesan N. Real time implementation of gain scheduled controller design for higher order nonlinear system using LabVIEW. International Journal of Engineering and Technology. 2014; 6(5): 2031-2038.

[3] Soni S, Gagrani M, Rathore A, Chakravarthi MK. Study of different controller's performance for a real time non-linear system. International Journal of Advancements in Electronics and Electrical Engineering. 2014; 3(3):10-14.

[4] Kumar DD, Meenakshipriya B. Design and Implementation of non linear system using gain scheduled PI controller. Procedia Engineering. 2012; 38: 3105-3112, https://doi.org/10.1016/j.proeng.2012.06.361

[5] Rugh WJ and Shamma JS. Research on gain scheduling. Automatica. 2000; 36(10):1401-1425.

[6] Zaher AA. Design of model-based gain scheduling controllers for nonlinear systems. Journal of Physics: Conference Series. 2018; 1141. DOI 10.1088/1742-6596/1141/1/012117

[7] Venkataraman AP, Veerapathran V, Nokerov SM, and Venkataraman SP. Simple recursive technique to improve closed loop system time domain analysis. Advances in Machinery, Materials Science and Engineering Application IX, 2023; 40: 612-618. DOI 10.3233/ATDE230516

[8] Aravind P, Valluvan M, Saranya M. Simulation based modeling and implementation of adaptive control technique for non linear process tank. International Journal of Computer Applications. 2013; 68(16): 1-6. DoI 10.5120/11660-7242.

[9] Ravi VR, Thyagarajan T and Priyadharshni SY. Gain scheduling adaptive model predictive controller for two conical tank interacting level system. 2012 Third International Conference on Computing, Communication and Networking Technologies (ICCCNT'12), Coimbatore, India, 2012; p. 1-7, doi: 10.1109/ICCCNT.2012.6396058.

[10] Kala H, Aravind P and Valluvan M. Comparative analysis of different controller for a nonlinear level control process. 2013 IEEE Conference on Information & Communication Technologies, Thuckalay, India, 2013, p. 724-729, DoI: 10.1109/CICT.2013.6558189.