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On a General Method for Modeling the Controlled Dynamics of Manipulators with Parallel Kinematics as Systems with Geometrical Constraints

Aleksandr Ya KRASINSKIY [a,1]
[a] *Dept. of Computer Modeling and Probability Theory, Moscow Aviation Institute, National Research University); Information and Computing Center, Rosbiotech, Moscow, Russia*

Abstract. An adequate mathematical model including all components of an automatic system is a prerequisite for the most cost-effective implementation of the device's potential capabilities. Such a strict nonlinear mathematical model must make it possibility to minimize the actuator drives number, reduce the measurement information amount (the number of measuring sensors) and analyze transient processes in actuator drives. The traditional application of Lagrange equations with constraint multipliers in dynamics models of non-free systems leads to an increase in their dimensionality. The variables of such models are all coordinates, all (including dependent) velocities and also constraint multipliers. The article develops the application of the theoretical mechanics rigorous methods non- free systems to minimize the dimensionality of nonlinear mathematical models of parallel manipulators as the systems with geometric constraints, taking into account the dynamics of actuator electric drives. The transition to equations in redundant coordinates, free from constraint multipliers, excludes from consideration not only multipliers, but also dependent velocities. The application of the proposed approach to modeling the stand Ball and Beam dynamics is considered.

Keywords. Mathematical dynamics model, geometric constraints, linear-quadratic problem

1. Introduction

In modern industrial robotic equipment, robot manipulators with parallel kinematics are widely used. The presence of parallel kinematics chains provides higher rigidity and accuracy, less weight in comparison with manipulators with sequential kinematics [1-3]. However, the closure of parallel kinematics chains in certain manipulator nodes leads to some conditional relationships between the nodes coordinates and the distances between the nodes. From the point of view of mechanics, they should be considered as non-free mechanical systems with geometric constraints.

[1] Aleksandr Ya KRASINSKIY, Corresponding author, Dept. of Computer Modeling and Probability Theory, Moscow Aviation Institute, National Research University), Information and Computing Center, Rosbiotech, Moscow, Russia; E-mail: krasinsk@mgupp.ru.

Analytical mechanics of systems with geometrical constraints is a rather narrow special section of theoretical mechanics with its own specific methods for the dynamics mathematical description of this systems class.

The mathematical training level necessary for the fundamental principles full understanding of its methods goes far beyond the scope of standard engineering education. This rather complex mechanics section has never been included in the curriculum of standard engineering training, since until recently technical devices with geometric constraints were not widely used in technical practice.

Therefore, in order to obtain a mathematical model necessary for the potential capabilities implementation of high-class automatic devices with developed information processing systems and actuator technology, specialists in technical practice use far from the most effective (obtained in the nineteenth century [4,5]) forms of equations with constraint multipliers [6-11].

But in accordance with the logic of further development of this analytical mechanics direction, theoretical specialists continued to develop its new methods [12-15]. At present, analytical mechanics of systems with geometrical connections offers specialists in technical practice a wide choice of various equation forms and variables types that best correspond to the nature of the problem under consideration.

It is precisely such dynamics nonlinear mathematical models obtained by strict analytical methods that make it possible to most fully use the properties of the proper (without applying controls) object motions. With this approach, the application of the general optimal systems theory [16] allows for the greatest possible minimization the actuator number (the controls vector dimension) while reducing the volume of measurement information (the sensors number).

However, in engineering practice, it is not only these results that do not find proper application. Modeling the dynamics of such systems requires a level of rigor that is so unusual for engineering practice that ignorance of long-known fundamentally important results often leads, when studying the specific devices dynamics, to performing operations that are not permissible in the systems with geometric constraints dynamics strict modeling.

As an example, we will cite the situation with geometric constraints. As early as the beginning of the twentieth century, Routh [17] pointed out that for the correct allocation of the first approximation, in the equations of dynamics, it is necessary to take into account the second-order terms in the equations of geometric constraints (compare with [15]).

The widely known Ball and Beam stand can be considered as an example of a manipulator with parallel kinematics. In node *A*, two cinematic chains are closed: a single-link chain *OA* and a two-link chain O_1BA (figure 1.):

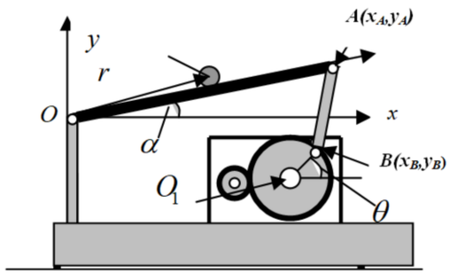

Figure 1. Scheme of the “Ball and Beam” stand.

In this system, the electric drive, due to the inclination of the chute OA, can roll the ball C to any predetermined position on the chute and stabilize this equilibrium. The chute is connected on one side at point O to a fixed supporting post, and on the other to a movable lever AB. The parameters of the system are such that at the angle θ=0, and the lever AB is vertical (figure 2).

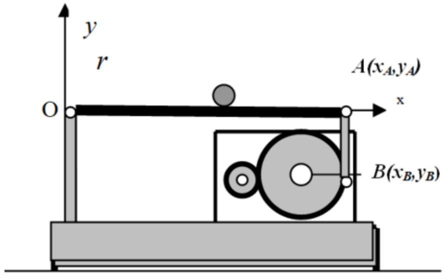

Figure 2. Zero equilibrium position.

The motion of the lever is controlled by a DC motor. A nonlinear geometric relationship is imposed on the system: the distance between points $A(x_A,\ x_B)$ and $B(x_B,\ y_B)$ is constant:

$$\left(L(cos\alpha-1)+d(1-cos\theta)\right)^2+(Lsin\alpha+l-dsin\theta)^2=l^2; \qquad (1)$$

$(L=OA$ − length of the trough, $l=AB$ − length of the lever, d − radius of the output drive wheel).

In most studies on the dynamics of the Ball and Beam stand with a geometric constraint [18-22] (except [21]), when constructing a mathematical model, immediately, starting with [18], a transition to a linear dependence is made, which is completely unjustified (see [22] with an extensive bibliography).the dependent coordinates are excluded from the linearized geometrical constraints equations $\alpha\approx\frac{d}{L}\theta$.

Due to the obvious condition $\alpha=0$ for the equilibrium of the ball, from this relation (1) for the rotation angle θ of the drive wheel only the value θ=0 is obtained, and a non-zero equilibrium position (figure 3) could not be obtained $\theta*=2\,arcsin\left(\frac{l}{\sqrt{l^2+d^2}}\right)$; and, accordingly, was not considered until 2014 [23].

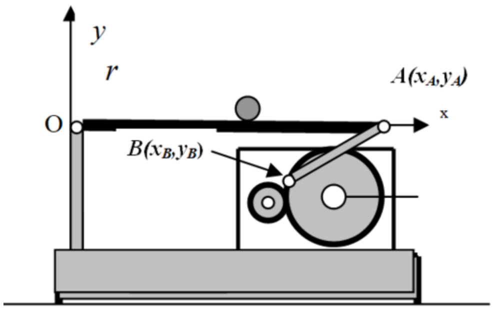

Figure 3. New equilibrium position.

Moreover, the linearization procedure of the constraints equations has been used in modeling the Ball and Beam stand dynamics of until very recently [24,25], despite the publications [26] on modeling the dynamics of the stand with full consideration of nonlinear geometric constraints (and the authors [24,25] refer in their work to the studies of [26]).

Moreover, attempts to apply control algorithms to a real technical device, found by using a variety of methods of mathematical control theory for considering inadequate mathematical models led to its behavior, which did not correspond at all to the model.

Therefore, a rigorous consideration of the dynamics of the Ball and Beam stand was required, taking into account the complete nonlinear constraint (1) from the standpoint of analytical mechanics of systems with geometric constraints [12-15], nonlinear stability theory [27-29] and mathematical control theory [30]. The basis of the consideration is the general methods of analytical mechanics of non-free systems with geometric constraints.

2. General Method for Modeling the Non-Free Systems Dynamics

Let us consider the algorithm for compiling Lagrange equations with constraint multipliers for a system with coordinates $q_1, \ldots, q_{n+m}$ and geometrical constraints

$$F(q) = 0; \quad F'(q) = \left(F_1(q), \ldots, F_m(q)\right); \tag{2}$$

$$\det \left\| \frac{\partial\left(F_1(q), \ldots, F_m(q)\right)}{\partial(q_{n+1}, \ldots, q_{n+m})} \right\| \neq 0; \tag{3}$$

with kinetic energy of the most general form

$$T(q,\ \dot{q}) = {}^1\!/_2\, a_{ik}(q)\dot{q}_i\dot{q}_k + a_i(q)\dot{q}_i + T_0; \tag{4}$$

and conditions

$$b_{\sigma i}(q)\delta q_i = 0;\ b_{\sigma i} = \frac{\partial F_\sigma}{\partial q_i}\ ;\ \sigma = \overline{1,m};\ i = \overline{1,n+m}; \tag{5}$$

imposed on the variations of coordinates due to the presence of constraints (2) (here and below, summation is performed over repeating indices) on which the forces related to the coordinates (potential and non-potential) act.

From the d'Alembert-Lagrange principle one can obtain [12,13] the equations of motion with constraints multipliers

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i + \lambda_\sigma b_{\sigma i};\ \ \sigma = \overline{1,m};\ i = \overline{1,n+m}; \tag{6}$$

The variables of the model are

$$q_i,\ \dot{q}_i,\ \lambda_s \quad i = \overline{1,n+m};\ \sigma = \overline{1,m};$$

In the general case, using rigorous methods of analytical mechanics, a complete nonlinear mathematical dynamics model for a system with geometric constraints is obtained, including $n+m$ second-order differential equations (6) and m nonlinear algebraic equations of geometric constraints (2). When passing to normal form, we obtain $2(n+m)$ first-order differential equations and m algebraic equations of constraints to determine all $n+m$ coordinates, all $n+m$ velocities and m constraint multipliers, so that the total dimension of the model is equal to $2(n+m)+m=2n+3m$.

This approach leads to high-dimensional models, since the mechanical component model of the mechatronical system includes all coordinates, all (including those dependent due to connections) velocities, and it is also necessary to take into account the relations (2) for determining the Lagrange multipliers. Therefore, there are practically no mathematical dynamics models for mechatronic systems with geometric connections, including models of actuators. The use of methods of analytical mechanics of non-free systems [12-15] reduces the dimension of the model due to the transition to equations in redundant coordinates [14,15] by excluding dependent velocities from consideration using differentiated equations of geometric connections and the expression for Lagrange multipliers found in the general case.

3. Some Method for Reducing the Dimension of the Mathematical Model in Modeling the Non-free Systems Dynamics

When considering the specific systems dynamics, the study complexity is of great importance, and, as a rule, there is no need to determine the reactions of the constraints. In this case, it becomes possible to simplify the mathematical model by excluding as many variables as possible from consideration, with the excluded variables traditionally being the constraints multipliers.

Historically, the algorithm for this procedure was first proposed by A.M. Lyapunov (pp. 354-357 [4]). This is not a general theoretical method, but the following extremely labor-intensive and cumbersome algorithm for studying the dynamics of a specific system includes the following operations:

1. From the motion equations with constraints multipliers compiled in the form (6), express the all accelerations $\ddot{q}_\iota$.
2. Having differentiated the equations of geometric constraints (1) twice in time, obtain a system of linear relations regarding accelerations.
3. Substituting the acceleration expressions from the motion equations into these relations, we will have a system of linear inhomogeneous algebraic equations for the constraint multipliers $\lambda_\sigma$with a non-singular matrix of coefficients.
4. By resolving these equations with respect to the constraint multipliers λ_σ , obtain expressions for them as functions of all coordinates and velocities $\lambda_\sigma(q,\dot{q})$.
5. By substituting the obtained expressions for the constraint multipliers into the original equations of motion (6), we obtain a mathematical model of the dynamics of the system with geometric constraint (2).

Despite the obvious extreme complexity of this method, this is how the dynamics of the Delta robot are still being studied [6]. As a result, the mathematical model, even without including a description of the dynamics of the drives, turns out to be so cumbersome that one has to limit oneself to only computer simulation.

In essence, the algorithm of A.M. Lyapunov was used without references to it. Moreover, in our opinion, in modern studies of the robotic devices dynamics, the behavior analysis of the mechanical robot component from the analytical mechanics standpoint is not carried out at all, but a transition is immediately made to computer simulation. And the conclusion about the model adequacy built using modern information technologies is made on the basis of the closeness of the model behavior and the real object dynamics which is completely unfounded: computer simulation shows the dynamics of the model only under very specific initial conditions.

It is not legitimate to draw a global conclusion about the general solution of the differential equations systems based on an analysis of the particular solution behavior.

4. Some General Methods for Reducing the Dimension of the Mathematical Model in Modeling the Non-free Systems Dynamics

G. K. Suslov (pp. 320-331 [13]), A. I. Lurye (pp. 320-331 [12]), M. F. Shulgin [14] developed general methods for such an exclusion, associated with a single or double differentiation with respect to time of the equation of geometric constraints (2).

In our opinion, the most effective way to reduce the dimensionality of the model is an alternative method of excluding from consideration the multipliers and dependent velocities by switching to differentiated equations of constraints and resolving taking into account (3) relatively dependent velocities:

$$b_{si}(q)\dot{q}_i = 0; b_{si}(q) = \frac{\partial F_s}{\partial q_i}; \ s = \overline{1,m}; \ i = \overline{1,n+m};$$

$$\dot{q}_\sigma = B_{\sigma j}(q)\dot{q}_j; \ \sigma = \overline{1,m}; \quad j = \overline{1,n}; \tag{7}$$

$$\|B_{\sigma j}(q)\| = -\left\|\frac{\partial(F_1,\dots,F_m)}{\partial(q_{n+1},\dots,q_{n+m})}\right\|^{-1}\left\|\frac{\partial(F_1,\dots,F_m)}{\partial(q_1,\dots,q_n)}\right\|;$$

Denoting the result of eliminating dependent velocities using (7) from the kinetic energy (4) through $T^*(q_1,\dots,q_{n+m},\dot{q}_1,\dots,\dot{q}_n)$, and from the acting forces through Q_i^*, determining [12,14]) the Lagrange multipliers from

$$\lambda_\sigma = \frac{d}{dt}\frac{\partial T}{\partial \dot{q}_\sigma} - \frac{\partial T}{\partial q_\sigma} - Q_\sigma; \quad \sigma = \overline{n+1,n+m};$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i + \lambda_\sigma b_{\sigma i}; \quad \sigma = \overline{1,m}; \ i = \overline{1,n+m};$$

We can obtain a mathematical model of the system dynamics in the form of M.F. Shulgin [14] in redundant coordinates a result

$$\frac{d}{dt}\frac{\partial T^*}{\partial \dot{q}_j} - \frac{\partial T^*}{\partial q_j} = Q_j^* + B_{\sigma j}(q)\left(\frac{\partial T^*}{\partial q_\sigma} + Q_\sigma^*\right);$$

$$\dot{q}_\sigma = B_{\sigma j}(q)\dot{q}_j; \ \sigma = \overline{n+1,n+m}; \ j = \overline{1,n}; \tag{8}$$

This method in general case significantly reduces the mathematical dynamics model (8) dimensionality of systems with geometrical constraints: in addition to the multipliers, the velocities of coordinates dependent on the constraints are also excluded from consideration. The total model dimensionality in normal form includes 2n+m first-order differential equations, i.e. it is reduced by 2 m compared to dimension 2n+3m (2) and (6).

The general theoretical algorithm for obtaining the mathematical model (8) includes the following operations:

1. Differentiate the geometric constraints equations (2) once with respect to time.
2. Obtain expressions for dependent velocities from this linear algebraic system.
3. Substitute these expressions into the kinetic energy (3) and force to exclude dependent velocities from consideration.
4. Create a mathematical model (8) of the dynamics of systems in redundant coordinates.

For the practical application of this method for dynamics mathematical modeling of systems with geometric constraints, mathematical training in the standard engineering education amount is quite sufficient.

In the general case, let us present a mathematical model of the dynamics of systems with geometric constraints as the Shulgin equation (7) in explicit scalar form [15]:

$$\begin{gathered} a_{lr}\ddot{q}_r + \frac{\partial a_{lr}}{\partial q_j}\dot{q}_r\dot{q}_j + \frac{\partial a_{lr}}{\partial q_\mu}B_{\mu j}\dot{q}_l\dot{q}_j - \frac{1}{2}\frac{\partial a_{lj}}{\partial q_l}\dot{q}_r\dot{q}_j - \\ -\frac{1}{2}\frac{\partial a_{rj}}{\partial q_\mu}B_{\mu l}\dot{q}_r\dot{q}_j + \left(\frac{\partial d_l}{\partial q_r} - \frac{\partial d_r}{\partial q_l} + B_{\mu l}\frac{\partial d_l}{\partial q_r} - B_{\mu l}\frac{\partial d_r}{\partial q_\mu}\right)\dot{q}_r + \\ + \frac{\partial W}{\partial q_l} + B_{\mu l}\frac{\partial W}{\partial q_\mu} = Q_l + B_{\mu l}Q_\mu; \quad \dot{q}_\sigma = B_{\sigma j}(q)\dot{q}_j; \\ \mu, \sigma = \overline{n+1, n+m}; \quad l, j, r = \overline{1, n}; \end{gathered} \tag{9}$$

Here $W(q) = \Pi(q) - T_0(q)$ is the changed (reduced) potential energy, $\Pi(q)$ - potential energy, $Q_l, Q_\mu$and now denotes the non-potential forces corresponding to the coordinates $q_l, q_\mu$when they are introduced in excess.

The first problem, the which study required the development of a comprehensive application of not only the above-mentioned use the rigorous analytical mechanics methods for non-free systems [12-15], but also nonlinear stability theory [27-29] and mathematical control theory [30] to real modern technical devices, was a complete solution to the problem of stabilizing a given position in a ball in the "Ball and Beam" stand.

As a stabilizing control, an additional voltage on the armature winding of the actuator drive's commutator motor was considered, the which coefficients were determined [31] from the solution of the linear-quadratic stabilization problem by N.N. Krasovskii's method [30]. To conclude about the asymptotic (despite the presence of a zero root of the characteristic equation) stability in a complete nonlinear system closed by the found control, a proof of the general theorem [23] was required.

5. Application of the Developed Mathematical Model to the Stabilization Problem for Ball and Beam Stand

The kinetic and potential energies of the system are as follows:

$$T = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}m(r\dot{\alpha})^2 + \frac{1}{2}J\left(\frac{\dot{r}}{R}\right)^2 + \frac{1}{2}J_0\dot{\theta}^2;\ \ \Pi = mgr\ sin\alpha; \tag{10}$$

Here m and J, respectively, the mass and moment of inertia of the ball, R its radius. J_0 is the moment of inertia reduced to the axis of rotation of the actuator. From the time-differentiated equation of the connection (1) for the dependent velocity we have

$$\dot{\alpha} = \frac{d[(d-L)\sin\theta + Lsin\,(\theta-\alpha) + lcos\,\theta]}{L[-(d-L)\sin\alpha - dsin(\theta-\alpha) + l\cos\alpha]}\dot{\theta}; \tag{11}$$

After eliminating the dependent velocity using (11), the kinetic energy (10) will be transformed in

$$T^* =$$
$$\frac{1}{2}m\dot{r}^2 + \frac{1}{2}\left[m\left(r\frac{d((d-L)\sin\theta + Lsin\,(\theta-\alpha) + l\,cos\theta\,)}{L(-(d-L)\sin\alpha - dsin(\theta-\alpha) + l\cos\alpha)}\right)^2 + J_0\right]\dot{\theta}^2 +$$
$$+\frac{1}{2}J\left(\frac{\dot{r}}{R}\right)^2;\ \ \Pi = mgr\sin\alpha; \tag{12}$$

As a mathematical model (8) for the mechanical part of the stand, we obtain the equations

$$\left(m + \frac{J}{R^2}\right)\ddot{r} - mr\left(\frac{d}{L}\dot{\theta}\right)^2 + mgsin\left(\frac{d}{L}\theta\right) = 0;$$

$$\left(J_0 + m\left(r\frac{d}{L}\right)^2\right)\ddot{\theta} + 2\mathrm{mr}\left(\frac{d}{L}\right)^2\dot{r}\dot{\theta} + mgr\frac{d}{L}cos\left(\frac{d}{L}\theta\right) = -b_0\dot{\theta} + k_2 i_a; \tag{13}$$

To take into account the dynamics of the drive, we will add a mathematical model of the actuator motor anchor chain

$$L_a\frac{di_a}{dt} + R_a i_a + k_1\frac{d\theta}{dt} = e_v; \tag{14}$$

L_a is the induction of the anchor winding, R_a is its resistance, k_1 is the coefficient of back-EMF, k_2 is the electromechanical constant of the engine, i_a is the current in the winding, e_v is the voltage supplied to the winding.

For a given equilibrium position $r = r_0 \neq 0$ of the ball with $\theta = 0$ to exist, the following conditions must be met:

$$i = i_{a0} = const \neq 0;\ \ \theta = \theta_0 = 0;\ \ r = r_0 = const \neq 0;\ \ e_v = e_{v0} = const \neq 0; \tag{15}$$

$$i_{a0} = \frac{mgr_0 \frac{d}{L}}{k_2};\ \ e_{v0} = \frac{R_a i_{a0}}{k_1} = \frac{R_a mgr_0 \frac{d}{L}}{k_1 k_2};$$

Let us introduce perturbations for the model variables

$$r = r_0 + x_1;\ \dot{r} = x_2;\ \theta = x_3;\ \dot{\theta} = x_4;\ i_a = i_{a_0} + x_5;$$

$\alpha = x_6;\ e_v = e_{v_0} + u;$

Equations of perturbed motion with a distinguished first approximation

$$\dot{x_1} = x_2;$$

$$\dot{x_2} = -\frac{mg}{m+\frac{J}{R^2}} x_6;$$

$$\dot{x_3} = x_4;$$

$$\dot{x_4} = -\frac{mg\frac{d}{L}x_1 - \frac{2d^2 mgr_0}{l}(dx_3 - Lx_6) - b_0 x_4 + k_2 x_5}{J_0 + m\left(\frac{r_0 d}{L}\right)^2}; \tag{16}$$

$$\dot{x_5} = \frac{u - R_a x_5 - k_2 x_4}{L_a};\ \ \dot{x_6} = \frac{dx_4}{L};$$

General Aizerman-Gantmacher [31] replacement from analytical mechanics of systems with differential constraints

$$x_6 = f + \frac{d}{L}x_3; \tag{17}$$

to isolate the critical variable f, it changes the coefficients of the first approximation

$$\dot{x_1} = x_2;$$

$$\dot{x_2} = -\frac{mg\frac{d}{L}}{m+\frac{J}{R^2}} x_3 - \frac{mg}{m+\frac{J}{R^2}} f;$$

$$\dot{x_3} = x_4;$$

$$\dot{x_4} = -\frac{mg\frac{d}{L}x_1 + \frac{2d^2 mgr_0}{l}f - b_0 x_4 + k_2 x_5}{J_0 + m\left(\frac{r_0 d}{L}\right)^2}; \tag{18}$$

$$\dot{x_5} = \frac{u - R_a x_5 - k_2 x_4}{L_a};$$

$$\dot{x_6} = 0;$$

Stability in the complete system is possible only in the critical case of one zero root corresponding to the variable f. By virtue of the theorem [23] proved on the basis Kamenkov's theorem [29], asymptotic stability takes place in the complete system if the real parts for all nonzero roots of the characteristic equation in the first approximation system are negative. To ensure the specified arrangement of the roots, it is necessary to determine the stabilizing control from the solution by the method of N.N. Krasovskii [30] of the linear-quadratic problem for the linear controlled subsystem:

$$\dot{x} = Px + Qu; \tag{19}$$

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{mg\frac{d}{L}}{m+\frac{J}{R^2}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\frac{mg\frac{d}{L}}{J_0+m\left(\frac{r_0 d}{L}\right)^2} & 0 & 0 & -\frac{mg\frac{d}{L}}{m+\frac{J}{R^2}} & -\frac{k_2}{m+\frac{J}{R^2}} \\ 0 & 0 & 0 & \frac{-k_2}{L_a} & \frac{-R_a}{L_a} \end{pmatrix};$$

$$Q' = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{L_a} \end{pmatrix}.$$

For the controlled subsystem (19) the controllability condition

$$rankW = rank(Q \quad PQ \quad P^2Q \quad P^3Q \quad P^4Q)=5; \tag{20}$$

is satisfied. The stabilizing control as a linear function of the variables of only the controlled subsystem is determined from the solution of the corresponding Lyapunov-Bellman-Riccati equation.

A full detailed study of this problem is given in [26], and graphs of the transition processes are also given taking into account the dynamics of the actuator. Further, on the basis of the results obtained for Ball and Beam, a general algorithm for reducing the dimensionality of the control problem during stabilization of steady-state motions of systems with geometric connections is developed.

By applying the developed method to the Delta robot, an analytical, rigorous, nonlinear mathematical model of its dynamics was obtained [32] with the inclusion of transient processes in three actuators.

6. Conclusion

The full strict rationale for the proposed modeling method cannot be presented at a level understandable to a person with standard engineering training. However, the practical application of the method does not require an understanding of the general theoretical results used in it. The simplicity of the algorithm, the use of which ensures reliable obtaining of results justified in the method, is demonstrated by the complete solution of the problems of stabilization of a given configuration of the Ball and Beam stand.

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