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Reliability-Based Optimization of Sinusoidal-Web Steel Beams: Integrating Experimental and Numerical Analyses for Enhanced Structural Performance

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Abstract. A new method for optimizing the design of nonlinear sinusoidal-web steel beams is introduced in this paper, focusing on reliability-based design principles. Utilizing a custom-written code with a reliability index as a key control factor, the study incorporates stochastic variables such as flange thickness, sinus wave width, sinusoidal-web plate thickness, and applied load magnitudes. Employing Finite Element Analysis (FEA) through ABAQUS software conducting experimental testing on sinusoidal HEA beams, this research showcases the successful application of reliability-based design in modifying beam design to meet the reliability requirements. The adoption of the suggested approach, which incorporates Monte Carlo simulation, is crucial in guiding the development of structural configurations that are robust to random nature in both load and manufacturing conditions. Results showcase improved structural integrity, and the successful convergence of optimized values, emphasizing the potential for enhanced steel structure design.

Keywords. Reliability-based design, sinusoidal-web steel beams, structural optimization, Monte Carlo simulation

1. Introduction

In the field of civil engineering, recent research works have dealt with diverse facets of structural design, incorporating innovative methodologies and materials [1–3]. Among these, steel beams play a crucial role in structural engineering, providing the necessary strength and durability to support the weight of various structures. They are designed to withstand the complex forces and loads involved in different engineering projects [4]. In recent years, there has been a growing focus on the creative design of sinusoidal-web steel beams, which deviate from traditional beam structures [5]. In their study, Guo et al. [6] presented an approach for designing steel arches featuring a sinusoidal web part to calculate the strength of when it is under vertical load.

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Engineering systems' structural integrity may now be ensured with the use of reliability-based design techniques [7–9]. When considering steel beams, especially those with sinusoidal webs, it is crucial to conduct a thorough assessment of their dependability. Bärnkopf et al. [10] conducted a comparison between deterministic and stochastic nonlinear calculations to determine the exact buckling resistance of steel featuring corrugated web girders. Building upon the knowledge gained from analyzing different variables and designing for reliability, the process of improving structures, particularly by using steel beams with sinusoidal-web sections, has become a key area of concentration in modern structural engineering research [11].

The aim of the proposed research is to assess the implications of integrating probabilistic optimization algorithm into the numerical modelling of sinusoidal-web steel beams via a thorough examination. The process of determining the most suitable design parameters for the beam is achieved via a thorough sequence of analysis and comparisons. One major challenge is the intricate complexity of the sinusoidal geometry, providing the opportunity for new insights into the most effective design parameters. Moreover, the study examines the outcomes obtained from four-point bending tests conducted on the beam. A probabilistic analysis is carried out using a nonlinear code in order to accomplish the intended goal. In light of random nature of the considered variables such as applied loads and geometric dimensions, the paper claims that the reliability index functioned as an effective limit. Moreover, in accordance with the statistical characteristics of sinusoidal-web features, the Monte Carlo method is employed in the computation of reliability indices.

2. Theoretical framework

In accordance with the foundational principle of reliability analysis, the design of this investigation is predicated on reliability. Under the assumption that X_S and X_R are independent random variables governed by probabilistic density functions $f_R\ (X_S)$ and $f_R\ (X_R\ )$, respectively, one can approximate the failure criterion as $X_R\ \leq\ X_S$, where X_R denotes the non-negative threshold for X_S. In order to approximate the failure probability (P_f), Equation (1) is utilized.

$$P_f\ =\ P[X_R \leq X_S] =\ \int\int_{X_R\leq X_S} f_R(X_R) f_S(X_S) dX_R dX_S \tag{1}$$

Additionally, P_f can be quantified as follows:

$$P_f\ =\ \int_{g(X_R,X_S)\leq 0} f(X)dX =\ \int_{D_f} f(X)dX \tag{2}$$

The Monte Carlo method is utilized in this work to approximate the P_f distribution. The underlying principle of this method is to produce the random vector X through the utilization of the probability joint density function $f_X(x)$. Estimating P_f is possible through the utilization of the Monte Carlo method, which computes the ratio between the number of points produced and the total number of points present in the failure domain. The following results from the formulation of this hypothesis which incorporates the indicator function of D_f:

$$\chi_{D_f}(x) = \begin{Bmatrix} 1 & if\ \ x \in\ D_f \\ 0 & if\ \ x \notin\ D_f \end{Bmatrix} \tag{3}$$

Therefore, the P_f formula can be reconstructed in the following manner:

$$P_f = \int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{+\infty} \chi_{D_f}(x) f_X(x) dx \tag{4}$$

Therefore, $\chi_{D_f}(X)$ is a random variable that demonstrates a two-point distribution:

$$\mathbb{P}\left[\chi_{D_f}(X) = 1\right] = P_f \tag{5}$$

$$\mathbb{P}\left[\chi_{D_f}(X) = 0\right] = 1 - P_f \tag{6}$$

P_f is defined as $\mathbb{P}[X \in D_f]$. Given the correlation between $\chi_{D_f}(X)$ and the mean value and variance, which can be determined as follows:

$$\mathbb{E}\left[\chi_{D_f}(X)\right] = 1 \cdot P_f + 0 \cdot \left(1 - P_f\right) = P_f \tag{7}$$

$$\mathbb{V}ar\left[\chi_{D_f}(X)\right] = \mathbb{E}\left[\chi_{D_f}^2(X)\right] - (\mathbb{E}\left[\chi_{D_f}(X)\right])^2 = P_f - P_f^2 = P_f(1 - P_f) \tag{8}$$

To determine P_f using the Monte Carlo, the following represents the mean's estimation:

$$\widehat{\mathbb{E}}\left[\chi_{D_f}(X)\right] = \frac{1}{Z}\sum_{z=1}^{Z} \chi_{D_f}(X^{(z)}) = \widehat{P}_f \tag{9}$$

$X^{(z)}$ represents probability density functions that are linked to independent random vectors Z, where z = 1,...,Z. Considering uncertainty, various random variables are taken into account, such as the magnitude of applied loads, flange thickness of the beam, web thickness of the sinusoidal part, and width of the sinus wave profile. These variables are assumed to follow a Gaussian distribution, which comprises a mean value (E) and variance (Var). Standard structural engineering practices incorporate the target reliability index. The reliability index (β) involves the following equation:

$$\beta_{\text{target}} - \beta_{\text{calc}} \leq 0 \tag{10}$$

The following equations are executed in order to determine β_{target} and β_{calc}:

$$\beta_{\text{target}} = -\Phi^{-1}\left(P_{f,target}\right) \tag{11}$$

$$\beta_{\text{calc}} = -\Phi^{-1}\left(P_{f,calc}\right) \tag{12}$$

Therefore, the suggested reliability-based optimization technique is outlined as follows:

$$Minimize{:}\ W(x) = \rho L\left(2A_f + A_w\right) = \rho L\left(2b_f t_f + b_w t_w\right) \tag{13.a}$$

$$\sigma_i^C \leq \sigma_i \leq \sigma_i^T \tag{13.b}$$

$$\mathrm{u}_{\text{target}} - \mathrm{u}_{\text{calc}} \leq 0 \tag{13.c}$$

$$\beta_{\text{target}} - \beta_{\text{calc}} \leq 0 \tag{13.d}$$

The symbol L denotes the length of the beams, while $W(x)$ represents their total weight. ρ represents the material's density, which in this instance is 78.5 kN/m^3. A_f and A_w represent the cross-sectional areas of the flange and web, respectively. In addition, b_w and t_w denote the width and thickness of the web, respectively, whereas b_f and t_f represent the width and thickness of the flange, respectively. Considered random variables that adhere to a normal distribution, these geometric parameters each have a mean and standard deviation. Notably, upper stress σ_i^T and lower stress σ_i^Cconstraints are implemented to ensure the structural integrity remains intact within the fully elastic zone. Preventing plastic deformation in the resultant beams is the purpose of implementing this precaution. Furthermore, the displacement value functions as an essential limitation. Significantly, Equation (13.c) establishes the termination criterion pertaining to displacement. In this equation, $\mathrm{u_{target}}$ denotes the specified limit of displacement, which must not surpass the computed value of displacement $\mathrm{u_{calc}}$ between iterations. Figure 1 illustrates the workflow, starting from the MATLAB script execution, which subsequently interfaces with ABAQUS, leading to the final step in the process.

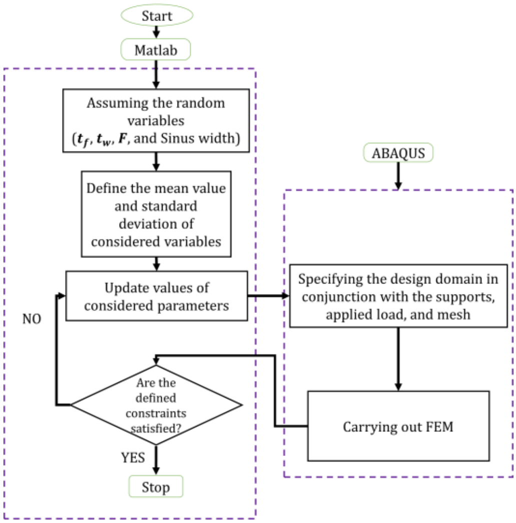


Figure 1. Workflow Diagram

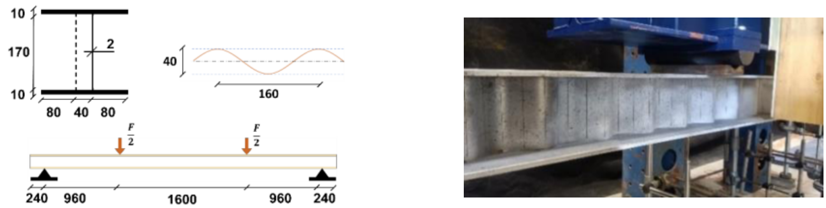
3. Experimental work

HEA beams with sinusoidal-web configurations served as the primary subjects during the experimental phase of this work. By means of a four-point bending experiment, which was chosen for its ability to comprehensively examine the flexural responses of the beams, the structural behavior of these objects was rigorously evaluated. Furthermore, Table 1 provides the properties of the materials utilized in this work. The experimental methodology employed in this study lays the groundwork for the subsequent interpretation and analysis of the observed outcomes, thereby aiding in the achievement of the research's goals.

Three identical steel beams featuring sinusoidal-web configurations were selected throughout the experimental investigations. The geometric characteristics of each SHEA-200 beam are as follows: a flange thickness (t_f) of 10 mm, a flange width (b_w) of 200 mm, a thickness (t_w) of 2 mm for the sinusoidal-web plate, a height (h) of 190 mm, and an overall length (L) that is standardized at 4000 mm. In Figure 2, a detailed explanations of the experimental configuration, geometry, the sinusoidal wave geometry, and cross-section of SHEA-200 beam are presented.

Table 1. Material properties of the considered sections

Part	**Elastic modulus** (E) GPa	**Yield strength** (f_y) MPa	**Ultimate strength** (f_u) MPa
Flange	200	288	636.60
Sinusoidal-web plate	210	282	639

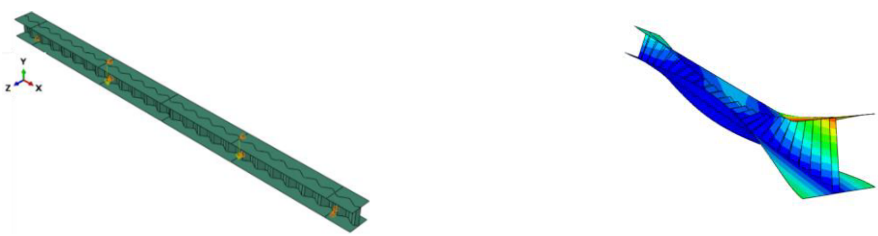


(a) (b)

Figure 2. Considered SHEA-200 beam: (a) geometry of the beam (b) test layout

4. Results and discussion

The section addresses the implementation of FEA. To do this, S4 elements are used to simulate the beam, providing more accurate results of how the beam reacts to changing conditions. The applied load acts in two positions as depicted in Figure 3 of the ABAQUS model. In order to replicate the experimental test conditions precisely, lateral supports are added to the pinned-fixed connection model that represents the boundary conditions. The analysis of linear buckling has been updated to include a perturbation of imperfection value L/1000. Figure 3 also illustrates the visual representation of the global buckling mode.



(a) **(b)**

Figure 3. SHEA-200 model: **(a)** part assembly **(b)** global buckling mode

The results of the experimental tests were used to confirm the accuracy of the model's findings. The examination showcases a notable level of consensus, as depicted in Figure 4.

To perform probabilistic analysis, a written code is constructed with the reliability index serving as a constraining assumption. Monte-Carlo utilizing 3×10^7 sample points. The presumed random variables are also detailed in Table 2. Observing that the applied load's average values fall below the maximum load-bearing capacity, it is imperative to clarify that the inclusion of the imperfection value (L/1000) as a stochastic variable will not significantly impact the results.

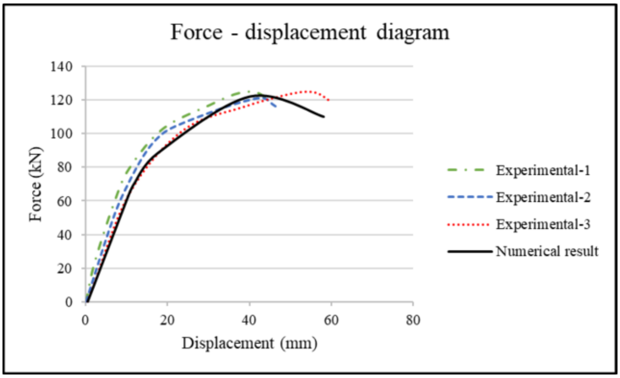


Figure 4. Force-displacement curves of SHEA-200

Table 2. Consideration parameters of the problem

Considered parameter	$t_f\ (mm)$	$t_w\ (mm)$	$F\ (kN)$	Sinus wave width
Mean	10	2	92	40
Standard deviation	40%	35%	40%	5%

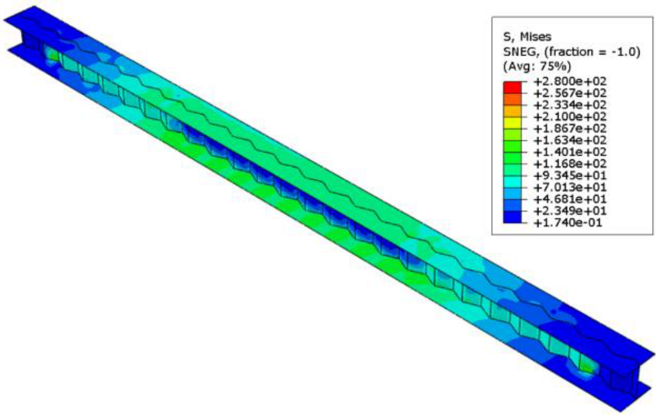


Figure 5. Stress distribution of the resulted optimized beam

Applying 92 kN as a mean load, as specified in Table 2, sets a baseline within the elastic behavior for the beam under consideration. After further analysis, it is evident that there is a slightly increased load of 93.5 kN, highlighting the natural variability that is often encountered in real-world situations. The significant displacement, resulting in the program ending at 18 mm, demonstrates the algorithm's susceptibility to unanticipated structural changes. The step-by-step approach towards achieving the best possible values, guided by a specific reliability index ($\beta_{target} = 3.00$), demonstrates how reliability-based optimization can be used to customize the design of the beam to meet strict reliability requirements. The calculated values for the web thickness, sinus wave width, and flange thickness are 5.5mm, 30mm, and 12mm, respectively. The significance of these values lies in the fact that they indicate an efficient use of materials, which is critical for the design of sustainable steel structures and guarantees stability under particular conditions. The stress distribution of the SHEA-200 beam in its optimized configuration as illustrated in Figure 5, demonstrates the effectiveness of proposed probabilistic approach. Significant advancements are apparent upon conducting a more thorough analysis of the stress distribution, specifically in regions where stress has been diminished. The adherence to strict stress bounds during the optimization procedure guarantees that complete elasticity is maintained in every zone of the resultant model.

5. Conclusions

This work introduced an innovative framework to optimize the design of nonlinear imperfect sinusoidal-web beams, leveraging the principles of reliability-based design. The methodology employs a custom-written code, wherein a reliability index serves as a pivotal control factor for delineating the analysis boundaries. This code accommodates the stochastic nature of variables such as flange thickness, sinus wave width, sinusoidal-web plate thickness, and applied load magnitudes, presuming a normal distribution. The integration of reliability-based design, emerges as a key driver in steering structural design towards robust configurations adept at withstanding uncertainties inherent in both load and manufacturing conditions.

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