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Urban Train Timetable Optimization Based on Multi-Objective Optimization

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Abstract. The urban rail transit industry in modern society is growing at an alarming rate, and in the rail transit of the network is the schedule is the basis of the train operation. Current research focuses on designing and optimizing train schedules to better meet passenger demands. In this paper, a multi-objective programming model is established on the basis of objective functions that are coupled and aimed at minimizing operating costs and passenger travel efficiency in order to optimize the train timetable, while considering meeting the demand of passenger flow. One objective function is kept in the original problem according to the characteristics of the model, and other objective functions are transformed into constraint conditions by adding restricted domains, thus turning them into single-objective programming models. Genetic algorithm is used to obtain results and train operation is simulated through the dynamic programming algorithm to carry out dynamic search. The CSMA/CD (Carrier Sense Multiple Access/Collision Detection) protocol is introduced to optimize the constraint conditions. The waiting time is transformed into the minimum tracking time interval by sending the carrier monitoring code. As such, the departure time data of large and small routes are calculated dynamically, and equal interval parallel operation diagrams are drawn. The calculation results indicate that the multi-objective optimization model improved by genetic algorithm can effectively solve practical cases, and its train timetable can highly match the spatiotemporal distribution of passenger flow demand and obtain satisfactory feasible solutions within a reasonable time.

Keywords. Train timetable, multi-objective optimization, genetic algorithm, dynamic programming, CSMA/CD

1. Introduction

Optimizing train schedules [1] is a well-established challenge within the realm of rail transit operation management, and it remains a prominent area of study in logistics and transportation management. Since 1971, when Amit and Goldfarb[2] initially employed mathematical programming to depict and enhance the operational strategy of rail transit trains, numerous studies have been published on the operation strategy of train systems in urban transit. Shafahi and Khani [3] proposed two constant time headway models, minimizing the transfer time in the transportation network. With the aim of reducing the waiting time for passengers, Barrena [4,5] introduced an extensive approach involving adaptive large neighborhood search and a branch-and-cut algorithm, aimed at addressing the optimization model for rail transit operation planning. Li et al. [6] proposed a demand-

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oriented mixed integer nonlinear programming model (HH-ST) that combines heterogeneous time headway and short radius turn.

Apparently, although previous studies aimed to optimize urban rail transit train schedules with the goal of reducing operating costs and improving passenger travel efficiency, they exclusively focused on either cost reduction or passenger travel enhancement during the optimization process. Therefore, in this paper, with the dual objective of reducing operating expenses and enhancing passenger travel efficiency, in view of the operation mode of urban rail transit with large and small routes, a multi-objective model is transformed into single-objective model according to the constraint method, other objective functions are turned into constraint conditions by adding restricted domains, the genetic algorithm is used to obtain the optimal solution, dynamic search is carried out by dynamic programming, CSMA/CD [7] is used to send carrier monitoring codes with waiting time as the minimum tracking time interval, and equal interval parallel operation diagrams are drawn dynamically.

2. Description

Under the operation mode of urban rail transit with large and small routes, large-route trains and small-route trains operate alternately. A large-route train completes the entire route, while a small-route runs between one temporary starting station and one temporary ending station along the route. T_1 and T_2 represent the turnaround time of large-route train and small-route train respectively. It is stipulated that only stations with turnback capability can function as the initial or terminal point on the line. i is the set of train routes, defined as i=1 representing a large route and i=2 representing a small route. The trains along the route are presented as an ordered set s={1, 2, 3..., n} along the operation planning direction. It is stipulated that overtaking of adjacent trains is forbidden, and the train operation sequence should remain unchanged. The rated passenger capacity of each train is C_z. a indicates that the ceiling of the train's load factor is 100%.

Train timetable is generally an equal interval parallel operation diagram, meaning the departure interval (a train departs every 5 minutes) corresponds to the time spent at the same station.t_{1d} and t_{2d} represent the typical passenger wait times for S_1 and S_2. Departure interval is restricted. The docking duration of a train at a station is directly proportional to the passenger count involved at that station. t_{run}, j represents the pure running time of a train in section j, while t_{stop}, j corresponds to the train's station docking duration at station j. In addition, when two trains run in the same tracking section, a certain safety interval (tracking interval time, I_o) shall be maintained.

3. Model Building and Solution

The compilation, evaluation, and optimization of train operation are characterized by multiple constraint conditions, large scale, and complex calculations. Factors such as passenger flow demand, enterprise operating costs, and service levels lead to the typical problem of multi-objective constrained optimization.

The ε-constraint method is one of the main solutions for multi-objective optimization problems, with the core idea of retaining one objective function in the original problem and transforming other objective functions into constraint conditions by adding restricted domains. Essentially, this method transforms multi-objective problems into single-objective optimization problems. Therefore, in this paper, the model is transformed using the ε-constraint method; a genetic algorithm is employed for solutions, considering the characteristics of the transformed single-objective models; dynamic search is carried out based on dynamic programming; CSMA/CD is used to send carrier monitoring codes to convert the waiting time into the minimum tracking time interval; and finally equal interval parallel operation diagrams are drawn. The depicted framework of the solution algorithm can be observed in Figure 1.

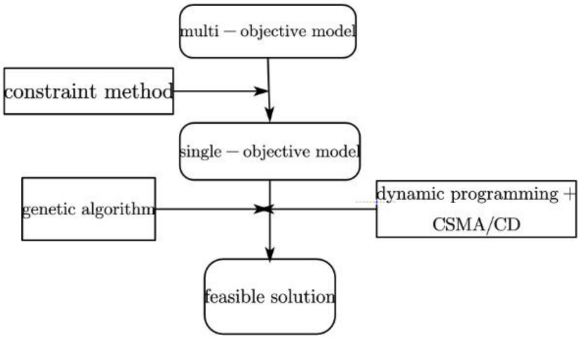

Figure 1. Solution algorithm framework.

3.1. Building of Multi-objective Optimization Model

Minimizing enterprise operating costs and maximizing passenger travel efficiency are equivalent to minimizing train travel time and maximizing passenger travel quality. The docking cost encompasses expenses such as energy consumption, train losses, and other costs incurred during train station docking. By minimizing the travel time and docking cost of trains can the operating costs be reduced and the passenger travel quality be improved. Z represents the minimum train travel time and docking cost, and is taken as an objective function.

Objective function:

$$\min\ Z = S_1 t_{1d} + S_2 t_{2d} \tag{1}$$

3.1.1. Train quantity constraint

$$\left[\frac{T_1}{60}\cdot f_1\right] + \left[\frac{T_2}{60}\cdot f_2\right] \leqslant \left[\frac{T_1}{60}\cdot f\right] \tag{2}$$

$$\text{Wherein}\ \begin{cases} T_1 = 2\cdot\left(\sum\limits_{j=1}^{n-1} t_{run,j} + \sum\limits_{h=1}^{n} t_{stop}\right) \\ T_2 = 2\cdot\left(\sum\limits_{j=a}^{b-1} t_{run,j} + \sum\limits_{h=a}^{b} t_{stop}\right) \end{cases}$$

Train quantity constraint ensures the upper limit of train arrival time. Constraint (1) ensures that the required trains under the large-and-small routing mode do not exceed the departure frequency under the single routing mode.

3.1.2. Load factor constraint

$$\max j\left(\frac{\sum_{d=j+1}^{n}\sum_{o=1}^{j} q_{od}}{\sum_{i=1}^{2} f_i}, \frac{\sum_{d=1}^{j}\sum_{o=j+1}^{n} q_{od}}{\sum_{i=1}^{2} f_i}\right) \leqslant \alpha \cdot C_z \tag{3}$$

The load factor constraint [8] of a train ensures that the loaded members of the train do not exceed the rated upper limit. If safety is ensured, the closer to the upper limit, the higher the selectivity is.

3. Minimum tracking interval

$$f_1 + f_2 \leqslant \frac{60}{I_0} \tag{4}$$

When the train operation sequence diagram is fixed, by ensuring that the adjacent intervals meet the tracking interval can the train safety interval constraint be achieved. The following equation is used to determine the minimum tracking interval constraint after conversion to frequency.

4. Departure interval constraint

$$f_{\min}^2 \leqslant (f_1, f_2) \leqslant f_{\max}^2 \tag{5}$$

Departure interval constraint reduces operating costs while ensuring the safety of train operation. It improves the service level and ensures the uniformity of train departure time.

5. Docking time constraint

$$20 \leqslant \frac{1}{T_i} = \max\left(f_i, \frac{q_{od}}{t}\right) \leqslant 120 \tag{6}$$

Docking time constraint ensures the safe arrival and departure interval of adjacent trains at the station.

6. Other constraints

(1) $3 \leq OD \leq 24$, the constraint on the number of stations along a small route ensures the effectiveness of the operation mode.

(2) The constraint on the nodes along a small route, represented by g, and defined as g=1 representing turnback station; g=∞ representing non-turnback station. This ensures that the selected node is a turnback station, in accordance with the actual circumstances. Wherein, t represents the average boarding and alighting time of passengers.

To sum up, in this problem, the multi-objective model can be described as single objective programming models with constraints[9]:

$$\min Z = (S_1 t_{1d} + S_2 t_{2d}) \cdot g$$

$$S.T\begin{cases} \left[\frac{T_1}{60} \cdot f_1\right] + \left[\frac{T_2}{60} \cdot f_2\right] \leqslant \left[\frac{T_1}{60} \cdot f\right] \\ \max j\left(\frac{\sum_{d=j+1}^{n}\sum_{o=1}^{j} q_{od}}{\sum_{i=1}^{2} f_i}, \frac{\sum_{d=1}^{j}\sum_{o=j+1}^{n} q_{od}}{\sum_{i=1}^{2} fi}\right) \leqslant d \cdot C_z \\ f_1 + f_2 \leqslant \frac{60}{I_0} \\ f_{\min}^2 \leqslant (f_1, f_2) \leqslant f_{\max}^2 \\ 20 \leqslant \max\left(f_i, \frac{q_{od}}{t}\right) \leqslant 120 \\ 3 \leqslant OD \leqslant 24 \end{cases} \tag{7}$$

3.2. Genetic Algorithm Solving

Genetic algorithm is a random search optimization algorithm based on evolution theory, biological selection theory, and population genetics theory [10]. It represents a computational model that imitates the natural evolution process to establish and solve problem extremum. Thanks to its solid biological foundation and intelligent simulation generation process, genetic algorithm is not limited to the advantages of specific conditions, for which it is widely used in solving combinatorial optimization problems, nonlinear problems [11], and multi-dimensional space optimization problems.

Here are the detailed steps: (Figure 2):

Step 1: Initialize and set the evolution iterator, iter → 0; set the maximum evolution belt speed to 10^4; randomly generate 200 individuals as the initial population.

Step 2: Selection operation: employ the selection operator within the group.

Step 3: Cross operation: implement the crossover operator within the group.

Step 4: Mutation operation: introduce the mutation operator within the population.

Step 5: Individual evaluation: Individual assessment involves determining the individuals' fitness within the population by considering both constraint conditions and objective functions.

Step 6: The population produces the next generation after steps 2, 3, 4, and 5; if $i < iter$, then i → i+1; if $i > iter$, the highest fitness achieved during the evolutionary process designates the individual to be the optimal solution, which is then presented as the output. The flowchart can be found in Figure 2.

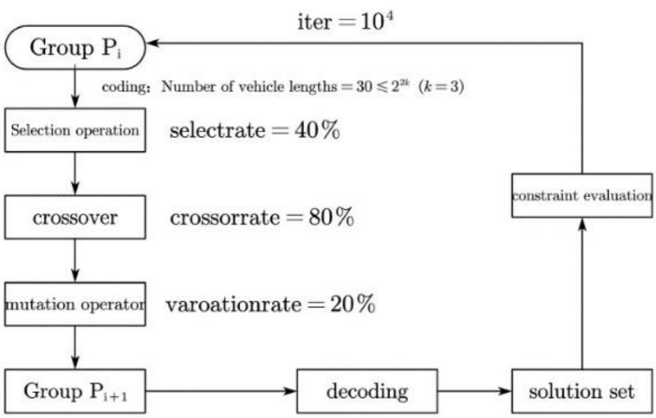

Figure 2. A flowchart for solving single-objective optimization using genetic algorithm.

The algorithm fitness function is determined by the objective function and is used to calculate and evaluate each individual's fitness value of [12], which represents the current individual's adaptation to the living environment. The individual's performance improves as their fitness value increases, leading to a greater likelihood of the individual being selected. The following represents the fitness function:

$$fit[f(x)] = \frac{1}{f(x)} \tag{8}$$

3.3. Dynamic Programming Simulation

Optimizing the train schedule for urban rail transit is a highly intricate task, as it requires taking into account the passenger flow at each station, the docking time and distance at each station, and the departure time of passengers. When optimizing the timetable, one must consider a range of practical situations. Dynamic programming, as a common optimization algorithm [13], can solve a problem by dividing it into smaller sub-problems. It can also solve complex rail transit train optimization problems by obtaining the approximate optimal solution based on a dynamic programming model [14].

In order to better achieve the goal of minimizing economic costs and maximizing service effectiveness at the same time, Problem 1 Constraint (4) has been changed to the CSMA/CD protocol. The waiting time is taken as the minimum tracking interval [15]. The upper limit of Constraint (5) is lifted in order to ensure the passenger flow demand is met as much as possible, that is, $20 \leqslant \frac{1}{T_i} = \max\left(f_i, \frac{q_{od}}{t}\right)$.

Under the above constraints, the train time sequence state transition equation of train O is updated as follows:

$$dp[i] = \begin{cases} dp[i-1] + T_{\left\lfloor \frac{i-1}{2} \right\rfloor \left\lfloor \frac{i}{2} \right\rfloor} \\ dp[i-1] + \max\left(h(x) + \frac{q_{\left\lfloor \frac{i}{2} \right\rfloor}}{t}, t_{\min}\right) \end{cases} \tag{9}$$

The trains passing through a station can be divided into docking trains and departing trains. [i/2] represents the station where the train is located. When i is an even number, it represents that the train has just entered station [i/2]. Its inbound time sequence state can be changed to $dp[i-1] + T_{\left\lfloor \frac{i-1}{2} \right\rfloor \left\lfloor \frac{i}{2} \right\rfloor}, T_{\left\lfloor \frac{i-1}{2} \right\rfloor \left\lfloor \frac{i}{2} \right\rfloor}$, representing the time it takes for the train to depart from station [(i-1)/2] to station [i/2]. When i is an even number, it represents that the train is departing from the station. Its outbound time sequence state can be changed to the inbound time sequence + the optimal docking time sequence. Wherein, h(x) is obtained from CSMA/CD, $q\left\lfloor \frac{i}{2} \right\rfloor \frac{1}{t}$ represents the time it takes for passengers to get on/off, and t_{min} is the minimum docking time of the train.

4. Simulation Example

In order to confirm the feasibility and efficiency of the proposed method, the rail transit route in question is supported by actual data, comprising a total of 30 stations, with passenger flow data collected between 7:00 and 8:00. The Matlab on a computer with a Gen Intel Core TM i5-11400H 2.70GHz CPU and 16GB of memory is used for programming and execution.

4.1. Basic data

Table 1 lists the station numbers, station spacing, section running time, cross-sectional passenger flow of 30 stations, and whether they can serve as the starting/ending stations of the route under the large-and-small urban rail transit operation mode.

4.2. Result Analysis

In the above examples, the optimal value-evolution generation number of the parameters for each generation of the model (as shown in Figure 3) is calculated through single objective programming models, genetic algorithm, and dynamic programming. It can be seen that the longitudinal coordinates first fluctuate and then gradually stabilize, and there is no significant downward or upward trend in the values that remain relatively stable in subsequent evolution generation numbers. Therefore, it can be determined that the number of trains running in the operating section of a large route should be 10, while for a small route, the operating segment consists of 8-25 stations, and there should be 10 trains in operation. The maximum fitness calculated according to equation (8) is: 0.00000045684. The lower the fitness, the higher the fitness value, the greater the likelihood of population alignment, the more an individual can conform to the environment, and the improved the algorithm's performance.

Table 1. Passenger flow and operational data of stations

Section	Section running time (s)	Station Spacing (km)	Coss-sectional passenger flow (person)	Whether the departure station can serve as a starting/ending station along the route
Station 1->Station 2	120	1.38	3169	Yes
Station 2->Station 3	97	1.15	5613	Yes
Station 3->Station 4	101	1.318	7331	No
Station 4->Station 5	89	0.964	11179	No
Station 5->Station 6	102	1.096	15802	Yes
Station 6->Station 7	144	1.945	21502	No
Station 7->Station 8	129	1.506	30650	No
Station 8->Station 9	162	2.267	30939	Yes
Station 9->Station 10	103	1.241	33990	No
Station10->Station11	96	1.012	36824	Yes
Station11->Station12	139	1.589	45749	No
Station12->Station13	133	1.865	46535	No
Station13->Station14	82	0.779	45402	No
Station14->Station15	94	1.233	46049	Yes
Station15->Station16	193	2.428	33568	No
Station16->Station17	128	1.738	32233	No
Station17->Station18	111	1.612	31248	Yes
Station18->Station19	90	1.14	30524	Yes
Station19->Station20	106	1.034	30579	No
Station20->Station21	159	1.8	30681	No
Station21->Station22	104	1.161	30411	Yes
Station22->Station23	80	0.827	30815	Yes
Station23->Station24	114	1.592	10435	No
Station24->Station25	79	0.721	10493	No
Station25->Station26	86	1.026	9648	Yes
Station26->Station27	113	1.35	9130	Yes
Station27->Station28	86	0.792	8003	Yes
Station28->Station29	121	1.491	6611	No
Station29->Station30	154	2.111	4597	No
Station 30	0	0	0	Yes

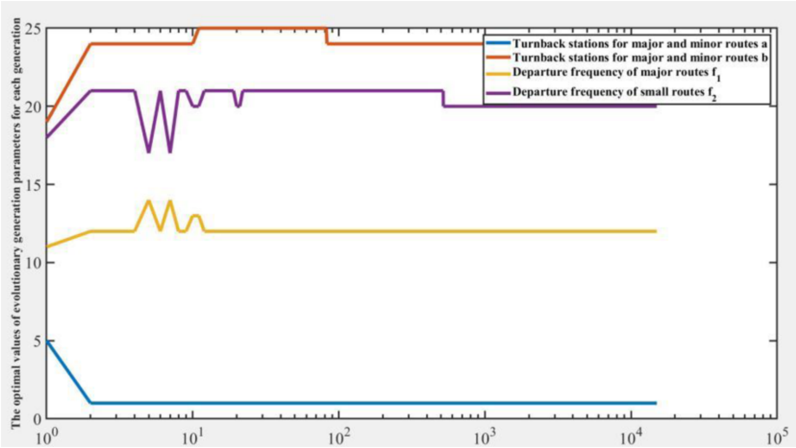

Figure 3. Optimal value-evolution generation number of the parameters for each generation.

The equal interval parallel operation diagram of large route at stations 1-30 and small route at stations 8-25 are obtained (as shown in Figure 4). It can be seen from the diagram that the train timetable can well match the spatiotemporal distribution of passenger flow demand; compared to other time periods, there is a large passenger flow demand between 7:30 and 8:00, resulting in more train operation lines during this period. This indicates that with the goal of maximizing operational benefits and improving passenger travel efficiency, the effectiveness of the method proposed in this paper is confirmed by the successful alignment of passenger flow demand across the train timetable in terms of its spatiotemporal distribution.

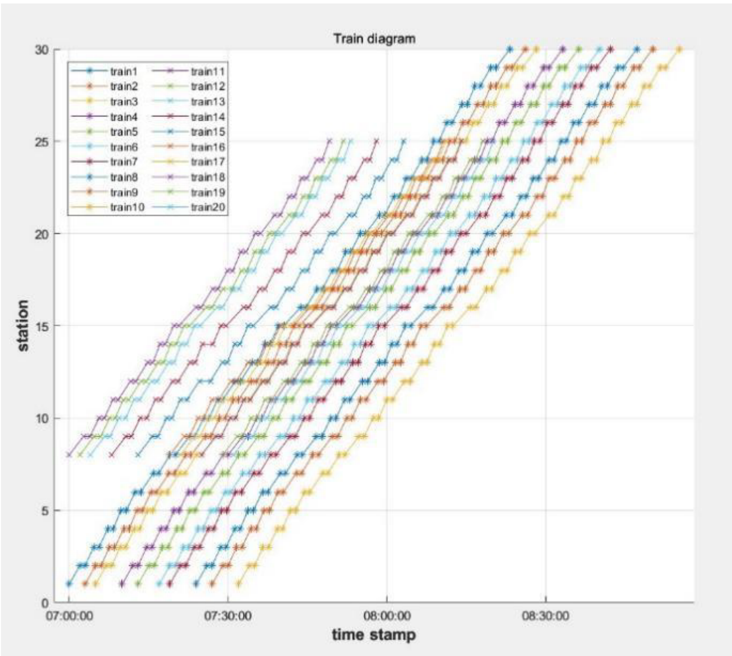

Figure 4. Train diagram.

5. Conclusions

In this paper, centered on the goal of both minimization of the costs of operation and maximizing the quality of services provided, a multi-objective programming model is established, train operation is simulated with the dynamic programming algorithm to carry out dynamic search, the CSMA/CD protocol is introduced to optimize constraint conditions, the goal of cost minimization and service quality maximization is better represented in an abstract manner on the basis of mathematical models, thereby obtaining ideal results in the simulation experiment. Further studies are required on how to better represent abstract requirements such as minimizing operating costs and maximizing service quality from multiple dimensions and directions, as well as on the performance of relevant algorithms in the application processes.

Acknowledgement

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