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A Simplified Semi-Analytic Method to Calculate Notch Stresses at V-Notch Tips

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Abstract. In this thesis, based on the theoretical derivation of notch mechanics, a simplified semi-analytical method has been established for the analysis of V-notch stress fields, thereby providing a theoretical foundation for rapid computation of notch stresses in welded joints. By comparing the notch stress results obtained through the semi-analytical method and those derived from finite element analysis (FEM), the accuracy of the semi-analytical method in analyzing notch stress fields has been verified. The findings indicate that the semi-analytical approach offers a distinct advantage in addressing notch stress analysis due to its mesh insensitivity.

Keywords. V-notch; semi-analytic method; notch stresses; welded joint.

1. Introduction

The majority of engineering structures consist of steel welded components and are consistently subjected to alternating environmental loads. [1,2]， fatigue failure is a main failure mode in marine engineering. As is well known, localized stress concentrations result in severe notch stress gradients at weld toes, which leads to the initiation and propagation of fatigue cracks. [3,4]. The notch stresses analysis around the weld corners is prerequisite when employing the fatigue assessment approaches. This brings about the analysis of the corner stress distribution of welded structures, which is a necessary condition for accurate fatigue design of welded structures.

The profiles of weld toes are often regarded as a distinct scenario, similar to sharp V-notches, bearing similarity to notches with un-machined welds and assuming conventional arc-welding technologies [5,6], as illustrated on the Figure 1. The initial method for assessing stress fields at V-notch tips was through eigenvalue expansion. [7,8], as introduced by Williams [9]. Williams confirmed that the stresses around V-notch exhibits singularity, and the grade of singularity is determined by the notch angle in linear elastic stress fields. In the theory of linear elasticity, the stress at the notch is infinite and the concept of stress concentration is not applicable to the notch. Therefore Gross and Mendelson [10] introduced the Notch Stress Intensity Factors (NSIFs) to characterize stress fields around the V-notch, extending the established Stress Intensity Factors typically used for cracks. At that time, only the notch stress intensity factor data of a specific shape notch could be given, and the stress intensity factor of the weld toe gap

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of the quasi-welded joint could not be calculated, which also limited the development of early fatigue strength evaluation methods. Subsequently, Qian and Hasebe [11] established the definition of NSIFs under mode III shear loading.

Nonetheless, due to the stress singularity of the V-notch, a drawback lies in the requirement for highly refined finite element meshes when analyzing the notch stress distributions. In this thesis, a simplified semi-analytical method is proposed for estimating the stress distributions around the V-notch tips.

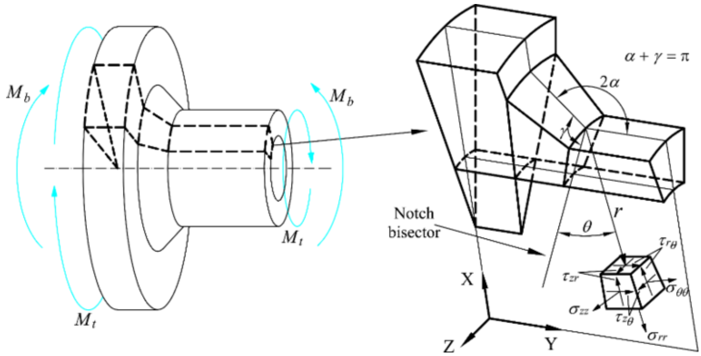


Figure 1. Polar reference coordinates system at a typical weld toe notch.

2. Theoretical Background

Williams [9] asserted that in linear elastic stress fields, the classical definition of stress intensity factors (SIF) used to describe stress singularities at crack tips can be applied to pointed V-notches. However, the stress singularity level is diminished and contingent upon the opening V-notches angles. In a polar reference coordinate system (r,θ), the stress fields of V-notch tips can be resolved into mode I and mode II stresses:

$$\begin{Bmatrix} \sigma_{\theta\theta} \\ \sigma_{rr} \\ \tau_{r\theta} \end{Bmatrix} = \lambda_1 r^{\lambda_1-1} d_I \begin{Bmatrix} f_{I,\theta\theta}(\theta) \\ f_{I,rr}(\theta) \\ f_{I,r\theta}(\theta) \end{Bmatrix} + \lambda_2 r^{\lambda_2-1} d_{II} \begin{Bmatrix} f_{II,\theta\theta}(\theta) \\ f_{II,rr}(\theta) \\ f_{II,r\theta}(\theta) \end{Bmatrix} \tag{1}$$

where r represents the distance to V-notch tip., λ_1,λ_2 are eigenvalues of the 'eigenfunction expansion' in the solution, see Eq. (2) and Eq. (3), $d_{I,II}$ are the complex constants, $f_{I,II}(\theta)$ are the angular distribution functions describing the stress fields near the V-notch, q in Eqs. (2), (3) is correlated with the notch opening angle 2α . The eigenvalue equation of stress field of V-notch is obtained as follows:

$$\sin(\lambda_1 q\pi) + \lambda_1 \sin(q\pi) = 0 \tag{2}$$

$$\sin(\lambda_2 q\pi) - \lambda_2 \sin(q\pi) = 0 \tag{3}$$

$$2\alpha=2\pi-q\pi \tag{4}$$

$$\sin\lambda_3\left(2\pi-2\alpha\right)=0,\ \lambda_3=\frac{\pi}{2\pi\text{-}2\alpha} \tag{5}$$

Let us consider a more complex V-notch subjected to mixed mode I, II and III loading (Figure.1). Based on the concept of the NSIFs proposed by Gross and Mendelson [10], the stress components due to mode I and II are derived with the help of curve-assisted coordinate system by Lazzarin and Tovo [5,6], see Eqs. (6), (7). Then, Qian and Hasebe [11] obtain the formula of mode III stress component, see Eq. (8).

$$\begin{Bmatrix}\sigma_{\theta\theta}\\ \sigma_{rr}\\ \tau_{r\theta}\end{Bmatrix}=\frac{1}{\sqrt{2\pi}}\frac{r^{\lambda_1-1}K_1}{[(1+\lambda_1)+\chi_{b1}(1-\lambda_1)]}\times\left[\begin{Bmatrix}(1+\lambda_1)\cos(1-\lambda_1)\theta\\ (3-\lambda_1)\cos(1-\lambda_1)\theta\\ (1-\lambda_1)\sin(1-\lambda_1)\theta\end{Bmatrix}+\chi_{b1}(1-\lambda_1)\begin{Bmatrix}\cos(1+\lambda_1)\theta\\ -\cos(1+\lambda_1)\theta\\ \sin(1+\lambda_1)\theta\end{Bmatrix}\right] \tag{6}$$

$$\chi_{b1}=-\frac{\sin\left(1-\lambda_1\right)q\pi/2}{\sin\left(1+\lambda_1\right)q\pi/2}$$

$$\begin{Bmatrix}\sigma_{\theta\theta}\\ \sigma_{rr}\\ \tau_{r\theta}\end{Bmatrix}=\frac{1}{\sqrt{2\pi}}\frac{r^{\lambda_2-1}K_2}{\left[(1-\lambda_2)+\chi_{b2}(1+\lambda_2)\right]}\times\left[\begin{Bmatrix}(1+\lambda_2)\sin(1-\lambda_2)\theta\\ (3-\lambda_2)\sin(1-\lambda_2)\theta\\ (1-\lambda_2)\cos(1-\lambda_2)\theta\end{Bmatrix}+\chi_{b2}(1+\lambda_2)\begin{Bmatrix}\sin(1+\lambda_2)\theta\\ -\sin(1+\lambda_2)\theta\\ \cos(1+\lambda_2)\theta\end{Bmatrix}\right]. \tag{7}$$

$$\chi_{b2}=-\frac{\sin\left(1-\lambda_2\right)q\pi/2}{\sin\left(1+\lambda_2\right)q\pi/2}$$

$$\begin{Bmatrix}\tau_{z\theta}\\ \tau_{zr}\end{Bmatrix}=\frac{K_3}{\sqrt{2\pi}}r^{\lambda_3-1}\begin{Bmatrix}\cos\left(\lambda_3\theta\right)\\ \sin\left(\lambda_3\theta\right)\end{Bmatrix} \tag{8}$$

$$\sin\lambda_3\left(2\pi-2\alpha\right)=0,\ \lambda_3=\frac{\pi}{2\pi\text{-}2\alpha} \tag{9}$$

In the above equations, $\lambda_1,\lambda_2,\lambda_3$ and χ_1,χ_2 are constants related to the opening angle, see Table 1. It's important to note that mode I and mode III stresses are typically coupled with mode II stress, When the notch angle exceeds 102°, mode II stress becomes nonsingular. K_1,K_2,K_3 are the Notch Stress Intensity Factors (NSIFs) for mode I, II, and III. The definitions for NSIFs are as follows:

$$K_1=\sqrt{2\pi}\lim\left(\sigma_{\theta\theta}\right)r^{1-\lambda_1},r\to 0 \tag{10}$$

$$K_2=\sqrt{2\pi}\lim\left(\tau_{r\theta}\right)r^{1-\lambda_2},r\to 0 \tag{11}$$

$$K_3=\sqrt{2\pi}\lim\left(\tau_{z\theta}\right)r^{1-\lambda_3},r\to 0 \tag{12}$$

Table 1. Values of constants in Eqs. (6)-(8)

2α (°)	λ_1	χ_{b1}	λ_2	χ_{b2}	λ_3
0	0.500	1.000	0.500	1.000	0.500
30	0.501	1.071	0.598	0.921	0.545
45	0.505	1.166	0.660	0.814	0.571
60	0.512	1.312	0.731	0.658	0.600
90	0.545	1.841	0.909	0.219	0.666
120	0.616	3.003	1.149	-	0.750
135	0.674	4.153	1.302	-	0.800
150	0.752	6.362	1.486	-	0.857

3. The Modified Semi-Analytic Method

Lazzarin and Tovo [5,6] conducted a study on the scale effect of the NSIFs and concluded that NSIFs can be conveniently represented in the form of non-dimensional parameters. The calculation of these parameters is as follows:

$$K_1 = k_1 \sigma_n t^{1-\lambda_1} \tag{13}$$

$$K_2 = k_2 \sigma_n t^{1-\lambda_2} \tag{14}$$

$$K_3 = k_3 \tau_n t^{1-\lambda_3} \tag{15}$$

where σ_n and τ_n are the nominal stresses depending on the applied remote loads. k_i are geometry parameters that are contingent on the overall welded geometry.

The stress distributions equations become:

$$\sigma_{\theta\theta} = \frac{\sigma_n}{\sqrt{2\pi}} \times \frac{1}{x^{1-\lambda_1}} \times \left[C_1(\alpha,\theta) k_1^{1/1-\lambda_1} t\right]^{1-\lambda_1} \tag{16}$$

$$\tau_{r\theta} = \frac{\sigma_n}{\sqrt{2\pi}} \times \frac{1}{x^{1-\lambda_2}} \times \left[C_2(\alpha,\theta) k_2^{1/1-\lambda_2} t\right]^{1-\lambda_2} \tag{17}$$

$$\tau_{z\theta} = \frac{\tau_n}{\sqrt{2\pi}} \times \frac{1}{x^{1-\lambda_3}} \times \left[C_3(\alpha,\theta) k_3^{1/1-\lambda_3} t\right]^{1-\lambda_3} \tag{18}$$

where x represents the distance to the notch tip.

For a given V-notch angle 2α at a specific direction θ, λ_i and $C_i(\lambda_i,\theta)$ can be treated as constants. Consequently, a new equivalent parameter '*as*' can be defined as follows:

$$as_i = C_i(\lambda_i, \theta) \cdot f_i(h, t, L)^{1/(1-\lambda_i)} \pi^{-1/2(1-\lambda_i)} t \tag{19}$$

Then stress components of the V-notch can be simplified as follow forms:

$$\sigma_{\theta\theta} = \frac{\sigma_n}{\sqrt{2}} \left(\frac{as_1}{x} \right)^{1-\lambda_1} \tag{20}$$

$$\tau_{r\theta} = \frac{\sigma_n}{\sqrt{2}} \left(\frac{as_2}{x} \right)^{1-\lambda_2} \tag{21}$$

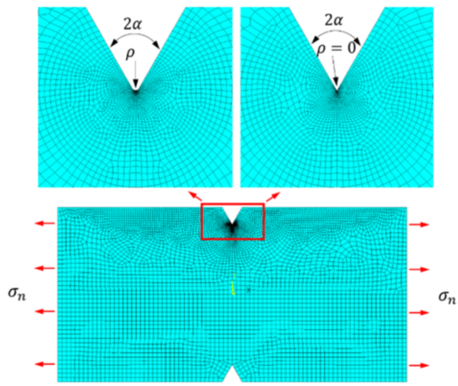
$$\tau_{z\theta} = \frac{\tau_n}{\sqrt{2}} \left(\frac{as_3}{x} \right)^{1-\lambda_3} \tag{22}$$

It is evident that the equivalent parameter *as* governs the singularity level. It's important to note that '*as*' is expressed in units of length (with the same unit as the distance "*x*").

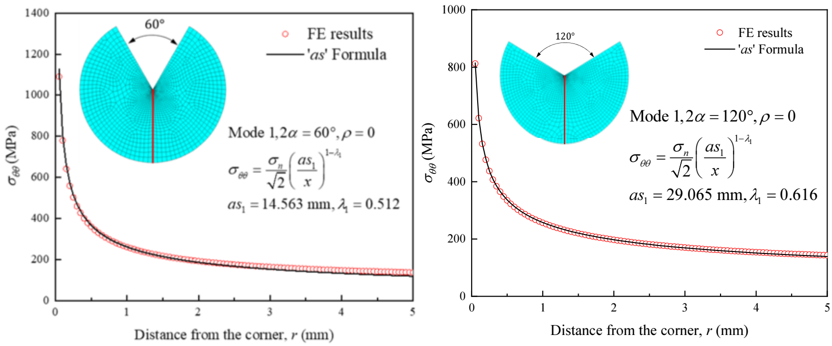
Through a series of theoretical analyses and derivations, the stress field formula for notches has been simplified into a semi-analytical formula in the form of an exponential function. In this formula, the characteristic parameters and can be fitted using finite element analysis results. Consequently, with this simplified semi-analytical notch stress formula, it becomes feasible to compute the stress distribution for V-notches.

4. Verification of the Semi-Analytic Method

In this section, typical notch models for mode 1,2 and 3 loading conditions are established. For each type of loading, notch models with 60° and 120° opening angles are considered. To validate the semi-analytical method, the results derived from the semi-analytical formula are compared with those obtained from finite element analysis to confirm the methodology's precision.

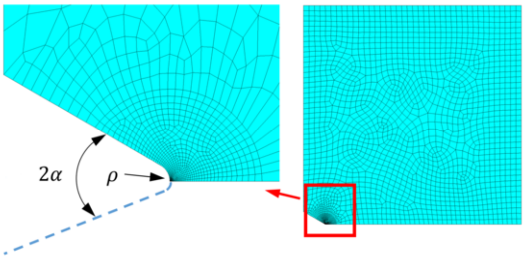


(a) notch model for mode 1

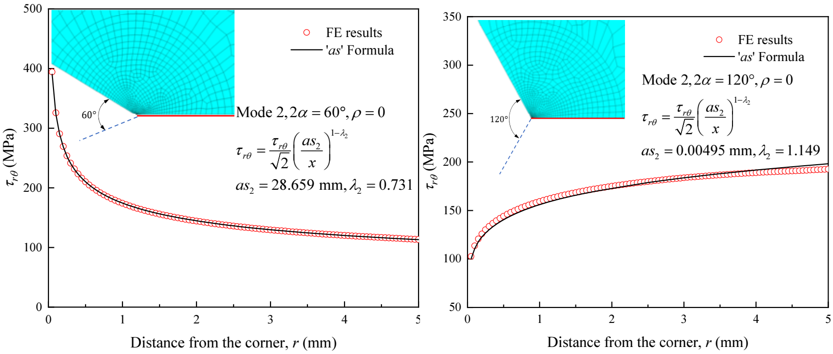


(b) 2α=60° (c) 2α=120°

Figure 2. Comparison results of two methods for mode 1

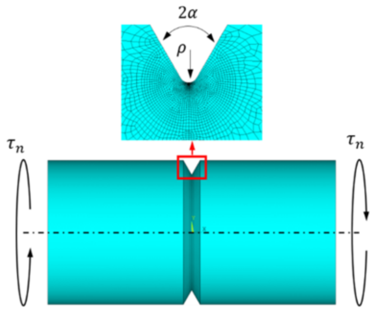


(a) notch model for mode 2

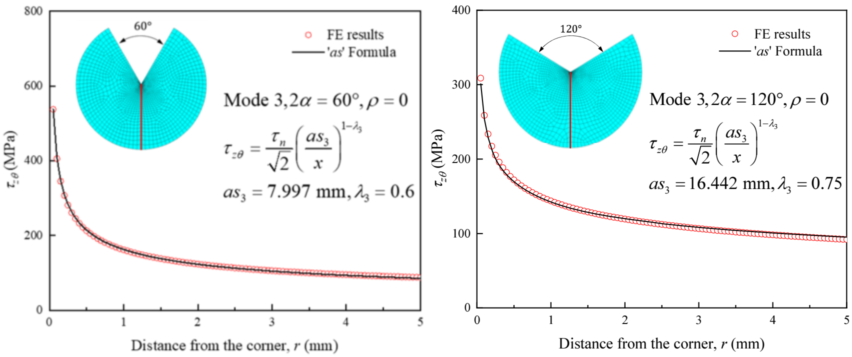


(b) 2α=60° (c) 2α=120°

Figure 3. Comparison results of two methods for mode 2



(a) notch model for mode 2

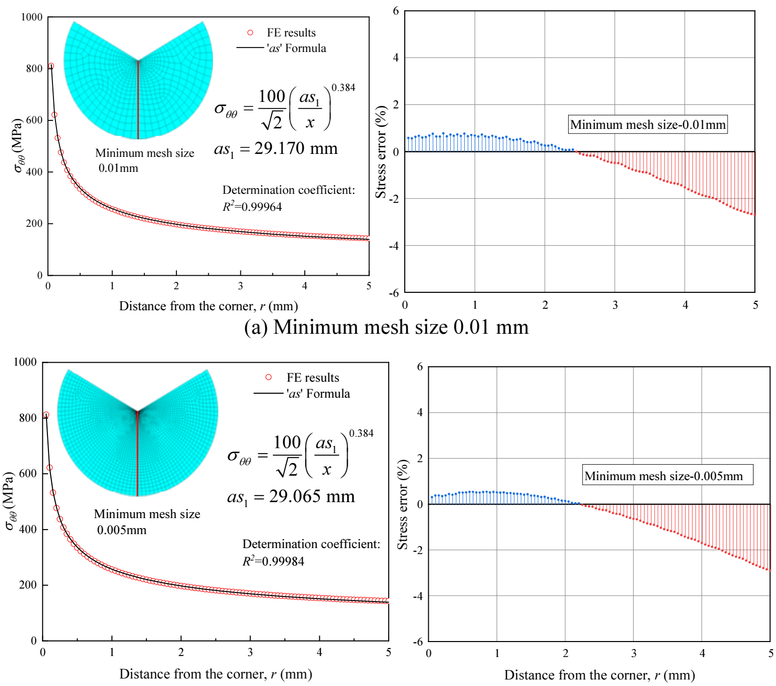


(b) 2α=60° (c) 2α=120°

Figure 4. Comparison results of two methods for mode 3

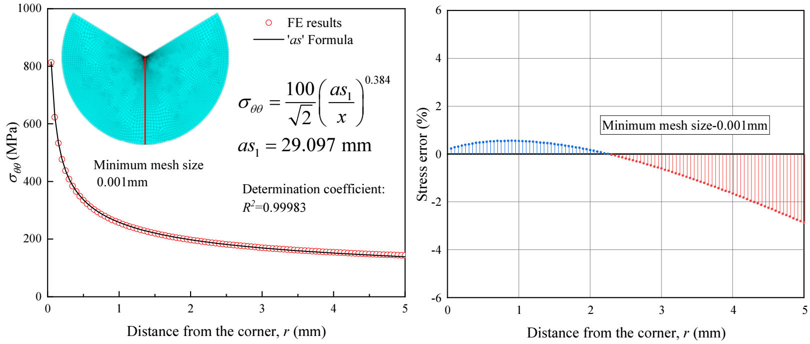
The notch models subjected to mode 1,2 and 3 loading are respectively illustrated in Figures 2, 3 and 4. To ensure the accuracy of computational results near the notch, a circular progressively refined mesh region has been implemented in the vicinity of the notch corner.From Figures 2, 3 and 4, the stress results obtained from the semi-analytic formula exhibit a strong correlation with the finite element results, both in terms of stress variation trends and values. This confirms the accuracy of the semi-analytic method for V-notch analysis.

The semi-analytic formula is only related to the stress distribution of theV-notch and a fine mesh is not necessary in this method, as shown in Figure. 5. Take the notch mode of 120° opening angle under mode 1 load as an example, minimum mesh size ranges from 0.01 mm to 0.001mm, the fitting *as* values barely have change, and the stress errors are all within 3%. The semi-analytic method has an advantage of mesh insensitivity.



(a) Minimum mesh size 0.01 mm

(b) Minimum mesh size 0.005 mm



(c) Minimum mesh size 0.001 mm

Figure 5. Stress errors between semi-analytic method and FE results with different mesh sizes

5. Conclusion

Based on theoretical derivation of notch mechanics, a simplified semi-analytical method has been established for analyzing the V-notch stress fields, providing a theoretical foundation for the rapid computation of notch stresses in welded joints. By comparing notch stress results of the semi-analytic method and those from FEM, the accuracy of the semi-analytic method to analyze notch stress fields is verified. The results of notch models with different mesh densities show little change in the semi-analytic formula, which indicates that the semi-analytic formula has the advantage of grid insensitivity in analyzing notch stress.

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