

Study on Vibro-Acoustic Characteristics and Suppression of Plate-Cavity System

Jinpeng SU^a, Yiqiang JIANG^a, Qiang ZHANG^{a,1}, Zhengmin HU^b

^a *College of Mechanical and Electronic Engineering, Shandong University of Science and Technology, Qingdao 266590, China*

^b *School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China*

Abstract. Plate is widely used structure in marine, aerospace, and vehicle engineering. It is usually suffered from fluctuated forces and will vibrate and radiate noise. Coupled with acoustic cavity, there will be a complex vibro-acoustic coupling phenomenon, which will aggravate the vibration and acoustic radiation. In this paper, a negative capacitance piezoelectric shunt damping technique is used to suppress the vibro-acoustic response of the plate-cavity system. The multi-field coupling control model is established, and the acoustic vibration response of the system is studied. Through numerical analysis, it is concluded that the negative capacitance piezoelectric shunt technology can effectively suppress the vibro-acoustic response of the plate-cavity system.

Keywords. Composite plate, Cavity, Piezoelectric shunt damping, Vibro-acoustic response

1. Introduction

Composite plates have excellent mechanical properties and designability such as high strength, light weight and fatigue resistance[1],[2], and are widely used in aerospace, naval and other equipment as basic units[3],[4]. When subjected to external interference, the structural unit of the plate generates vibration and radiates noise to the outside, which is coupled with the cavity, resulting in complex vibro-acoustic coupling phenomenon and more significant vibration acoustic radiation. Therefore, it's indeed crucial to investigate the coupling mechanism and control of composite plate-cavity systems in order to enhance the performance of related equipment[5].

The primary approaches for mitigating the noise of the plate-cavity system's internal and external sound fields involve structural optimization design for reduced noise and the implementation of vibration noise control techniques[6]. The traditional active and passive vibration and noise control techniques often face challenges when it comes to achieving effective control at low and wide frequency ranges while introducing minimal additional mass, which limits the application in engineering. In recent years, piezoelectric shunt damping technology has attracted much attention because it is easy to realize low frequency and multi-mode control, and researchers have done a lot of research and made important progress. Currently, there is limited research on the use of

¹ Corresponding Author: Qiang Zhang, College of Mechanical and Electronic Engineering, Shandong University of Science and Technology, e-mail: 415564476@qq.com.

piezoelectric shunt circuits in damping plate-cavity systems. There is a significant lack of experimental research to verify the effectiveness of piezoelectric shunt damping technology in suppressing the acoustic response of the coupling system[7].

2. Vibro-acoustic response control of plate-cavity system

Based on a semi-analytical model, the impact of various excitation conditions on the acoustic vibration response in different systems is calculated, and how to suppress the acoustic vibration response of system under different working conditions is studied.

Therefore, the text utilizes the negative capacitance piezo shunt damping technology to mitigate the acoustic vibration response of the system. The piezoelectric plate connected with the negative capacitance shunt circuit is pasted on the surface of the plate, and by controlling the plate's vibration, the acoustic response of both the internal and external sound fields in the plate-cavity system can be effectively attenuated[8].

3. Vibro-acoustic response control technology based on negative capacitance piezoelectric shunt damping

Figure 1(a) shows the schematic diagram of the plate acoustic cavity coupling system with multiple shunt piezoelectric plates pasted on the surface[7], and each piezoelectric plate is externally connected with an independent shunt circuit. Because of the positive piezoelectric impact of piezoelectric materials, a voltage difference is generated at the two ends of the piezoelectric plate, thereby creating an electrical current in the parallel circuit. The mechanical energy of structural vibration is converted into electrical energy and finally dissipated in the form of heat energy, resulting in piezoelectric shunt damping, which inhibits the vibration of the plate, then attenuates the response of the plate-cavity system.

The shunt circuit used in this paper is the negative capacitance shunt circuit shown in Figure 1(b)[8], which is formed by the RL resonant circuit in parallel with the negative capacitance. The damping coefficient and natural frequency of the electromechanical vibration absorber are changed by adjusting the resistance and inductance values in the shunt circuit respectively, to achieve the best control effect of vibro-acoustic response at the mode frequency to be controlled.

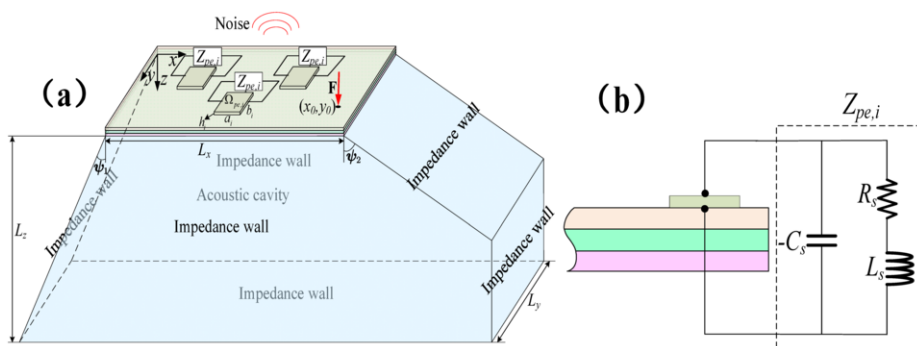


Figure 1. Laminated cavity coupling system: (a)The laminated plate-cavity system with multiple shunted piezoelectric patches; (b) Negative capacitance shunt circuit.

The acousto-solid-electric multi-field coupling control model of laminate - irregular acoustic cavity coupling system is established. The stress-electric displacement constitutive equation of piezoelectric plate is as follows[9],[10],[11]:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ D_3 \end{Bmatrix} = \begin{bmatrix} \bar{C}_{11}^E & \bar{C}_{12}^E & 0 & -\bar{e}_{31} \\ \bar{C}_{12}^E & \bar{C}_{11}^E & 0 & -\bar{e}_{32} \\ 0 & 0 & \bar{C}_{66}^E & 0 \\ \bar{e}_{31} & \bar{e}_{32} & 0 & \bar{\epsilon}_{33}^s \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \\ E_3 \end{Bmatrix} \quad (1)$$

where the superscripts E and s denote the constant electric field strength and constant strain, \bar{C}_{ij}^E ($i,j=1,2,6$) represents the elastic stiffness of the piezoelectric material, \bar{e}_{31} , \bar{e}_{32} and $\bar{\epsilon}_{33}^s$ represent the piezoelectric constant and dielectric constant of the piezoelectric material. The electric field strength is expressed as $E_3 = -\nu_i/h_i$, according to the potential drop ν_i generated by the i th piezoelectric sheet.

Assuming that there are k piezoelectric plates on the laminated plate, the expression of the total strain energy of the piezoelectric plate is:

$$U_{pe} = \sum_{i=1}^k \frac{1}{2} \iiint_{V_{pe,i}} (\sigma_{xx}\epsilon_{xx} + \sigma_{yy}\epsilon_{yy} + \tau_{xy}\gamma_{xy} - D_3E_3)_{pe,i} dV_{pe,i} \quad (2)$$

Here $V_{pe,i}$ represents the volume of the i piezoelectric plate, substituting formula (1) into formula (2), the arrangement is:

$$U_{pe} = \sum_{i=1}^k \frac{1}{2} \iiint_{\Omega_{pe,i}} \left(\begin{aligned} & A_{11}^i \left(\frac{\partial u}{\partial x}\right)^2 + 2A_{12}^i \left(\frac{\partial u}{\partial x}\right) \left(\frac{\partial v}{\partial y}\right) + A_{11}^i \left(\frac{\partial v}{\partial y}\right)^2 + A_{66}^i \left(\frac{\partial u}{\partial y}\right)^2 + A_{66}^i \left(\frac{\partial v}{\partial x}\right)^2 \\ & + 2A_{66}^i \left(\frac{\partial u}{\partial y}\right) \left(\frac{\partial v}{\partial x}\right) - 2B_{11}^i \left(\frac{\partial u}{\partial x}\right) \left(\frac{\partial^2 w}{\partial x^2}\right) - 2B_{12}^i \left(\frac{\partial u}{\partial x}\right) \left(\frac{\partial^2 w}{\partial y^2}\right) - 2B_{12}^i \left(\frac{\partial v}{\partial y}\right) \left(\frac{\partial^2 w}{\partial x^2}\right) \\ & - 2B_{11}^i \left(\frac{\partial v}{\partial y}\right) \left(\frac{\partial^2 w}{\partial y^2}\right) - 4B_{66}^i \left(\frac{\partial u}{\partial y}\right) \left(\frac{\partial^2 w}{\partial x \partial y}\right) - 4B_{66}^i \left(\frac{\partial v}{\partial x}\right) \left(\frac{\partial^2 w}{\partial x \partial y}\right) \\ & + D_{11}^i \left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2D_{12}^i \left(\frac{\partial^2 w}{\partial x^2}\right) \left(\frac{\partial^2 w}{\partial y^2}\right) + D_{11}^i \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 4D_{66}^i \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \\ & + 2(\bar{e}_{31} \frac{\partial u}{\partial x} + \bar{e}_{32} \frac{\partial v}{\partial y}) \nu_i - 2 \frac{h+h_i}{2} (\bar{e}_{31} \frac{\partial^2 w}{\partial x^2} + \bar{e}_{32} \frac{\partial^2 w}{\partial y^2}) \nu_i - \bar{\epsilon}_{33}^s \frac{\nu_i^2}{h_i} \end{aligned} \right)_{pe,i} d\Omega_{pe,i} \quad (3)$$

The piezoelectric sheet possesses a combined kinetic energy sum is:

$$\begin{aligned} T_{pe} &= \frac{1}{2} \sum_{i=1}^k \iiint_{V_{pe,i}} \rho_{pe} (\dot{U}^2 + \dot{V}^2 + \dot{W}^2)_{pe,i} dV_{pe,i} \\ &= \frac{1}{2} \sum_{i=1}^k \iint_{\Omega_{pe,i}} \left\{ \begin{aligned} & I_0^i [(\dot{u})^2 + (\dot{v})^2 + (\dot{w})^2] - 2I_1^i \left(\dot{u} \frac{\partial \dot{w}}{\partial x} + \dot{v} \frac{\partial \dot{w}}{\partial y} \right) \\ & + I_2^i \left[\left(\frac{\partial \dot{w}}{\partial x} \right)^2 + \left(\frac{\partial \dot{w}}{\partial y} \right)^2 \right] \end{aligned} \right\}_{pe,i} d\Omega_{pe,i} \quad (4) \end{aligned}$$

According to Hamiltonian principle, the energy variational equation of plates with multiple piezoelectric plates attached to the surface of the system can be written as follows:

$$\delta \int_{t_0}^{t_1} \left[(T_p + T_{pe}) - (U_p + U_{pe}) + W_{pe} - U_{BC} + W_f + W_{ep} + \sum_{i=1}^{R_c} W_{icp} \right] dt = 0 \quad (5)$$

where the T_p , U_p , U_{BC} and W_f represent the kinetic energy, strain potential energy, elastic potential energy and the work done by the external excitation, W_{ep} represents the work done by the external acoustic field on the upper side of the coupling system on the laminate, and W_{icp} represents the work done by the sound pressure in the i th sub-cavity on the laminate.

For the irregular cavity shown in Figure 1, it is necessary to first divide the cavity into R_c sub-cavities by region decomposition technology, and then convert each sub-cavity into a regular rectangular cavity by coordinate transformation technology. According to the modified variational principle, the energy variational equation of the irregular cavity is established as follows:

$$\delta \left\{ \int_{t_0}^{t_1} \left[\sum_{i=1}^{R_c} (T_{ic} - U_{ic} + W_{ic} + W_{ipc}) + W_{source} \right] dt + \int_{t_0}^{t_1} \sum_{i,i+1} (W_{i,i+1}^\lambda - W_{i,i+1}^c) dt \right\} = 0 \quad (6)$$

where the T_{ic} , U_{ic} , W_{ic} and W_{source} are the work done by the kinetic energy, potential energy, and acoustic impedance boundaries of the i th sub-cavity, and the work done by the point sound source inside the cavity.

According to the given formula, the governing equation for the multi-field coupling of acousto-solid-electric interaction in the system can be derived as:

$$\begin{bmatrix} \mathbf{M}_s & 0 \\ \mathbf{C}_{cs}^T & \mathbf{M}_c \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_s \\ \ddot{\mathbf{q}}_c \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_s + \sum_{i=1}^k Z_{spe,i} \Theta_{pe,i}^T \Theta_{pe,i} & 0 \\ 0 & \mathbf{Z}_c \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}}_s \\ \dot{\mathbf{q}}_c \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_s & -\mathbf{C}_{cs} \\ 0 & \mathbf{K}_c \end{bmatrix} \begin{Bmatrix} \mathbf{q}_s \\ \mathbf{q}_c \end{Bmatrix} = \begin{Bmatrix} \tilde{\mathbf{F}}_s \\ \tilde{\mathbf{F}}_c \end{Bmatrix} \quad (7)$$

4. Vibro-acoustic response control analysis of plate-cavity system

Based on the established control model, the response of this coupled system is studied. The influence of installation position of piezoelectric plate on the control effect of piezoelectric shunt circuit is studied. In order to precisely control its response of the coupling system, taking the peak response corresponding to the second order and the third order coupling patterns of this control system coupled with the plate-cavity as an example, the cloud map of the modal strain function can be depicted in Figure 2. Four positions A, B, C and D are defined in (a), and the modal strain function value $|\Gamma(x, y)|$

at the four positions meets $|\Gamma(A)| > |\Gamma(D)| > |\Gamma(B)| > |\Gamma(C)|$. And, four positions E, F, G and H are defined in (b), and the strain function value $|\Gamma(x,y)|$ at these four positions satisfies $|\Gamma(E)| \approx |\Gamma(H)| > |\Gamma(F)| > |\Gamma(G)|$.

First, as per the manner in which the resonant frequency of the circuit and the frequency of the mode to be regulated are aligned, the initial inductance value of the shunt circuit is roughly determined and a small initial resistance value is taken, and then the circuit parameters are further adjusted to achieve the best control effect. Figure 3(a) shows the response of the mean square sound pressure in the coupling system, while Figure 3(b) displays the radiant sound power response. The responses are observed under two scenarios: when the piezoelectric plate is not present, and it is positioned at different locations. As depicted in the diagram, the piezoelectric shunt circuit has the best control effect when the piezoelectric plate is arranged at position A, followed by position D, and the control effect at position B is weaker than that at position D. However, the piezoelectric shunt circuit has almost no control effect at the C position, because the strain function value at the C position is the smallest and close to the region where the strain function sign changes.

Next, the piezoelectric plates attached to the negative capacitance shunt circuit are arranged at positions E, F, G and H respectively, and the other steps are the same as above. The optimal control effect of the shunt circuit is observed in Figure 3 (c) and (d) when positioning the piezoelectric plate at E and H, with the subsequent favorable arrangement being at position F. However, when the piezoelectric plate is arranged at the G position, the piezoelectric shunt circuit has almost no control effect, for the same reason as the C position.

From the above analysis, the placement of the piezoelectric plate plays a crucial role in the effectiveness of the piezoelectric shunt circuit control. The larger the modal strain function value of the installation position of the piezoelectric plate, the better the control effect. Therefore, in the vibro-acoustic response control test, the piezoelectric plate is arranged in the area where the modal strain function value of the coupling mode to be controlled is larger.

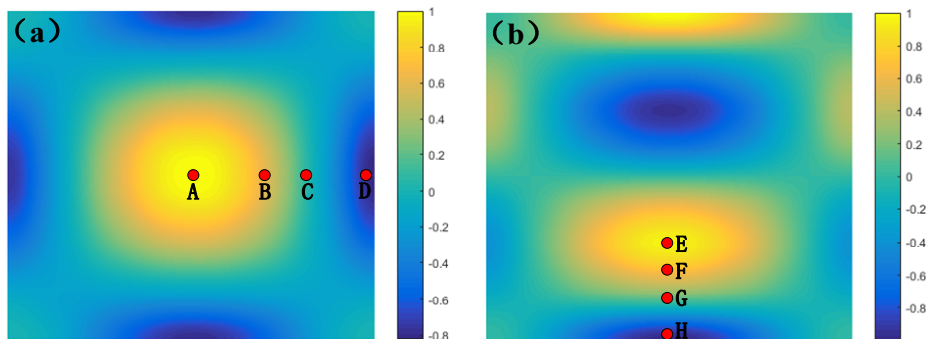


Figure 2. Modal strain function contours of the laminated plate-cavity system: (a) the modal strain function contour of the second mode; (b) the modal strain function contour of the third mode.

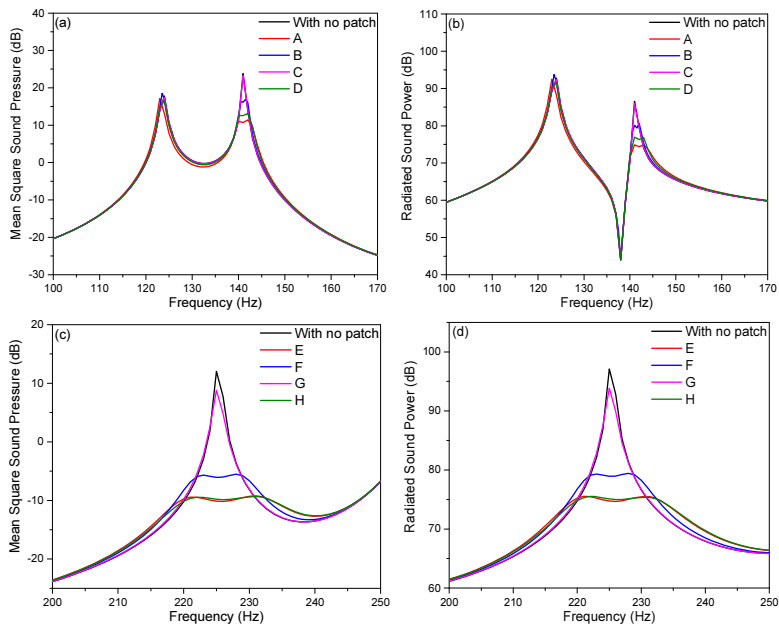


Figure 3. Vibroacoustic responses of the system with the piezoelectric patch placed at various locations: (a)(c) indicate mean square sound pressure, (b)(d) represent radiated sound power.

5. Conclusion

Theoretical values of the optimal circuit parameters, as derived from the control model in this paper, exhibit remarkable proximity to the experimental values obtained through test modifications. This result serves as evidence that the acoustic-solid-electric multi-field coupling control model presented in this paper can offer valuable guidance for control tests centered around piezoelectric shunt damping technology. Additionally, the utilization of negative capacitance piezoelectric shunt technology proves to be highly effective in suppressing the vibro-acoustic response of coupled plate systems. In fact, this approach demonstrates greater effectiveness compared to traditional active and passive vibration and noise control technologies.

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