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A Novel Formation Control Design of the Artificial Swarm System

Rongrong YU, Si LU, Weiyong ZHU, Mingfei MU, Chenming LI[1]
College of Mechanical and Electronic Engineering, Shandong University of Science and Technology, Xin'an Street, Qingdao, 266590, Shandong, China

Abstract. For the artificial swarm system, this article proposes a novel formation control design. First, an accurate dynamic model is established based on dynamic swarm performances. Further, the collaborative formation can be formed, and safety zones are guaranteed at the same time. Then, the controller merging properties of the swarm system is devised. By designing the nominal control item and the adaptive robust control item, the resulting design can achieve the swarm formation movement and uncertainty compensation perfectly. The deterministic performances are guaranteed, such that uniform boundedness and uniform ultimate boundedness. The superiority of the design proposed is illustrated by the simulations of heterogeneous mobile robots system.

Keywords. Artificial swarm system, Collaborative formation, Dynamic mechanistic model, Safety zones.

1. Introduction

The swarm system, as a focal point, has captured the attention of lots of researchers nowadays [1, 2]. To pursue the great system performance, the innovative control design is urgently needed [3]. First, a reasonable model is the basis for controller design. Many classic modeling methods have been widely applied [4, 5]. From the perspective of mechanistic modeling, a swarm dynamic model is established by considering the swarm properties in this article. Furthermore, the control design of the swarm system have been studied. In [6], a novel guidance control based on trajectory tracking error was introduced for underactuated USVs system. In [7], Sun proposed an adaptive robust control to form a compact formation and realize collision avoidance simultaneously.

Furthermore, the uncertainty, as inevitable factors, has a significant impact on the system control design [8]. Many control methods have been proposed to compensate for it. Base on output feedback, an adaptive control was proposed for MIMO nonlinear systems [9]. In [10], an innovative fuzzy swarm control approach was introduced to effectively solve the chattering issue in OMRs system. In this article, a formation control design is conducted to render the desired system movement and the issue of uncertainty is addressed simultaneously.

Three salient contributions are presented in this paper. First, by considering the swarm dynamic performances, a dynamic model is established to render the collision avoidance and collaborative formation. Specifically, the diffeomorphism operation is

[1] Corresponding Author: Chenming LI, College of Mechanical and Electronic Engineering, Shandong University of Science and Technology, e-mail: chenmingli@sdust.edu.cn

applied to render the collision avoidance, which consistently establishes safety zones. On the other hand, the resulting deterministic swarm performances of the proposed control can effectively achieve the collaborative formation. Second, the nominal control item is designed to render the formation movement. Third, the adaptive robust control item is designed to effectively compensate for uncertainty.

2. Dynamics analysis and control design

2.1. Safety zones: collision avoidance

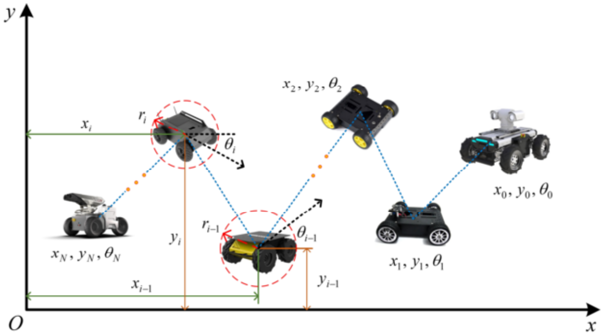


Figure 1. Dynamic swarm model.

As shown in Figure 1, a heterogeneous mobile robots swarm system is studied. Considering the motion equation of the swarm system

$$M_i(\mathrm{X}_i,t)\ddot{\mathrm{X}}+D_i(\mathrm{X}_i,\dot{\mathrm{X}}_i,t)=\tau_i \tag{1}$$

where $t \in \mathbf{R}$ is the time, $M_i \in \mathbf{R}^{3\times3}$ is the inertia matrix, $D_i \in \mathbf{R}^3$ represents the gravitational force, Coriolis force, and the external disturbance. $\tau_i \in \mathbf{R}^3$ represents the control inputs. $\mathrm{X}_i \in \mathbf{R}^3$ represents the state, $\mathrm{X}_i=[x_i,y_i,\theta_i]^T$, where x_i and y_i are the horizontal and vertical positions, respectively, θ_i is the steering angles. The speed is $\dot{\mathrm{X}}_i \in \mathbf{R}^3$, $\dot{\mathrm{X}}_i=[\dot{x}_i,\dot{y}_i,\dot{\theta}_i]^T$. The acceleration is $\ddot{\mathrm{X}}_i \in \mathbf{R}^3$, $\ddot{\mathrm{X}}_i=[\ddot{x}_i,\ddot{y}_i,\ddot{\theta}_i]^T$. In the case that $i=0$, the virtual agent 0 is used to determine the desired trajectory. The actual agents 1 to N are designed to follow the leader.

The safety zones are designed to ensure collision avoidance, the radii can be obtained

$$r_{i-1}=\frac{(f_{i-1}^2+g_{i-1}^2)^{\frac{1}{2}}}{2}, r_i=\frac{(f_i^2+g_i^2)^{\frac{1}{2}}}{2}, \tag{2}$$

where f_{i-1}, f_i and g_{i-1}, g_i represent the lengths and widths of the respective agents.

Let $\xi_{xi}=\ln(\hat{s}_{xi}/\bar{s}_{xi})$, $\xi_{yi}=\ln(\hat{s}_{yi}/\bar{s}_{yi})$, where $\hat{s}_{xi}=\Delta x_i^2-R_i^2$, $\hat{s}_{yi}=\Delta y_i^2-R_i^2$, $\Delta x_i=x_{i-1}-x_i$, $\Delta y_i=y_{i-1}-y_i$, $R_i=r_i+r_{i-1}$. For desired situation, $\bar{s}_{xi}=\Delta\bar{x}_i^2-R_i^2$, $\bar{s}_{yi}=\Delta y_i^2-R_i^2$, $\Delta\bar{x}_i=\bar{x}_{i-1}-\bar{x}_i$, $\Delta\bar{y}=\bar{y}_{i-1}-\bar{y}_i$.

Then, we have $x_i = x_{i-1} - \left(e^{\xi_{xi}}\bar{s}_{xi} + R_i^2\right)^{1/2}$, $y_i = y_{i-1} - \left(e^{\xi_{yi}}\bar{s}_{yi} + R_i^2\right)^{1/2}$.

Let $\chi_i = [\xi_{xi}, \xi_{yi}, \xi_{\theta i}]^T, \xi_{\theta i} = \theta_i$, then, the matrix form can be derived

$$\ddot{\mathrm{X}}_i = \ddot{\mathrm{X}}_{i-1} + \mathrm{H}_i \ddot{\chi}_i + \mathrm{Z}_i \tag{3}$$

Based on (1), the swarm dynamic model can be established

$$M_i \mathrm{H}_i \ddot{\chi}_i + M_i \mathrm{Z}_i + M_i M_{i-1}^{-1}(u_{i-1} - D_{i-1}) + D_i = \tau_i \tag{4}$$

2.2. Constraint following: collaborative formation

The kinematic aggregation performance, as a constraint, is used to design the controller [11]. The resulting collaborative formation can be achieved. Further, define the constraint P_i

$$P_i(\hat{\chi}) = -\sum_{j=1, j\neq i}^{N} \nabla_{\Phi_i} \Theta_{ij}(\chi_i, \chi_j), \tag{5}$$

where $\hat{\chi} = [\chi_1, \chi_2, ..., \chi_N]^T$, $\Theta_{ij}(\cdot)$ is the potential function, $\Theta_{ij}(\cdot)$: $\mathbf{R^n} \times \mathbf{R^n} \to \mathbf{R}$. The dynamic error measure β can be obtained

$$\beta_i = \dot{\chi}_i - P_i(\hat{\chi}), \tag{6}$$

where $\beta_i = [\beta_{xi}, \beta_{yi}, \beta_{\theta i}]^T$. Further, taking the derivative of (6)

$$\dot{\beta}_i = \ddot{\chi}_i - \Phi_i(\hat{\chi}). \tag{7}$$

The constraint force of agent i can be solved [12]

$$\tau_i = M_i \mathrm{H}_i \Phi_i + M_i \mathrm{Z}_i + D_i + M_i M_{i-1}^{-1}(\tau_{i-1} - D_{i-1}). \tag{8}$$

Consider the uncertainty in practice, and the matrices M_i, D_i can be decomposed

$$\mathrm{B} = \overline{\mathrm{B}} + \Delta\mathrm{B} \tag{9}$$

Where $\overline{\mathrm{B}}$ represents nominal parts, $\Delta\mathrm{B}$ represents uncertainty part. $\overline{\mathrm{B}}$ includes $\overline{M}_i(\chi_i, t)$ and $\overline{D}_i(\chi_i, \dot{\chi}_i, t)$, $\Delta\mathrm{B}$ includes $\Delta M_i(\chi_i, \zeta_i, t)$ and $\Delta D_i(\chi_i, \dot{\chi}_i, \zeta_i, t)$, ζ_i represents the uncertainty.

Let $\mathrm{E}_i(\chi_i, \zeta_i, t) = \overline{M}_i(\chi_i, t) M_i^{-1}(\chi_i, \zeta_i, t) - I_i$, $\Delta G_i(\chi_i, \zeta_i, t) = \overline{M}_i^{-1}(\chi_i, t)\mathrm{E}_i(\chi_i, \zeta_i, t)$. Assumptions are proposed as following

(1) Let $W_i(\chi_i, \zeta_i, t) = P_i \mathrm{H}_i^{-1}(\chi_i, t)\overline{M}_i^{-1}(\chi_i, t)\mathrm{E}_i(\chi_i, \zeta_i, t)\overline{D}_i^{-1}(\chi_i, t)\mathrm{H}_i(\chi_i, t)P_i^{-1}$, where the constant matrix $P_i \in \mathbf{R}^{3\times 3}$, $P_i > 0$.

(2) For all (χ_i, t), $\frac{1}{2}\min \lambda_m\left(W_i(\chi_i, \zeta_i, t) + W_i^T(\chi_i, \zeta_i, t)\right) \geq \rho_{Wi}$, where $\rho_{Wi} > -1$ is a constant scalar.

(3) For all $\left(\mathrm{X}_{i-1}, \dot{\mathrm{X}}_{i-1}, \chi_i, \dot{\chi}_i, t\right)$, $\left(1+\rho_{Wi}\right)^{-1}\max[\|\aleph\|] \leq \Pi\left(\alpha_i, \mathrm{X}_{i-1}, \dot{\mathrm{X}}_{i-1}, \chi_i, \dot{\chi}_i, t\right)$, where $\alpha_i \in (0,\infty)^{ki}$ is an unknown constant vector $\aleph = P_i \mathrm{H}^{-1}\left(\Delta G_i\left(\tau_{1i} - \overline{M}_i \mathrm{H} P_i^{-1}\beta_i - D_i\right) - \overline{M}_i^{-1}\Delta D_i - \Delta G_{i-1}\left(u_{i-1} - D_i\right) + \overline{M}_{i-1}^{-1}\Delta D_{i-1}\right)$,

$\Pi_i(\cdot) : (0,\infty)^{ki} \times R^3 \times R^3 \times R^3 \times R^3 \times R$ is a known function.

(4) For all $\left(\mathrm{X}_{i-1}, \dot{\mathrm{X}}_{i-1}, \chi_i, \dot{\chi}_i, t\right)$, the function $\Pi\left(\cdot, \mathrm{X}_{i-1}, \dot{\mathrm{X}}_{i-1}, \chi_i, \dot{\chi}_i, t\right)$ exhibits the following properties: 1) C^1, 2) $\Pi_i(\alpha_{1i},\cdot) - \Pi_i(\alpha_{2i},\cdot) \leq \frac{\partial \Pi_i}{\partial \alpha_i}(\alpha_{2i},\cdot)(\alpha_{1i} - \alpha_{2i})$, 3) as for parameter α_i, it is non-decreasing.

The adaptive law is as follow to emulate the parameter α_i.

$$\dot{\tilde{\alpha}}_i = \frac{\partial \Pi_i^T}{\partial \alpha_i}(\tilde{\alpha},\cdot)\|\beta_i\| - \varpi, \tag{10}$$

where $\varpi = \vartheta_i \dfrac{\dot{\alpha}_i^2}{\dot{\alpha}_i + \delta_i}$ is the leakage part. $\vartheta_i, \delta_i \in \mathbf{R}^+$.

Based on (8) and (11), a novel adaptive robust control input can be proposed

$$\tau_i = \underbrace{\overline{M}_i \mathrm{H}_i \Phi_i + \overline{D}_i + \overline{M}_i \mathrm{Z}_i + \overline{M}_i \overline{M}_{i-1}^{-1}\left(\tau_{i-1} - \overline{D}_{i-1}\right)}_{\tau_{1i}} + \underbrace{\overline{M}_i \mathrm{H}_i P_i^{-1}\beta_i\left(1+\Pi_i^2\right)}_{\tau_{2i}}. \tag{11}$$

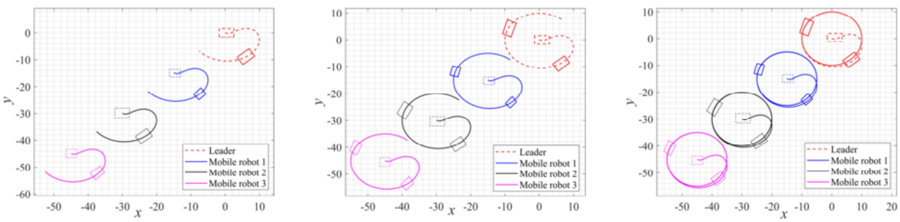
Theorem 2 Let $\Delta\alpha_i = \tilde{\alpha}_i - \alpha_i$, $\tilde{\delta}_i = \left[\beta_i^T (\Delta\alpha_i)^T\right]^T$ and $\tilde{\delta} = \left[\tilde{\delta}_1^T, \tilde{\delta}_2^T, \ldots, \tilde{\delta}_N^T\right]^T$. For the artificial swarm system with uncertainty (4), the asymptotic stability can be achieved by utilizing the control input (12) [13].

3. Illustrative example and results

Based on the ode15i integrator in the MATLAB environment, the simulations application of a heterogeneous mobile robots artificial swarm system with four agents is studied. The controller in simulation is designed as: $\tau_i = \tau_{1i} + \tau_{2i}$, corresponding to the control input (12). Specifically, the simulation parameters are set as: $f_0 = 3$, $g_0 = 4$, $f_1 = 3$, $g_1 = 3.5$, $f_2 = 3.5$, $g_2 = 4$, $f_3 = 3$, $g_3 = 3$, $m_0 = 1000$, $m_1 = 950$, $m_2 = 900$, $m_3 = 850$, $d_{xi} = 10$, $d_{yi} = 10$, $\mathrm{c}_{xi} = \mathrm{c}_{yi} = c_{\theta i} = 0.1$. Based on the equation (1), the related items $D_x = \left[D_{xi}, D_{yi}, D_{\theta i}\right]^T$, $D_{xi} = c_{xi}\|x_i\|\dot{x}_i$, $D_{yi} = c_{yi}$, $\|y_i\|\dot{y}_i$,

$D_{\theta i} = c_{\theta i} \left\| \theta_i \right\| \dot{\theta}_i$ and c_{xi}, c_{yi}, $c_{\theta i}$ are the resistance coefficients in generalized coordinates, respectively. Consider the uncertainty during the actual moving, the mass of agents and resistance coefficients can be decomposed: $m_i = \bar{m}_i + \Delta m_i(t)$, $c_{xi} = \bar{c}_{xi} + \Delta c_{xi}(t)$, $c_{yi} = \bar{c}_{yi} + \Delta c_{yi}(t)$, $c_{\theta i} = \bar{c}_{\theta i} + \Delta c_{\theta i}$, $\Delta m_i = 0.2\bar{m}_i \sin t$, $\Delta c_{xi} = 0.2\bar{c}_{xi} \sin t$, $\Delta c_{yi} = 0.2\bar{c}_{yi} \sin t$, $\Delta c_{\theta i} = 0.2\bar{c}_{\theta i} \sin t$. Initial conditions are: $x_0 = 0$, $\dot{x}_0 = 10$, $x_1 = -15$, $\dot{x}_1 = 10.5$, $x_2 = -30$, $\dot{x}_2 = 11$, $x_3 = -45$, $\dot{x}_3 = 11.5$, $y_0 = 0$, $\dot{y}_0 = -10$, $y_1 = -15$, $\dot{y}_1 = -10.5$, $y_2 = -30$, $\dot{y}_2 = -11$, $y_3 = -45$, $\dot{y}_3 = -11.5$, $\theta_0 = 0.1$, $\theta_1 = 0.2$, $\theta_2 = 0.3$, $\theta_3 = 0.4$, $\dot{\theta}_0 = \dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = 1$.

Figure. 2 displays the trajectories of heterogeneous mobile robots swarm system. Obviously, the leader can achieve desired constraint following despite incompatible initial conditions, and the collaborative formation between the leader and followers is commendably formed. At the same time, collision avoidance performance is ensured, with mobile robots in the swarm system always maintaining a safety distance.



(a) $t = 4s$ (b) $t = 7s$ (c) $t = 60s$

Figure 2. The trajectories of swarm system.

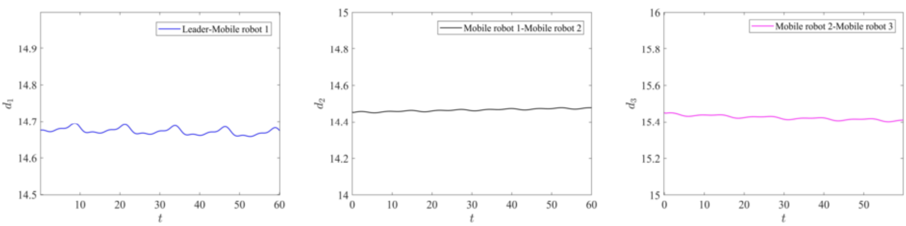
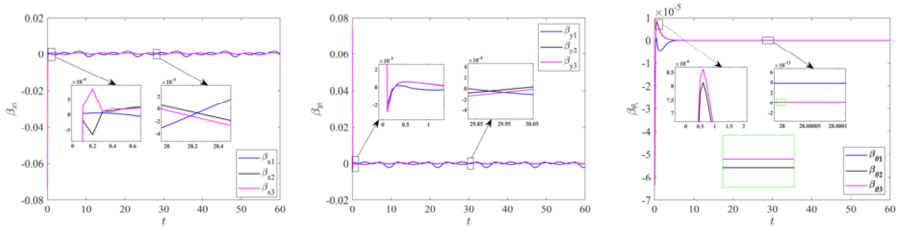


Figure 3. The spatial distances between adjacent mobile robots.

Figure. 3 shows the spatial distances between adjacent heterogeneous mobile robots. Maintaining a steady state, the safety zones are effectively established. In addition, the value of spatial distance meets the requirements between the set size of the agents and the safety distance. A stable safety zones are effectively ensured.

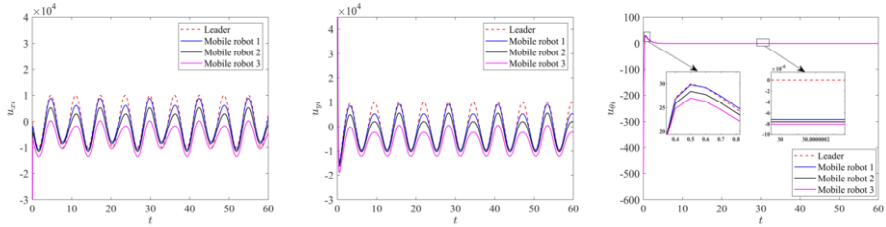
Figure. 4 represents the constraint following errors, which measures the deviation between the actual state and desired state. It can be seen that the errors decrease to zero in just 0.1 seconds, indicating that the control design in this paper exhibits superior overall performance.



(a) Horizontal error (b) Vertical error (c) Steering error

Figure 4. The constraint following errors.

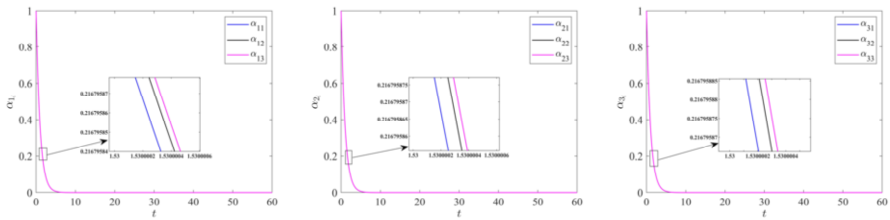
The simulation result of the control forces are shown in Figure. 5. Obviously, the control forces exhibit regular variations, reflecting the stability of the control method.



(a) Horizontal constraint force (b) Vertical constraint force (c) Steering constraint force

Figure 5. The constraint forces.

Figure. 6 illustrates the adaptive parameter variation curve related to adaptive constraint forces. It monotonically decreases to zero within 5 seconds, indicating that the designed controller can address uncertainty issues timely and efficiently.



(a) Adaptive parameter α_{1i} (b) Adaptive parameter α_{2i} (c) Adaptive parameter α_{3i}

Figure 6. The adaptive parameters.

4. Conclusion

A new control design is proposed to achieve the stable collaborative formation of the artificial swarm system. Three significant contributions are presented. First, an accurate dynamic model with stronger adaptability is established. The swarm performances including collision avoidance and collaborative formation, are taken into consideration simultaneously. Thus, the safety zones are guaranteed through the diffeomorphism transformation and the size of the safety zones can be adjusted with the relationship between the set safety distance and the size of the agents. On the other hand, the collaborative formation can be formed by rendering the desired dynamic swarm performance. Second, the nominal control item τ_{1i} is devised to achieve the collaborative formation. Third, the adaptive robust control item τ_{2i} is devised to solve the issue of uncertainty and the constraint forces can be adjusted in real-time. Furthermore, the superior performance of the proposed control design has been well demonstrated by application of a heterogeneous mobile robots swarm system.

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