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Sensitivity Analysis of Colding's Equation in Side Milling of Medium Carbon Steel C45 E

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Abstract. This work focuses on reducing the experimental need for creating a reliable tool life model for a data set of 46 tool data points with its resulting tool life for a single-tooth side milling application in medium carbon steel, C45 E.

Based on the data set, 615 180 unique tool life models are created using Colding's equation.

This is achieved by creating models using different unique subsets of the complete data set where the cardinality is varied from 7 to 43.

The paper shows that the improvement from adding more data points to the modelling are neglectable after 34 data points are included in the modelling if a maximum absolute model error $\leq 9\%$ is sought.

Furthermore, it is shown that the prediction error increases when extrapolating outside the range of equivalent chip thickness and cutting speed used for the modelling work compared to an interpolative error within the range.

By carefully planning the experimental set-up by maximising the cutting speed and feed range decreases the risk of creating a non-relevant model where the prediction error increases when located outside the modelling range.

Keywords. Machining, Colding's equation, Tool life, Milling

1. Introduction

Machining operations such as turning, milling, drilling, boring, reaming and tapping are commonly used when producing shapes and surfaces on products. Approximately 80% of all products are machined in one way or another before it's finalized. [1]

When forming the workpiece, factors such as: process parameters, workpiece material, tool geometry and tool material all influences the tool life. Understanding, describing and improving the machining process has been a subject of research for over a century [2].

A common method to increase the understanding is by using models [3]. Over the years, different approaches have been suggested: empirical-, numerical-, soft computing- and hybrid models. [2,4,5,6,7,8]

The ability to predict the tool life is an important aspect to optimize the tool cutting process.

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Even though advances have been made in several areas, empirical models, such as the Taylor formula [2] and Colding's equation [9,10] are still relevant in both academia and industry. Some advantages identified with empirical models are:

- Low amount of test data needed.
- Low calculation time.
- The wear degradation does not need to be fully evaluated to create a model.

[11,12]
Although, one limitation of empirical models is that the model is only valid for the machine - tool - workpiece combination used for the data acquisition. [13]

The Colding model proposed by Dr. B. Colding [9,10] has shown as a viable candidate when creating empirical models both in turning Johansson et. al. [14] and milling Kantojärvi et. al. [15]. To minimize the need for experimental data Johansson [12] proposed an experimental set-up to increase the probability of receiving a relevant model based on five experimental data points. It has also been shown that it is possible to calibrate a known tool life model with a closely related machine - tool - workpiece set-up using a reduced experimental trial [16].

Kantojärvi et. al. [15] published a data set of tool life for a milling application which includes a total of more than 50 hours of machining time. With an extensive testing the cost of producing the model becomes several times higher than for a turning application.

The aim of this paper is to investigate the possibilities to reduce the experimental set-up when using Colding's equation by analysing the data set published by Kantojärvi et. al. [15].

2. Background

2.1. Tool life modeling

It is showed that Colding's equation can be utilized for tool life modeling in milling applications [15]. The Coldings equation for predicting tool life is shown in Eq. (1). Later, B. Linström [17], showed that Colding's equation in fact is an extended Taylor formula, shown in Eq. (2).

$$v_c = e^{K - \frac{(\ln(h_e) - H)^2}{4M} - (N0 - L\ln(h_e))\ln T} \tag{1}$$

$$v_c \cdot T^{\alpha(h_e)} h_e^{m(h_e)} = C_0 \tag{2}$$

Where: $\alpha(h_e) = \alpha_0 + \alpha_1 \ln(h_e)$ and $m(h_e) = m_0 + m_1 \ln(h_e)$.

The relationship between Lindströms set of constants and Coldings are:

$$C_0 = e^{K-(H^2/4M)}, m_0 = -2H/4M$$

$$m_1 = 1/4M, \alpha_0 = N_0, \alpha_1 = -L$$

Where v_c is the cutting speed $[m/min]$, h_e the equivalent chip thickness $[mm]$ and T is the tool life $[min]$. $K, H, M, N0$ and L are model constants that are fitted against the experimental data set. The equivalent chip thickness for milling is calculated as shown in Eq. (3) [18].

$$h_{e,S} = \frac{a_p f_{ze}}{\frac{a_p - r_\varepsilon(1-\cos\kappa)}{\sin\kappa} + \kappa r_\varepsilon + \frac{f_{ze}}{2}} \tag{3}$$

Where: a_p is the depth of cut $[mm]$, f_{ze} the equivalent feed $[mm]$, r_ε the nose radius $[mm]$ and κ the setting angle $[rad]$. a_p, r_ε and κ is known based on selected process parameters and cutting tool. The equivalent feed, f_{ze}, is related to the geometetry of the tool and can be approximated using Eq. (4) [18].

$$f_{ze} \approx \frac{a_e * f_z}{l_{arc}} \tag{4}$$

Where: l_{arc} is the arch length $[mm]$ and can be determined geometrically usisng Eq. (5) and (6), a_e is width of engagement $[mm]$ and f_z is the feed per tooth $[mm]$.

$$l_{arc} = \frac{\varnothing_w}{2}\varphi_{ae} \tag{5}$$

$$\varnothing_w = \varnothing + 2\frac{\frac{2}{3}a_p}{\tan\kappa} \tag{6}$$

The tool life for a milling application is defined as the cutting edge's time engaged in the workpiece and can be calculated according to Eq. (7).

$$T = T_{tool} * T_{ratio}, T_{ratio} = \frac{l_{arc}}{\pi\varnothing_w} \tag{7}$$

Where: T_{tool} is the machining time $[min]$ from engagement to exit of workpiece.

To solve the model constants numerically, it were proposed by Kantojärvi et. al. [15], to solve for the constants identified by Lindström [17] on matrix form, as shown in Eq. (8), for a dataset of n experiments.

$$\underbrace{\begin{pmatrix} \ln(T_1) & \ln(h_{e,1}) & \ln(T_1)\ln(h_{e,1}) & \ln(h_{e,1})^2 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \ln(T_n) & \ln(h_{e,n}) & \ln(T_n)\ln(h_{e,n}) & \ln(h_{e,n})^2 & -1 \end{pmatrix}}_{\bar{A}} \underbrace{\begin{pmatrix} \alpha_0 \\ \alpha_1 \\ m_0 \\ m_1 \\ \ln(C_0) \end{pmatrix}}_{\bar{x}} = \underbrace{\begin{pmatrix} -\ln(v_{c,1}) \\ \vdots \\ -\ln(v_{c,n}) \end{pmatrix}}_{\bar{b}} \tag{8}$$

$$\bar{A}\bar{x} = \bar{b}$$

In order to determine the model constants, minimizing the least square error are performed on matrix form, Eq. (9).

$$\min ||\bar{A}\bar{x} - \bar{b}||_2 \tag{9}$$

$$\bar{x} = (\bar{A}^T\bar{A})^{-1}\bar{A}^T\bar{b}$$

2.2. Milling experiment

Kantojärvi et. al. [15] published a data set consisting of 46 experimental trials for a single tooth milling application. The milling was performed as a down milling operation, varying width of cut, cutting speed and feed in C45 E medium carbon steel. The data set consist of a combination of tool - process parameters for five different milling cutter configurations, shown in Table 1. [15]

In Figure 1 the equivalent chip thickness and cutting speed combination included in the data set by Kantojärvi et. al. [15] is shown for the tools presented in Table 1.

Table 1. Cutter configurations used for retrieving the tool life data.[15]

Cutter	Ø [mm]	r_ε [mm]	κ [°]	Rake angle [°]	Clearance angle [°]
A	157.6	1.0	46	11	12
B	312.6	1.0	46	11	12
C	176.4	1.0	89	11	12
D	330.6	1.0	89	11	12
E	32.0	0.8	90	10	7

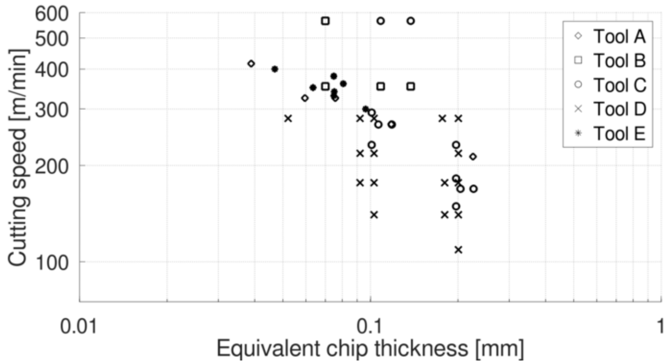
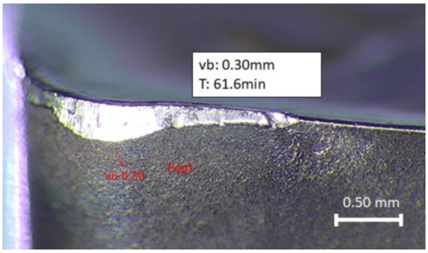


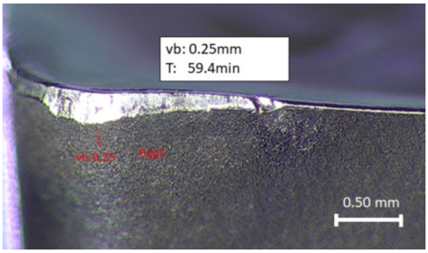
Figure 1. All experimental data points included in the experiment by Kantojärvi et.al. [15].

All tool life results published by Kantojärvi et. al. [15] were determined by measuring the maximum wear on the flank with the criterion $max\{V_b, V_n\} \leq 0.3mm$. Where: V_b is the flank wear and V_n is the notch wear. In Figure 2, the wear evaluation published by Kantojärvi et. al. [15] is shown.

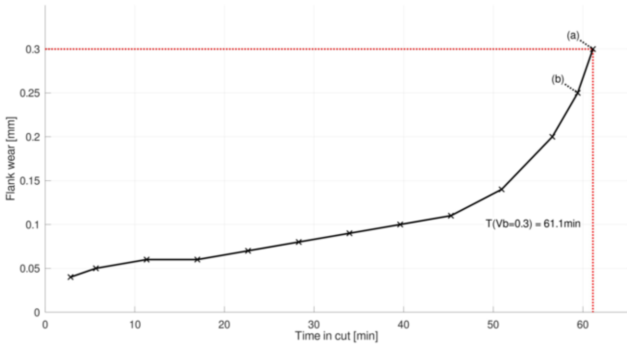
For the milling experiments published by Kantojärvi et. al [15] the Colding constants were determined to: $K = 6.192$, $H = -2.921$, $M = 1.404$, $N0 = 0.367$ and $L = -0.067$ with a resulting average mean model error of 7.96% for the experiment. [15]



(a) Measured flank wear after an engaged time of 61.1 minutes.



(b) Measured flank wear after an engaged time of 59.4 minutes.



(c) Example of flank wear development.

Figure 2. Determination of tool life for a cutting edge. (a) flank wear when tool life criterion is met and in (b) flank wear at latest point before tool life criterion is met. (c) Flank wear development with tool life shown for T(vb=0.3mm).[15]

3. Method

The modelling work, using Colding's equation Eq. (1), are performed on a subset $B_m \subseteq A$ containing m randomly selected tool performance points, satisfying: $|A| \geq |B_m| = m$. Where: $A = \{1, 2, 3 \dots n\}$ is all tool performance points in the data set published by Kantojärvi et. al. [15].

Regardless of which subset $|B_m|$ that are used, the model error is deteremined as the absolute model error, AME, over the whole data set A, shown in Eq. (10). This creates a statistical data set of AME for subsets of different cardinality to be used in further analysis.

Even though the Colding constants is not directly solved towards Eq. (10), Kantojärvi et. al. [15] showed that the solved constants often gives an improved solution compared to using an non-linear optimizer directly on Eq. (10). Something that is likely due to the removed risk of ending up in local optimums. [15]

$$AME = \frac{1}{|A|} \sum_{A} \frac{|v_{c,model,n} - v_{c,n}|}{v_{c,n}} \tag{10}$$

Analysing characteristics of the subset can be determined by calculating the ratio between maximum and minimum value of a certain property, as shown in Eq. (11).

$$x_R = \frac{max\{x(B_m)\}}{min\{x(B_m)\}} \tag{11}$$

Johansson [12] proposed to analyse the influence of error based on three identified types of data points when evaluating a turning application:

- **Approximative data point:** A data point which is a part of the model creation.
- **Interpolative data point:** A data point which are positioned within the $h_e - v_c$ range that the approximative data points span over.
- **Extrapolative data point:** A data point which are positioned outside the $h_e - v_c$ range that the approximative data span over.

The data set A can be divided into three subsets as shown in Eq. (12), a graphical representation is shown in Figure 3.

$$A = B_m \cup B_i \cup B_e \tag{12}$$

Where: $B_m = \{n \in A | n \text{ used in model creation}\}$

$B_i = \{n \in A | min\{v_c(B_m)\} \leq v_c(n) \leq max\{v_c(B_m)\} \text{ and } min\{h_e(B_m)\} \leq h_e(n) \leq max\{h_e(B_m)\}\}$

$B_e = A \setminus (B_i \cup B_m)$

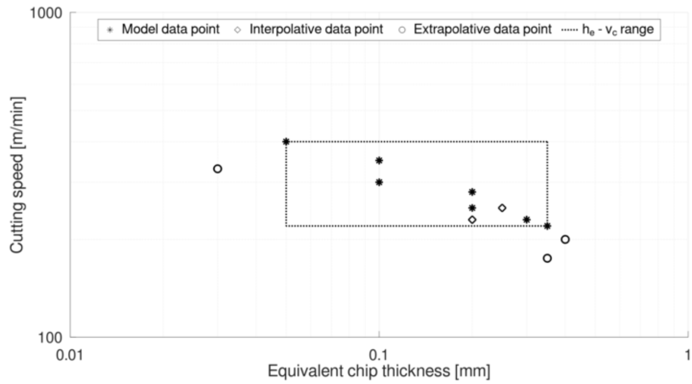


Figure 3. Definition of model, interpolative and extrapolative data points. Where: $(*)$: represents data points used for the modeling, $(\diamond)$: represents interpolative data points and $(\circ)$: represents extrapolative data points.

4. Results and discussion

Colding constants are solved, using Eq. (9), for multiple randomly selected data sets, containing $m = \{7,10,13,16,19,22,25,28,31,34,37,40,43\}$ data points. The distribution of AME is calculated according to Eq. (10) for each created model and is shown in Figure 4. In Table 2 the total number of models created for each size of subspace are presented. It can be observed that the number of models created for the subset with 43 out of 46 data points all unique subsets are solved.

Table 2. Number of models created for each subset $B_m \subseteq A$, $m = \{7,10,13,16,19,22,25,28,31,34,37,40,43\}$.

m	7	10	13	16	19	22	25
# of models	50 000	50 000	50 000	50 000	50 000	50 000	50 000
m	28	31	34	37	40	43	
# of models	50 000	50 000	50 000	50 000	50 000	15 180	

As can be expected, increasing the number of data points included in the modelling decreases spread in AME between models. The major improvement of spread between models can be observed by moving from subsets of 7 up to subsets of approximately 22 data points.

Looking at the ratio between models that has an AME exceeding a certain threshold, $AME \geq \{9,12,15\}\,\%$, and all models is shown in Figure 5. As can be observed in Figure 5, depending on the quality requirements set on the model, the point where the improvement becomes negligible with a subset of $\{37,19,16\}$ data points for $AME \geq \{9,12,15\}\,\%$.

Furthermore, it can be observed that all subsets, regardless of cardinality, there are models that perform equally good with respect to AME, shown in Figure 4. It is mainly the spread in-between the models that are affected by the cardinality, where increasing the number of data points included decreases the spread.

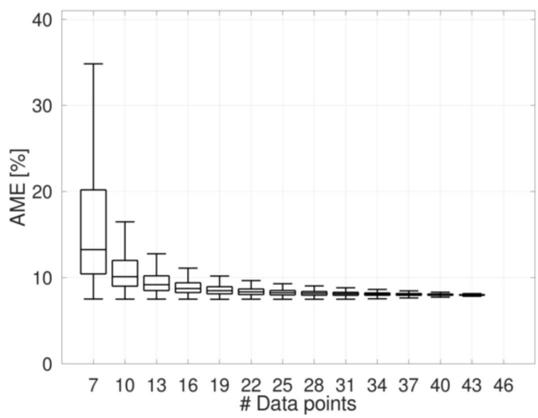


Figure 4. Model distribution with $\top$ and $\perp$ showing the 1.5x interquartile range (IQR). Outliers, models outside 1.5 IQR are not shown.

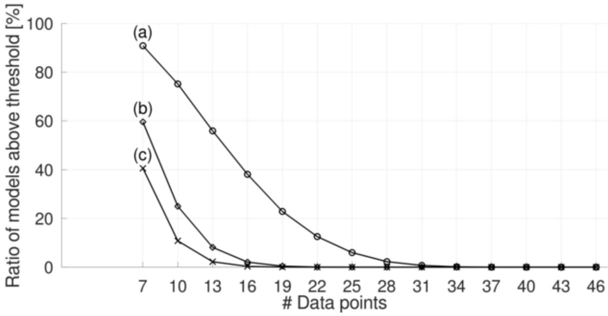


Figure 5. The ratio of models that shows an absolute mean error larger than the threshold. (a) $AME \geq 9\%$, (b) $AME \geq 12\%$ and (c) $AME \geq 15\%$

By dividing the model error into three different types of error: interpolative, extrapolative and approximative error, as proposed by [16], the error contribution from the different types is shown in Figure 6. It is no surprise that the AME of model data points are quite low and are increasing as the number of data points are included in the model. Both the extrapolative and interpolative errors have a larger AME compared to the approximative errors. It is also quite clear that extrapolative data points will generate a larger AME than the data points positioned inside the $h_e - v_c$ span, especially at the lower count of data points included in the modeling.

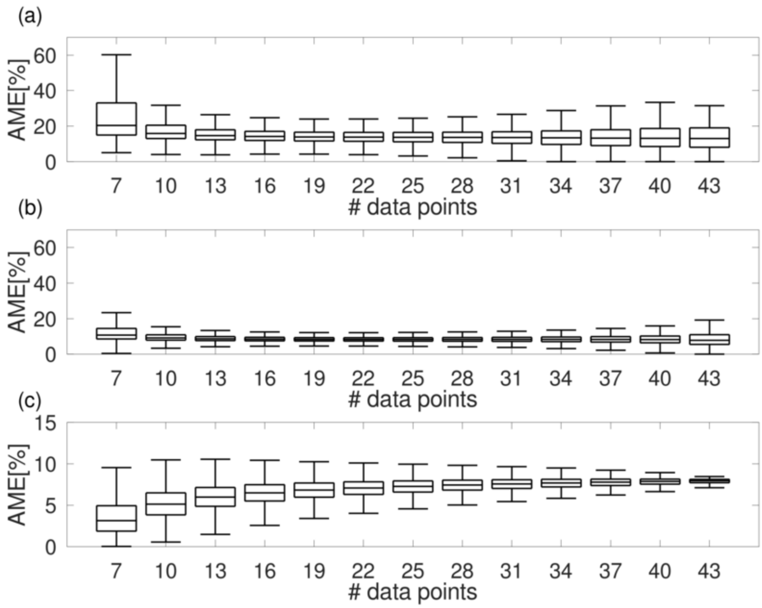


Figure 6. Model distribution with $\top$ and $\perp$ showing the 1.5x interquartile range (IQR). Outliers, outside 1.5 IQR are not shown. Where (a) AME on extrapolative data points, (b) AME on interpolative data points and (c) AME on approximative data points.

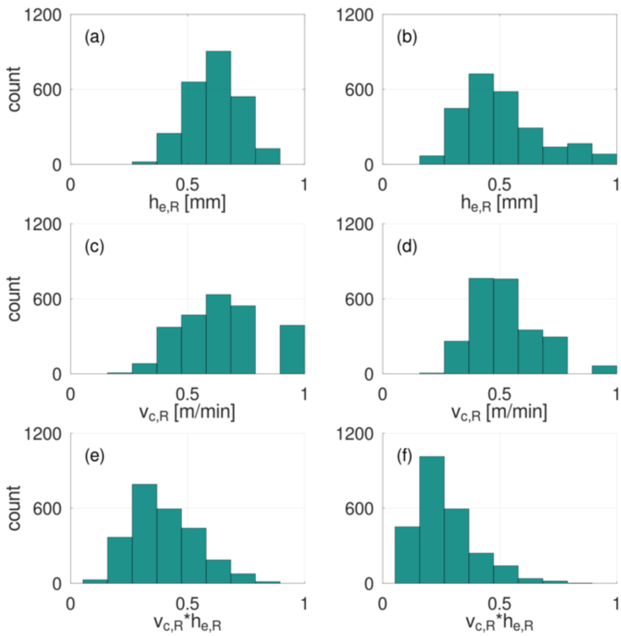


Figure 7. Histogram for the 2500 models with lowest $\{(a),(c)\&(e)\}$ and highest $\{(b),(d)\&(f)\}$ AME created using 7 data points. $(a-b)$ Count of models with an ratio between largest and smallest h_e in model. $(c-d)$ Count of models with an ratio between largest and smallest v_c in model and $(e-f)$ Count of models with an ratio between largest and smallest $h_{e,R} * v_{c,r}$ in model.

There seems to be a clear connection between the model error and the spread in approximative error. In order to quantify the $h_e - v_c$ range of certain models, the ratio between maximum and minimum value for both h_e, v_c and the product of both is calculated, using Eq. (11). This calculation is done for 2500 models with lowest and highest AME and are shown in Figure 7 normalized with the maximum ratio of the complete data set of each attribute, presented in Table 3.

The models with a low AME, has a majority of the model counts in the upper end of $h_{e,R}$ and $v_{c,R}$. For the models with a high AME there is still generally quite high differences on the h_e but a low difference in v_c. It is worth noting that the product of both $h_{e,R}$ and $v_{c,R}$ is generally low for the 2500 models with largest AME. This should be compared to the models with low AME where the count is slightly shifted towards higher $h_{e,r} * v_{c,r}$, as can be seen in Figure 7-e and f.

Table 3. Ratio between the maximum and minimum propertiy ion data set.

Property	Maximum ratio
h_e	5.8
v_c	5.2
$h_e * v_c$	30.2

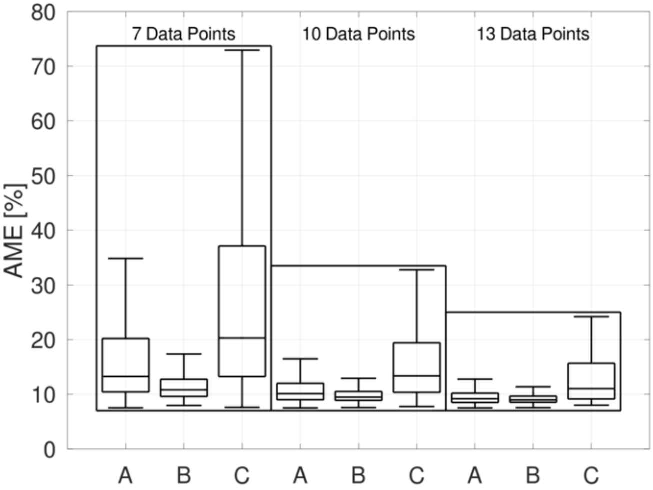


Figure 8. Box plots describing the *AME* distribution of models where: A - Includes all created models, B - includes all models which satisfy $v_{c,R} * h_{e,R} \geq 0.80 * max_A \{v_{c,R}\} * max_A \{h_{e,R}\}$ and C - includes all models which satisfy $v_{c,R} * h_{e,R} \leq 0.20 * max_A \{v_{c,R}\} * max_A \{h_{e,R}\}$.

In Figure 8, the *AME* distribution for all models is compared to models with a subset, C_m, containing all models which satisfy $h_{e,R} * v_{c,R} \geq 0.8 * max_A \{h_{e,R} * v_{c,R}\}$ and D_m which satisfy $h_{e,R} * v_{c,R} \leq 0.2 * max_A \{h_{e,R} * v_{c,R}\}$.

The spread for $m = \{7, 10\}$ in *AME* is lowered for the subset C_m compared to all models as well as models included in D_m. Where models in subset D_m have a greater spread than all models created.

Increasing the number of data points included in the model, $m = 13$, the difference between all models and C_m is neglectable. There are still a discrepancy between D_m and all created models. Further increasing the number of data points included in the model do not show an improvement with C_m. This is related to the fact that when increasing the number of data points included in the modelling, it will increase the minimum $h_{e,R} * v_{c,R}$ decreasing the influence from extrapolative errors, as shown in Table 4.

The number of models that fulfil the requirement of C_m increases as the number of data points included in the model increases. A opposite trend can be seen observed with the D_m subset, where an increasing cardinality decreases the probability of having models that fulfils the requirement.

Table 4. Median, quartile 1 and 3 and number of models with $m = \{7, 10, 13, 16, 19, 22\}$ for the data sets: All models, C_m which includes all models which satisfy $v_{c,R} * h_{e,R} \geq 0.80 * max_A \{v_{c,R}\} * max_A \{h_{e,R}\}$ and D_m which includes all models which satisfy $v_{c,R} * h_{e,R} \leq 0.20 * max_A \{v_{c,R}\} * max_A \{h_{e,R}\}$.

m	Attribute	All	C_m	D_m	m	Attribute	All	C_m	D_m
7	Median	13.3	10.8	20.3	10	Median	10.1	9.5	13.4
	Q1	10.4	9.6	13.2		Q1	9.0	8.9	10.4
	Q3	20.2	12.7	37.1		Q3	12.0	10.5	19.5
	# Models	50 000	352	5 669		# Models	50 000	1 271	833
13	Median	9.2	9.0	11.0	16	Median	8.7	8.7	11.5
	Q1	8.5	8.5	9.1		Q1	8.3	8.4	10.2
	Q3	10.2	9.7	15.7		Q3	9.4	9.1	14.9
	# Models	50 000	3 042	84		# Models	50 000	5 593	9
19	Median	8.5	8.5	-	22	Median	8.3	8.4	-
	Q1	8.1	8.2	-		Q1	8.0	8.1	-
	Q3	8.9	8.9	-		Q3	8.7	8.6	-
	# Models	50 000	9 070	-		# Models	50 000	13 167	-

5. Conclusion

The study aimed to increase the understanding of influencing factor to the experimental set-up when creating Colding tool life models for milling. It can be concluded that:

- The sensitivity of the modeling increases as the number of data points included in the modeling decreases.
- Data points outside the $h_e - v_c$-range generally is predicted with a larger error than data points inside the model's $h_e - v_c$-range.
- With a requirement of $AME \leq 9\%$ the model improvement is neglectable after 34 data points are included in the modelling.
- By increasing the ratio between maximum and minimum $h_{e,R} * v_{c,R}$ improves the probability of having a model with a low AME.

5.1. Future work

In milling, the time the tool is engaged in the workpiece is related to the width of engagement. Knowing if for instance cutter diameter and width of engagement influence the tool life could affect how the experimental set-up looks like. One would like to maximize the width of engagement to minimize time for testing.

All experiments that this investigation are based on single tooth machining in side-milling using a down-milling procedure, how applicable is the model for predicting tool lives using fully equipped milling tools, or using different milling method, such as up-milling.

Finally, investigations on the applicability on different workpiece materials should be performed to evaluate the applicability of empirical Colding models for milling applications.

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