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Distributed Network Improvement for Efficient Data Aggregation in Consensus-Based Sensor Networks

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Abstract. In the sensor networks considered in this paper, the topology of agents aggregating neighbors' data can be locally adjusted by each entity. In the sense of Fiedler value of network, this proper topological adjustment is easy to deploy and greatly benefits the increase of convergence rate in the tasks such as target localization, parameter estimation and data fusion. To this end, a distributed topology-based sensor consensus acceleration algorithm is proposed. By distributed estimating the eigenspaces of Laplacian matrix, this algorithm provides a direction for each sensor adjusting their aggregation weights, such that the Fiedler value is optimized towards a local maxima. Moreover, this method can also tackle the repeated eigenvalues and non-unique Fielder values. A numerical example is also provided to demonstrate the effectiveness of the proposed distributed strategy.

Keywords. Network consensus; Ad-hoc network; Distributed estimation.

1. Introduction

Advances in ad-hoc sensor networks have offered flexibility and adaptability, catering to the demands of complex real-world scenarios even in the absence of a central infrastructure. These networks consist of a collection of self-organizing sensors, each endowed with autonomous communication capabilities, enabling ad-hoc networks to collectively engage in intricate, real-time environmental monitoring. To this end, distributed information aggregation and cooperative algorithms come into play, serving diverse objectives such as consensus seeking, target localization, parameter estimation and data fusion within ad-hoc networks.

The efficiency of the entire system hinges significantly on the rate at which data is disseminated and aggregated throughout the network. Various factors influence this convergence rate, including chip processing speed, algorithmic efficiency, network scale,

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and more. However, the network topology, while easily adjustable, exerts substantial influence on the convergence process but has received relatively less investigation yet.

Throughout this paper, the network convergence rate is quantified using the well-known Fiedler value [1] or generalized algebraic connectivity [2] of a digraph. However, before acceleration of convergence, estimation the connectivity of entire network in a distributed manner already faces difficulty. Yang et al. [3] have proposed a decentralized estimation procedure that allows each agent to track the algebraic connectivity of a time-varying undirected graph. In the case of directed graphs, Asadi et al. [4] solves the generalized algebraic connectivity estimation problem by a modified distributed power iteration subspace consensus algorithm, for weighted digraphs.

Several techniques have been proposed in the literature to optimize the topology of networks to improve algebraic connectivity, but most of them are executed based on global network information. Xiao et al. [5] propose a prestigious semidefinite program framework for optimizing the edge weights of linear distributed averaging consensus problem. Shafi et al. [6] present a convex optimization method to adjust node and edge weights to impose individual constraints on several eigenvalues. Kim et al. [7] propose an iterative greedy-type algorithm for maximizing the Fiedler value of a state-dependent graph Laplacian with a guaranteed local convergence behavior.

The contribution of this work is the proposal of a distributed topology-based sensor consensus acceleration algorithm. This algorithm dynamically updates the weights of a sensor aggregating data among its neighbors based on local information, thus progressively accelerating the convergence of the entire network. The key idea is to elucidate the relationship between the Fiedler value and edge weights based on variational theory and then design distributed algorithms for estimating eigenspaces and optimizing the topology. The feasibility and efficiency of these methods have been verified through simulations.

2. Preliminaries

Throughout this paper, vectors are by default considered as column vectors. The N-by N real space and complex space are respectively denoted as $\mathbb{R}^{N\times N}$ and $\mathbb{C}^{N\times N}$. Let I_n represent the identity matrix with subscript indicating its dimension. For a vector x, denote $[x]_i$ as the i^{th} element of it. Define the function $\Re(\cdot)$ that maps a complex number to its real part, and the function $\operatorname{diag}(\cdot)$ that maps a square matrix to a vector whose i^{th} element is the i^{th} diagonal entry of it. The Kronecker product and Hadamard product are denoted as $\otimes$ and $\odot$, respectively.

2.1. Network Model

In a sensor network of N agents, each agent can only transfer information with a certain subset of the entire agents set $\mathcal{V}=\{1,2,\cdots,N\}$, due to the limitations of wireless sensing range, communication routes or protocols. To describe such information transferring topology, a communication network is defined as a boolean directed graph $\mathcal{G}_c=\{\mathcal{V},\mathcal{E},C\}$ with $C\triangleq[c_{ij}]_{N\times N}, c_{ij}\in\{0,1\}$ and $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$. An edge $(j,i)\in\mathcal{E}$ means that node j can send information to node i with $c_{ij} = 1$ and the neighbors of agent i is denoted as $\mathcal{N}_i \triangleq \{ j \mid c_{ij} = 1 \}$ containing all the agents that can transfer information to it.

In this work, each agent is allowed to customize the weights that it aggregates neighbors' information, in order to optimize the convergence rate of entire sensor network. Specifically, denote the aggregation network as a weighted directed graph $\mathcal{G}_a = \{ \mathcal{V}, \mathcal{E}, W \}$ and each agent maintains one row of $W = [W_{1*}, W_{2*}, \cdots, W_{N*}]^\top$, namely, $W_{i*} = [w_{11}, w_{12}, \cdots, w_{1,N}]^\top$ contains the control gains of agent i aggregating the information from neighbors. Then the system control input can be written as

$$u_i = \sum_{j=1}^{N} \frac{c_{ij} w_{ij}}{\sum_{k=1}^{N} c_{ik} w_{ik}} (x_j - x_i) \triangleq \sum_{j=1}^{N} a_{ij} (x_j - x_i), \tag{1}$$

where $u_i, x_i \in \mathbb{R}^m$ are respectively m-dimensional control input and state of agent i. Let $A \triangleq [a_{ij}]_{N \times N}$ be the adjacency matrix of the aggregation network and $x = [x_1^\top, x_2^\top, \cdots, x_N^\top]^\top$ be the concatenate system state then the compact form of system dynamics is

$$\dot{x} = [(-I_N + A) \otimes I_m] x \triangleq -(L \otimes I_m) x . \tag{2}$$

This system are typical consensus-based sensor networks and L is the Laplacian matrix. To ensure the information diffusion among this sensor network, a basic assumption on topology connectivity has to be made.

Assumption 1, the aggregation matrix A is irreducible and row stochastic. That is, the communication network is strongly connected, and each agent has to assign the weights that satisfy $\forall i, \sum_{j=1}^{N} c_{ij} w_{ij} = \sum_{j=1}^{N} a_{ij} = 1$.

3. Main Technical Results

To evaluate the speed of system (2) converging, the mostly used index is the second smallest real part of the eigenvalues of Laplacian matrix L. Define the function $\Lambda : \mathbb{R}^{N \times N} \mapsto \mathbb{C}^N$ that yields the spectrum of a square matrix and $\Lambda_i : \mathbb{R}^{N \times N} \mapsto \mathbb{C}$ that maps a square matrix to one of its eigenvalues that have the i^{th} smallest real parts. Moreover, define the function $\Lambda_i^+ : \mathbb{R}^{N \times N} \mapsto \mathbb{R}$ with $\Lambda_i^+(L) = \Re(\Lambda_i(L))$. In this way, the so-called Fiedler eigenvalue γ in directed graphs can be represented by

$$\gamma = \Lambda_2^+ (L). \tag{3}$$

By default, the eigenvalues of L are always in the ascending order of their real parts as $\lambda_1, \lambda_2, \cdots, \lambda_N$ and we call the eigenvalues that belong to $\{ \lambda_i \mid \lambda_i = \Lambda_2^+ (L) \}$ as active eigenvalues corresponding to function $\Lambda_2^+ (\cdot)$.

The basic idea of this work is to optimize the performance metric γ of the sensor network by allowing each agent to distributively finetune the aggregation weights among its neighbors.

Specifically, in a distributed manner, the i th agent only have access to the i th row C_{i*} of communication topology $C=\left[C_{1*},C_{2*},\cdots,C_{N*}\right]^{\top}$. After receive the neighbors' information, agent i designs the weight vector $W_{i*}\geq 0$ to generate the control input $u_i=\left[\left(e_i-C_{i*}\odot W_{i*}\right)^{\top}\otimes I_m\right]x_i$ where $\|C_{i*}\odot W_{i*}\|_1=1$ and $\boldsymbol{e_i}=\left[0,\cdots,1,\cdots,0\right]^{\top}$ is the i^{th} canonical basis vector of $\mathbb{R}^N$. Then we aim to give a direction $\Delta W_i=\left[\xi_{i,1},\xi_{i,2},\cdots,\xi_{i,N}\right]^{\top}$ distributively for each agent, to optimize

$$\begin{aligned}&\max_{\{\Delta W_1,\cdots,\Delta W_N\}}\lim_{\delta\to 0^+}\frac{\Lambda_2^+\left(L+\delta\Delta L\right)-\Lambda_2^+\left(L\right)}{\delta},\\ &s.t.,\quad\begin{cases}\forall i, W_i+\Delta W_i\geq 0\\ \forall i,\|C_{i*}\odot\left(W_i+\Delta W_i\right)\|_1=1\end{cases}\end{aligned}\quad. \tag{4}$$

By solving this optimization problem, agents can cooperatively adjust the network topology such that the Fiedler value of Laplacian matrix converges to a local maxima.

Assumption 2: All the active eigenvalues of Laplacian matrix L are non-defective.

With this assumption, let $\lambda_{\tau_i}\in\Lambda(L), i=1,2,\cdots,t$ be the active eigenvalues whose real parts equal to $\Lambda_2^+\left(L\right)$ and r_{τ_i} be the algebraic multiplicity of λ_{τ_i}. Under assumption 2, we can denote $P_{\tau_i}\in\mathbb{C}^{N\times r_{\tau_i}}$ and $Q_{\tau_i}\in\mathbb{C}^{N\times r_{\tau_i}}$ as two matrices whose columns are independent normalized left and right eigenvectors corresponding to λ_{τ_i}, with $P_{\tau_i}^{\top}Q_{\tau_i}=I_{r_i}$. According to the perturbation theory of Kato [8], we are ready to deduce the directional derivative of $\Lambda_2^+\left(L\right)$, although the function $\Lambda_2^+\left(L\right)$ is not Lipchitz everywhere.

Lemma 1: Given a square matrix L with spectrum $\Lambda(L)$ and perturbation matrix $\Delta W:=\left[\Delta W_1,\Delta W_2,\cdots,\Delta W_N\right]^{\top}$, then the directional derivative of $\Lambda_2^+\left(L\right)$ with respect to perturbation intensity $\delta>0$ is

$$\lim_{\delta\to 0^+}\frac{\Lambda_2^+\left(L+\delta\Delta L\right)-\Lambda_2^+\left(L\right)}{\delta}=\min_{i=1,2,\cdots,t}\left\{\Re\left(\mu_j\right)\mid\exists v_j,P_{\tau_i}^{\top}\Delta LQ_{\tau_i}v_j=\mu_j v_j\right\}. \tag{5}$$

Specifically, the set $\left\{\mu_j\right\}$ is the spectrum of $P_{\tau_i}^{\top}\Delta LQ_{\tau_i}$.

We denote $f_{qr}\left(\cdot\right)$ as the QR decomposition and propose the following algorithm.

Algorithm 1. Distributed Topology-based Sensor Consensus Acceleration

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Input: Communication topology C_{i*}.

Initialize: The neighbor set $\mathcal{N}_i\triangleq\left\{j\mid c_{ij}=1\right\}$ with $d_i=\left|\mathcal{N}_i\right|$, random weight vector $W_{i*}\left(0\right)\geq 0$ with $\left\|W_{i*}\left(0\right)\odot C_{i*}\right\|_1=\left\|W_{i*}\left(0\right)\right\|_1=1$.

At the k^{th} step, agent i computes $T_{i*}\left(k\right)=e_i+W_{i*}\left(k\right)$ and randomly initialize state matrix $S_i(0) \in \mathbb{R}^{N\times N}$ with $T_{i*}^{\top}(k)S_i(k_1=0)=e_i^{\top}$.

Update $S_i(k_1)$ by

$$S_i(k_1+1)=S_i(k_1)-\frac{1}{d_i}R_i\left(d_i S_i(k_1)-\sum_{j\in\mathcal{N}_i}S_j(k_1)\right). \tag{6}$$

Set the finial value of $S_i(k_1)$ as $H(0)$. Then update $H(k_2)$ by

$$\begin{aligned}&\omega(k_2)=\left[H(k_2)\right]_{N,N},\ Q_\omega(k_2),R_\omega(k_2)=f_{qr}\left(H(k_2)-\omega(k_2)I_N\right)\\&H(k_2+1)=R_\omega(k_2)Q_\omega(k_2)+\omega(k_2)I_N\end{aligned} \tag{7}$$

Set $\widehat{\lambda_2^r}(k_2)=\max\left\{\mathrm{diag}\left(\Re\left(H(k_2)\right)\right)\right\}$ and compute the orthogonal basis of cokernel and kernel of $H(k_2)-\widehat{\lambda_2^r}(k_2)I_N$ as $\left\{p_j^S(k_2)\right\},\left\{q_i^S(k_2)\right\}$.

Solve the linear programming problem to obtain $\Delta W_{i*}^{*}(k)$

$$\begin{cases}\max\limits_{\Delta W_{i*}(k)}\left[p_j\right]_i\Delta W_{i*}(k)q_j\\ s.t.\quad 1_N^{\top}\Delta W_{i*}(k)=0,W_{i*}(k)+\Delta W_{i*}(k)\ge 0,\Delta W_{i*}(k)\odot C_{i*}=\Delta W_{i*}(k)\end{cases} \tag{8}$$

Theorem 1: Under assumption 1 and 2, let $\Phi_a\left(L(k)\right)=\left\{\lambda_i \mid \lambda_i=\Lambda_2^{+}\left(L(k)\right)\right\}$ be the active eigenvalues set of the current Laplacian matrix, then for each agent, the solution $\Delta W_{i*}^{*}(k)$ to (8) exists and the topology difference $\Delta W(k)$ satisfies

$$\max_z\left\{\frac{\partial\Re(\lambda_z)}{\partial\Delta W(k)}\middle|\lambda_z\in\Phi_a\left(L(k)-\Delta W(k)\right)\right\}\ge 0, \tag{9}$$

if the directional derivative of $\partial\Re(\lambda_z)/\partial\Delta W(k)$ exists at $W(k)$.

Proof: In the algorithm, iteration (6) is designed for all the agents, such that the j^{th} column of $S_i(k_1)$ converges towards the same solution to the linear equation

$$\forall j,\quad T_{j*}^{\top}(k)s_j=e_i^{\top}, \tag{10}$$

Putting together the solutions to these N equations yields $\hat{S}:=[s_1,s_2,\cdots,s_N]$, which thus satisfies $T^{\top}(k)\hat{S}=I_N$. Therefore, $\hat{S}$ is the inverse of $T(k)$. Noted that $T(k)=2I_N-L(k)$ and $T(k)$ is nonsingular, $\hat{S}$ is uniquely determined and has the same eigenspaces as $L(k)$.

Then, before computing the left and right eigenvectors of active eigenvalues [9], one need to ensure the estimation matrix $S_i(k)$ can uniformly converges to $\hat{S}$. According to [10][11], under assumption 1 and 2, for each agent, if its state matrix $S_i(k_1=0)$ is initialized as $T_{i*}^{\top}(k)S_i(k_1=0)=e_i^{\top}$, the consensus protocol (6) yields

$$\forall i, \quad \lim_{k_1 \to \infty} S_i\left(k_1\right) = \hat{S}, \tag{11}$$

with exponential convergence rate.

After $\hat{S}$ is obtained, $H\left(k_2\right)$ in the next step (7) can converge to an upper triangular matrix within a few iterations, with the same eigenvalues as $\hat{S}$ on its diagonal. That is, the obtained $\left\{p_j^S\left(k_2\right)\right\},\left\{q_i^S\left(k_2\right)\right\}$ are good estimation of eigenvectors of $\hat{S}$ corresponding to active eigenvalues. According to Lemma 1, the optimizer of (8) is the greatest directional derivative of this active eigenvalue w.r.t $W_{i*}\left(k\right)$, namely, the i^{th} row of weights matrix.

Moreover, the constrains of (8) ensures that there always exists a trivial direction of $W\left(k\right)$, along which $\Lambda_2^+\left(L\left(k\right)\right)$ is undiminished, i.e.,

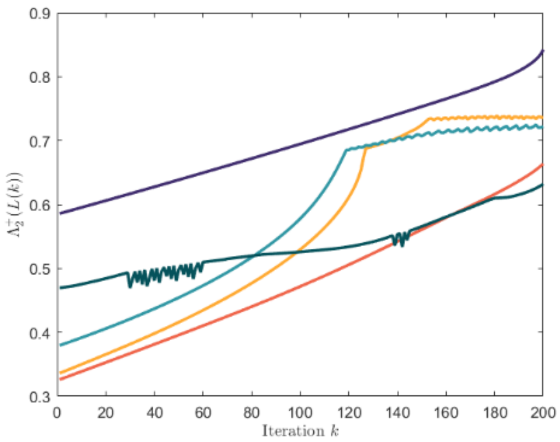
$$\begin{aligned}&\exists \Delta W^{\dagger} \in \left\{\Delta W \mid \forall i, 1_N^{\top} \Delta W_{i*}\left(k\right) = 0, W_{i*}\left(k\right) + \Delta W_{i*}\left(k\right) \geq 0\right\},\\ &\Lambda_2^+\left(L\left(k\right) - \Delta W\right) - \Lambda_2^+\left(L\left(k\right)\right) \geq 0\end{aligned}. \tag{12}$$

Therefore, the obtained $\Delta W_{i*}^*\left(k\right)$ from (8) satisfies (9), and does not break the property of $W\left(k\right)$ being irreducible and row stochastic. Hence, $\Delta W_{i*}^*\left(k\right)$ is a feasible direction for the Fiedler value of entire directed weighted network to grow in general, under assumption 1, 2 and provided the active eigenvalues of $L\left(k\right)$ are well separated.

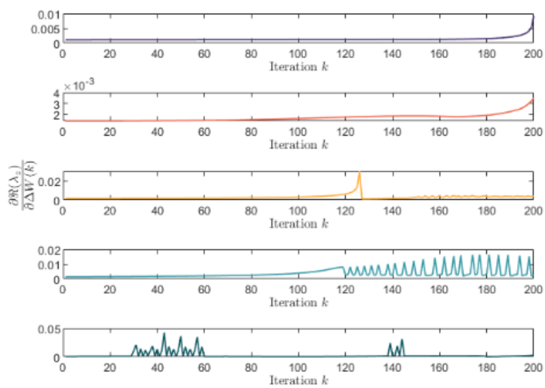
4. Numerical Example

This section aims to present a data aggregation process in consensus sensor network for supporting the designed algorithm. Consider $N = 10$ sensor entities that are connected through a strongly connected communication network C. Sensors maintain state vectors (e.g., measured values) and distributively fuse them in a consensus way, by local interaction. Regarding the problem this work investigates, we randomly initialize the weighted network $W\left(0\right)$, by which a sensor aggregates neighboring state vectors, then apply Algorithm 1 to simulate the progressively accelerating sensor consensus process.

Based on the information presented in figure 1, it is evident that Algorithm 1 typically provides a network weight direction that leads to an increase in the Fiedler eigenvalue of sensor networks. Corresponding to the two cases described in Theorem 1, the smooth segment depicted in figure 1(a) illustrates that the current weight distribution possesses a well-separated spectrum, and Algorithm 1 will enhance its Fiedler eigenvalue along the gradient. In contrast, the oscillating segment signifies that the algorithm selects a direction that separates the repeated values. This separation may temporarily result in a decrease in the Fiedler eigenvalue, but once the separation is complete, it will resume its upward trajectory.



(a)



(b)

Figure 1. (a): Evolution of Fiedler values versus iteration k in five different network structures and initial weights. (b): Evolution of the partial derivative of λ_z that belongs to the set in (9) w.r.t the change of network weights versus iteration k.

5. Conclusion

In this paper, we have addressed the problem of accelerating data aggregation for a weighted directed sensor network in a distributed approach. To this end, we focus on optimizing the network topology using local information. An integrated distributed acceleration algorithm is proposed that generates a direction for each sensor to adjust its aggregation weights, along which the second smallest eigenvalue of entire Laplacian matrix is guaranteed to have an increasing or undiminished real part in general. For the corner case where the Laplacian matrix exhibits repeated second smallest real parts, the designed algorithm is proven to separate them within a few iterations. The effectiveness of the proposed method is verified by a numerical example.

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