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Novel Tuning Rules for Adjusting the Parameters of the PI-PD Controller for Controlling Unstable Processes

Fadi ALYOUSSEF[a,1] and İbrahim KAYA[b]
[a] *Electrical and Electronics Eng. Dept., Institute of Natural and Applied Sciences, Dicle University, 21280 Diyarbakır, Turkey*
[b] *Electrical and Electronics Eng. Dept., Dicle University, 21280 Diyarbakır, Turkey*

Abstract. Unstable processes plus time delays are very frequent in the industry. PI-PD controllers are used for controlling unstable industrial operations as they give more robust performance relative to PID controllers. However, designing the parameters of PI-PD controllers is quite difficult. Recently, the centroid of the stability region based on the Weighted Geometrical Center (WGC) approach has been proposed for overcoming the tuning difficulty of the controller. Nevertheless, the current version of WGC available in the literature is time-consuming. Thus, this paper proposes new simple tuning rules to implement WGC when it is used for computing the parameters of the PI-PD controller for controlling unstable processes with time delays. An isothermal continuous stirred tank reactor is used for evaluating the performance of the proposed method.

Keywords. Stability region; PI-PD controller; Unstable processes; Centroid point.

1. Introduction

PI-PD controllers have been proposed as a method for overcoming the weaknesses of PID controllers and enhancing performance when controlling unstable industrial operations [1]. The PI-PD regulator is constructed using inner and outer feedback loops. The PD regulator is utilized in the inner loop while the PI regulator is employed in the outer loop [2]. The PI-PD regulator's primary benefit is that, relative to the PID controller, it gives improved closed-loop performance [3]. However, it is not easy task to design these parameters [4]. The PI-PD controller is therefore disadvantageous and has limited application because of the complexity of calculating these parameters [5].

Recently, calculating all the stabilizing settings of the PID/PI-PD controllers has gained increasing importance [6]. The Stability Region (SR) centroid, in particular, has been deemed as a suitable adjusting point for getting the PID/PI-PD controller settings [7-8]. Studies show that the centroid-calibrated controller's closed-loop performance is satisfactory [9-10]. Using the centroid to determine the control parameters can also lead to enhanced set-point tracking and fast disturbance rejection because it is placed in a safe zone away from every point of the Stability Boundary Locus (SBL). Moreover, the robustness against parameter variations can considerably be improved by employing

[1] Corresponding Author, Fadi AYOUSSEF, Electrical and Electronics Eng. Dept., Institute of Natural and Applied Sciences, Dicle University, 21280 Diyarbakır, Turkey. Emali: ENG.fadialyousef@hotmail.com.

the centroid for tuning purposes [11]. The centroid has also been regarded as a good option for overcoming the challenge of calculating the four adjusting variables of the PI-PD regulator [12]. In this respect, the Weighted Geometrical Center (WGC) technique for calculating the gains of the PI-PD regulator has recently been released [12-13]. The WGC method's primary drawback, however, is that it uses every point along SBL in accordance with a preset step size, which could result in a significant computing burden [10], particularly for a minute step size. It is critical to note that minute step sizes offer more exact outcomes since they allow for the usage of extra SBL points. To solve the above problem of WGC, an analytical version of WGC called AWGC has recently been revealed [14]. However, AWGC has been applied only for controlling integrating processes. Thus, the main goal of this work is to extend AWGC to be used for controlling unstable processes. The main contributions of this work can be summarized as follows:

1- Expanding AWGC method [14] to be employed for controlling unstable processes which are quite difficult to be controlled.
2- Providing simple running rules for tuning the parameters of the PI-PD controller which might improve the applicability of this controller on the industrial level as it reduces the tuning procedures required by control engineers.

The rest of this paper is organized as follows. The second section gives background about the PI-PD controller and the equations of the stability region. The next section is allocated for explaining the proposed method. A simulation example is given in the fourth section. The last section is devoted to conclusions.

2. Background

This section will be broken into two subsections. Background about the PI-PD controller is given in the first one. On the other hand, the stability reign equation is given in the second one.

2.1. PI-PD Controller

In a conventional closed-loop system, each of the PID regulator's acts is carried out in the forward route. This could result in a derivative kick which is an undesirable phenomenon [15]. To address this problem, the PD component of PI-PD regulators is transferred to an interior feedback loop to shift the process transfer function's poles to a more advantageous position where it could be governed more satisfactorily by the PI component performing in the onward route [2], [16]. Figure 1 depicts the architecture of the PI-PD regulator. The internal loop is composed of the transfer functions of the manufacturing process and the PD controller, whereas the external loop is composed of the PI regulator's transfer function and the inner loop's closed-loop transfer function. The PI-PD regulator's transfer functions $C_{PD}(s)$ and $C_{PI}(s)$ are expressed below:

$$C_{PD}(\mathrm{s}) = k_f + k_d s \tag{1}$$

$$C_{PI}(\mathrm{s}) = k_p + \frac{k_i}{s} \tag{2}$$

In above equations, k_f, k_d, k_p, and k_i are the modifiable PI-PD regulator gains. To get the controller's normalized representations, $\bar{s} = Ts$ has been inserted into the equations (1) and (2), yielding the following formulas:

$$C_{PD}(\bar{s}) = \frac{(k_f T + k_d \bar{s})}{T} \tag{3}$$

$$C_{PI}(\bar{s}) = \frac{(k_p \bar{s} + k_i T)}{(\bar{s})} \tag{4}$$

The above normalized forms will be used later for deriving the equations of the stability region.

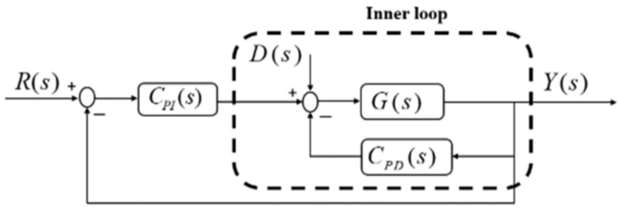

Figure 1. PI-PD controller.

2.2. Equations of the Stability Region

Several industrial applications such as industrial reactors are classified as unstable processes. Obtaining the required closed-loop performances for controlling such applications necessitates significant work on the part of control engineers due to the fact that unstable manufacturing operations have unstable poles at the right side of the S-plane. Furthermore, time delays influencing industrial processes may degrade the system performance by increasing system uncertainty [6]. In this paper, a SISO system is of concern. Also, the following low-order transfer function is utilized for describing unstable manufacturing operations with time delays:

$$G(s) = \frac{Ke^{-\theta s}}{Ts - 1} \qquad \forall \;\; \theta > 0 \;\;\&\;\; K > 0 \;\;\&\;\; T > 0 \tag{5}$$

In the above equations, T, K, and θ stand in for the time constant, process gain, and time delay, respectively. The normalized form of equation (5) is given below:

$$G(\bar{s}) = \frac{Ke^{-\tau \bar{s}}}{\bar{s} - 1} \tag{6}$$

The Generalized Stability Region (GSR) is determined using the normalized presentation of the regulator's and industrial process's transfer functions. The characteristic formula of the inner loop, which is displayed in figure 1, is calculated using the process transfer function, $G(\bar{s})$, stated in expression (6) and the transfer function of the PD regulator, $C_{PD}(\bar{s})$, provided in equation (3) as follows:

$$\Delta(\bar{s}) = 1 + C_{PD}(\bar{s})G(\bar{s}) \Rightarrow \Delta(\bar{s}) = (1 + \frac{Kk_d}{T}e^{-\tau\bar{s}})\bar{s} + Kk_f e^{-\tau\bar{s}} - 1 \tag{7}$$

Applying similar steps to that given in Refs [6], [17] on equation (7), the equations of the inner loop's SBL, $SBL(Kk_d / T, Kk_f, \bar{\omega})$, are sated as below:

$$\frac{Kk_d}{T} = -\cos(\tau\varpi) + \frac{1}{\varpi}\sin(\tau\varpi) \quad (8)$$

$$Kk_f = \cos(\tau\varpi) + \varpi\sin(\tau\varpi) \quad (9)$$

Equation (8), at $\varpi = 0$, is not defined. As a result, since SBL in this case has a discontinuity, and thus $\varpi = 0$ should not be employed in the computations of the stabilizing controller parameters. Yet, this has no impact on the accuracy of these calculations [18]. The right sides of equations (8) and (9) may be seen to be unrelated to the system variables i.e., θ, K, and T. SBL is hence also known as the generalized SBL which can be abbreviated as GSBL. Equations (8) and (9) is used for constructing GSBL by varying the values of ϖ in the limit $[\varepsilon, \varpi_{PD}]$ for a particular number of τ. With the intention of preventing division by 0 in equation (8), ε should be chosen to be an extremely small positive number. The upper limit of the bound $[\varepsilon, \varpi_{PD}]$ i.e., ϖ_{PD}, is obtained by solving the equation $\cos(\tau\varpi) + \varpi\sin(\tau\varpi) = 1$. The outer loop's stable characteristic equation, computed by $\Delta(\overline{s}) = 1 + C_{PD}(\overline{s})G(\overline{s}) + C_{PI}(\overline{s})G(\overline{s})$, is mathematically formulated using a random calibration point $C_{PD}(\overline{x}_{PD}, \overline{y}_{PD})$ taken from the inner loop's GSR (where $\overline{x}_{PD}$ and $\overline{y}_{PD}$ are supposed to be known) as expressed below:

$$\Delta(\overline{s}) = (1 + \overline{x}_{PD}e^{-\tau\overline{s}})\overline{s}^2 + (-1 + \overline{y}_{PD}e^{-\tau\overline{s}} + Kk_p e^{-\tau\overline{s}})\overline{s} + KTk_i e^{-\tau\overline{s}} \quad (10)$$

here, $\overline{x}_{PD} = Kk_d / T$ and $\overline{y}_{PD} = Kk_f$. Applying similar steps to that presented in Refs [6], [17] on equation (10), the equations of $GSBL(Kk_p, KTk_i, \varpi)$ are determined as follows:

$$Kk_p = \cos(\tau\varpi) + \varpi\sin(\tau\varpi) - \overline{y}_{PD} \quad (11)$$

$$KTk_i = \varpi^2\cos(\tau\varpi) - \varpi\sin(\tau\varpi) + \overline{x}_{PD}\varpi^2 \quad (12)$$

By swapping ϖ in the range $\varpi \in [0, \varpi_{PI}]$, where $\varpi_{PI} > 0$ is the first root resulting from solving: $\varpi^2\cos(\tau\varpi) - \varpi\sin(\tau\varpi) + \overline{x}_{PD}\varpi^2 = 0$, $GSBL(Kk_p, KTk_i, \varpi)$ is constructed using equations (11) and (12). In summary, the outer loop's GSR lies in the area between $GSBL(Kk_p, KTk_i, \varpi)$ and $KTk_i = 0$ [6].

3. Proposed Method

For controlling integrating processes, the following AWGC equations are proposed [14]:

$$\overline{x}_{AWGC} = \frac{1}{b-a}\int_a^b f(x)dx \quad (13)$$

$$\overline{y}_{AWGC} = \frac{1}{2(b-a)}\int_a^b g(x)dx \quad (14)$$

In the equations above, $f(x) \in R$ and $g(x) \in R$ are real functions which correspondingly denote the equations of SR on the x-axis and y-axis. In this paper, equations (13) and (14) are used to obtain tuning rules to adjust the parameters of the

PI-PD controller, which is used for controlling unstable processes. It should be observed, however, that the term $(1/\bar{\omega})\sin(\tau\omega)$ of equation (8) precludes the integral action shown in equation (13) from being done mathematically. To solve this problem, the equations $\frac{1}{(\omega_{PD}-\varepsilon)}\int_{\varepsilon}^{\omega_{PD}}(-\cos(\tau\omega)+\frac{1}{\omega}\sin(\tau\omega))d\omega$ and $\cos(\tau\omega)+\omega\sin(\tau\omega)=1$ are numerically solved for a step of 0.05 of τ in the range $\tau\in[0.1,1.9]$. Consequently, the centroid's x-axis coordinate of the inner loop, $\bar{x}_{AWGC\_PD}$, for the unstable process given in equation (6) is computed as stated below:

$$\bar{x}_{AWGC\_PD}=\begin{pmatrix}0.0002921\tau^5+0.0006498\tau^4+0.007269\tau^3+0.03707\tau^2\\+0.3869\tau-4.456\text{e-}06\end{pmatrix} \tag{15}$$

To calculate the centroid on the y-axis coordinate of the inner loop's SR, equation (9) should be modified to be $Kk_f=\cos(\tau\bar{\omega})+\bar{\omega}\sin(\tau\bar{\omega})+1$. This is because there is a shift by 1 on the y-axis of SR. Next, the integral action given in (14) is applied to the modified equation to get the following mathematical expression:

$$\bar{y}_{AWGC\_PD}=\begin{pmatrix}\frac{1}{2(\omega_{PD}-\varepsilon)}\int_{\varepsilon}^{\omega_{PD}}(\cos(\tau\omega)+\omega\sin(\tau\omega)+1)d\omega=\\\frac{\sin(\tau\omega_{PD})-\tau\omega_{PD}\cos(\tau\omega_{PD})}{\tau^2}+\frac{\sin(\tau\omega_{PD})}{\tau}+\omega_{PD}\end{pmatrix} \tag{16}$$

By numerically solving the equation $\cos(\tau\omega)+\omega\sin(\tau\omega)=1$ for a step of 0.05 of τ in the range $\tau\in[0.1,1.9]$, the value of ω_{PD} in the expression above is determined as shown below:

$$\omega_{PD}=\frac{123\tau^2-348.2\tau+829.6}{\tau^4+51.28\tau^3-63.08\tau^2+265\tau-0.04563}\qquad 0.1<\tau<=0.6 \tag{17}$$

$$\omega_{PD}=\begin{pmatrix}-0.4253\tau^8+3.184\tau^7-8.336\tau^6+3.734\tau^5+27.94\tau^4\\-73.21\tau^3+87.78\tau^2-58.6\tau+20.27\end{pmatrix}\qquad 0.6<\tau<=1.9 \tag{18}$$

The centroid point of the outer loop, $(\bar{x}_{AWGC\_PI},\bar{y}_{AWGC\_PI})$, are derived by applying the integral actions given in formulas (13) and (14) on formulas (11) and (12), correspondingly. As a result, the following expressions are obtained:

$$\bar{x}_{AWGC\_PI}=\left(\frac{\sin(\tau\omega_{PI})-\tau\omega_{PI}\cos(\tau\omega_{PI})}{\tau^2}+\frac{\sin(\tau\omega_{PI})}{\tau}-\bar{x}_{AWGC\_PI}\omega\right) \tag{19}$$

$$\bar{y}_{AWGC\_PI}=\left(\frac{\begin{pmatrix}\left(3\tau^2\omega^2{}_{PI}-3\tau-6\right)\sin\left(\tau\omega_{PI}\right)+\left((3\tau^2+6\tau)\omega_{PI}\right)\cos\left(\tau\omega_{PI}\right)\\+\tau^3\bar{x}_{AWGC\_PD}\omega^3{}_{PI}\end{pmatrix}}{6\tau^3\omega_{PI}}\right) \tag{20}$$

In the equation above, ω_{PI} can be linked to τ by numerically solving the equation $\omega^2\cos(\tau\omega)-\omega\sin(\tau\omega)+\bar{x}_{PD}\omega^2=0$ in the range $\tau\in[0.1,1.9]$ with a step of $\tau=0.05$ as stated below:

$$\omega_{PI}=\frac{-79.22\tau^2+179.2\tau+29.16}{\tau^4-32\tau^3+117.4\tau^2+18.51\tau+0.002902}\qquad 0.1<\tau\le 0.6 \tag{21}$$

$$\omega_{PI}=\begin{pmatrix}-0.8648\tau^7+7.893\tau^6-31.1\tau^5+68.94\tau^4\\ -93.93\tau^3+80.42\tau^2-42.32\tau+12.19\end{pmatrix}\qquad 0.6<\tau\le 1.9 \tag{22}$$

Figure 2 shows the steps needed to compute the PI-PD regulator's settings for unstable industrial operations using the proposed AWGC method.

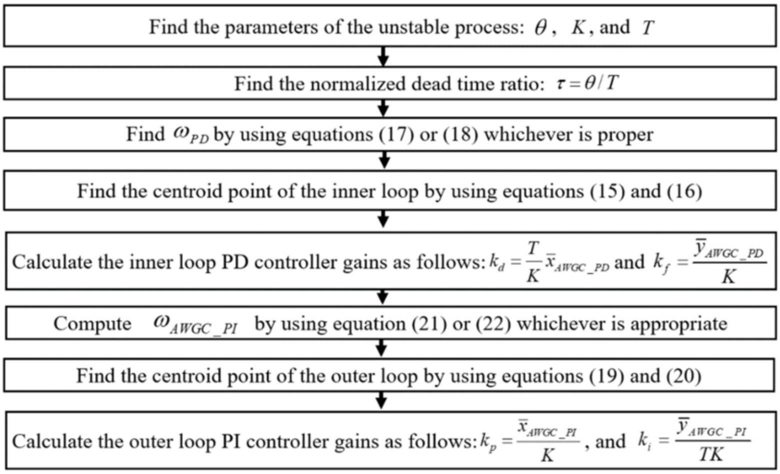

Figure 2. A flowchart for computing the parameters of the PI-PD controller using the proposed method

4. Simulation Example

A transfer function of an unstable industrial operation of the form $G(s)=3.433e^{-20s}/(103.1s-1)$ is used to describe an isothermal continuous stirred tank reactor [10]. The normalized dead time, τ, of this transfer function is computed to be $\tau=0.1940$. This value is adopted for computing the regulator gains via the provided AWGC technique by applying the procedures given in figure 2 to get the following settings: $k_f=0.9284$, $k_d=2.2974$, $k_p=0.1731$, and $k_i=0.0093$. The proposed method is compared to the PI-PD regulator of Kaya [19] and the ACCSRTR method [6], [17]. The step outcomes and related control efforts for all analyzed approaches to a unit step (given at t=0 s) and a nonnegative unity disruption introduced into the closed-loop structure at t=200 s are shown in figure 3 for the nominal case. Figures 4 (a) and (b) show the results for process parameter variations of +15% and -15%, correspondingly. To assess the endurance of the various techniques versus measurement noise, a band-limited white noise with an energy level of 0.00001 is included in the measured signals. Figure 5 depicts the system outputs under the measurement noise and the related control efforts. The following can be deduced from the above-mentioned figures:

1- ACCSRTR and the method of Kaya give faster transient response than the proposed method. However, they consume more control energy relative to the proposed method as clear from figure 3 (b).

2- In regard to robustness against process parameter fluctuations and measurement noise, AWGC appears to perform slightly better than ACCSRTR.

3- In comparison to the presented approach, Kaya's method [19] seems to yield less robust behavior in the face of parameter variations and measurement noise.

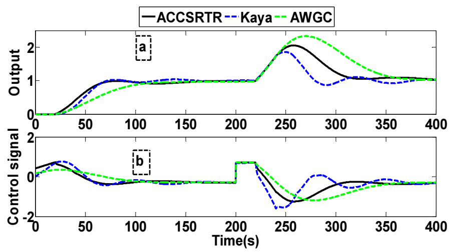

Figure 3. Nominal case results (a) Closed-loop system outputs, (b) Control efforts.

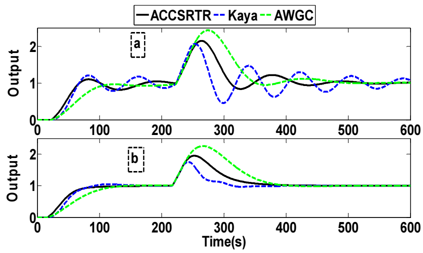

Figure 4. Closed-loop system outputs for variations in system parameters (a) +15%, (b) -15%

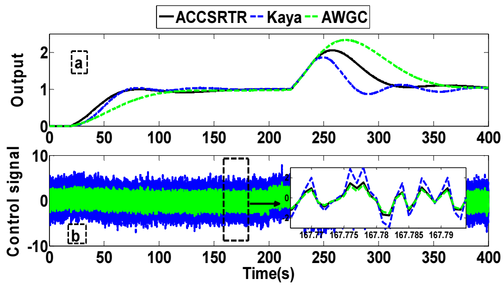

Figure 5. Closed-loop system outputs under measurement noise with a power of 0.0001 (a) Closed-loop system outputs, (b) Control efforts

5. Conclusions

In this paper, AWGC has been expended for tuning the parameters of the PI-PD controller for regulating unstable processes. Simple tuning rules were proposed to save the control engineer's time and overcome the PI-PD controller's tuning difficulty. Results showed that the proposed method is robust against parameter variation and measurement noise. Thus, it is convenient for applications that are working under severe conditions.

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