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New Tanimoto Similarity Measures Between Pythagorean Fuzzy Sets with Applications on Pattern Recognition

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Abstract. Pythagorean fuzzy sets (PFSs) are a versatile tool for handling uncertain problems and have proven effective in practical applications. However, many existing Pythagorean fuzzy similarity measures have counter-intuitive situations, making it challenging to measure the similarity or difference between PFSs accurately. To address this issue, we propose two similarity measures for PFSs inspired by the Tanimoto similarity measure. We also explore the properties of the proposed measures and provide several numerical examples to illustrate their superior performance in processing fuzzy information from PFSs. Finally, we demonstrate the applicability of the proposed measures in solving pattern recognition problem.

Keywords: Pythagorean fuzzy sets, Tanimoto similarity measure, uncertainty, pattern recognition

1. Introduction

Decision-making is a pervasive behavior, and the main factor hindering correct decision-making is uncertainty [1]. In various fields, uncertainty has become increasingly prevalent due to the complex nature of objective phenomena and the inherent limitations of human knowledge [2]. This obscure feature often takes place in random and indeterminate ways, making it difficult to accurately describe, leading to numerous difficulties in the decision-making stage [3]. As a result, various new theories and methods have surfaced to represent uncertain information in practical problems [4-8]. One prominent theory is fuzzy sets, which has gained significant attention since its introduction by Zadeh in 1965 [4]. Fuzzy sets theory serves as an extension of classical set theory to address situations where the boundaries between different categories are not well-defined. By assigning degrees of membership to each element of a set, fuzzy sets allow us to depict vague concepts, such as "tall" and "short", "hot" and "cold", in a more natural way. By addressing the shortcomings of traditional decision-making methods, fuzzy sets theory has enabled us to reason about uncertain information and make decisions based on incomplete or ambiguous data. This novel theory offers a new way of describing fuzzy and uncertain information and

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modeling, which has been widely applied in many fields, including control systems, pattern recognition, decision-making, and artificial intelligence [9-14].

As the complexity of decision-making problems increases, the traditional fuzzy sets theory falls short in accurately representing the uncertain information in the decision problems. To address this issue, various scholars have proposed extended forms of traditional fuzzy sets from diverse perspectives, including Intuitionistic fuzzy sets (IFSs), Pythagorean fuzzy sets (PFSs), and so on. Among these, IFSs have attracted consider- able attention due to their ability to represent fuzzy and uncertain information with the inclusion of both membership and non-membership degrees of elements. This salient feature has made IFSs a valuable tool in several fields for addressing uncertainties [5]. However, IFSs stipulate that the sum of membership degree and non-membership degree must not exceed one, which can be challenging to satisfy in some instances. Therefore, as an extension of IFSs, in 2013, Yager first introduced PFSs [6]. This model introduces the notion of Pythagorean membership function, which extends the concept of membership and non-membership degrees to a triplet of parameters, including membership, non-membership, and hesitation degrees.

PFSs offer a wider membership space than IFSs by relaxing the constraints on membership and non-membership degrees. Specifically, PFSs impose the restriction that the square sum of membership and non-membership degrees is less than or equal to one, enabling them to more effectively capture and represent uncertain information. Subsequently, various studies on PFSs were carried out with great enthusiasm. For example, Li [15] developed Pythagorean fuzzy Hamy mean operators for multi-attribute group decision-making to aid in supplier selection. Gao [16] proposed the Pythagorean Fuzzy Hamacher Prioritized aggregation operators for multi-attribute decision-making based on existing Prioritized aggregation operators. Wu and Wei [17] utilized Hamacher operations to develop Pythagorean fuzzy aggregation operators.

Similarity measures have always attracted much attention in fuzzy sets and related extensions. Currently, similarity measurement of Pythagorean fuzzy sets has shown great potential in a variety of applications [18-21]. Garg [18] devised a correlation measure grounded in PFSs to address the issue of multi-attribute decision-making. Li and Zeng [19] investigated the normalized Hamming distance and the normalized Euclidean distance of PFSs, seeking to improve the accuracy and efficiency. Additionally, other researches include similarity-based measures using the cosine similarity measure [20], exponential similarity measure [21], Dice similarity measure [22] and so on [23]. These measures aim to tackle challenges such as asymmetry, differences in intersection size, and complexity. Although progress has been made in developing these measures, there are still instances where they exhibit counter-intuitive behavior. Therefore, selecting suit- able similarity measures for specific applications remains a challenge.

In this paper, we propose two new similarity measures for PFSs based on the Tanimoto similarity concept. We prove and demonstrate the properties of the proposed measures through numerical examples. We also propose two models for using these measures in pattern recognition problem in Pythagorean fuzzy environments. We also conduct pat- tern recognition experiment in Pythagorean fuzzy environment to compare the proposed measures with the existing similarity measures. The results show that the proposed similarity measure can not only overcome many counter-intuitive situations in existing measures, but also help make more convincing decisions when distinguishing PFSs.

The following study is presented below. Specifically, in Section 2, we briefly review the fundamental concepts of fuzzy sets theory. In Section 3, we propose two novel similarity measures for PFSs and establish their properties. In Section 4, we test the performance of the proposed similarity measures on pattern recognition problem. Finally, in Section 5, we make the conclusion.

2. Preliminaries

This section will provide an introduction to the fundamental concepts concerning intuitionistic fuzzy sets, Pythagorean fuzzy sets, and Tanimoto similarity measure. These concepts will serve as the foundation for our future research endeavors.

2.1. Intuitionistic fuzzy sets

Definition 1 ([5]). *Suppose that Z is a finite universe of discourse. The intuitionistic fuzzy set I in Z is defined as:*

$$I = \{\langle z, \rho_I(z), \tau_I(z)\rangle \mid z \in Z\} \tag{1}$$

where $\rho_I(z)$ denotes the membership degree of $z \in Z$, and $\tau_I(z)$ expresses the non-membership degree of $z \in Z$. For any $z \in Z$, $\rho_I(z)$ and $\tau_I(z)$ meet the following conditions:

$$0 \le \rho_I(z) + \tau_I(z) \le 1 \tag{2}$$

For any $z \in Z$, the indeterminacy degree of the element z is:

$$\theta_I(z) = 1 - \rho_I(z) - \tau_I(z) \tag{3}$$

2.2. Pythagorean fuzzy sets

Definition 2 ([6]). *Suppose that Z is a finite universe of discourse. The Pythagorean fuzzy set P in Z is defined as:*

$$P = \{\langle z, \rho_P(z), \tau_P(z)\rangle \mid z \in Z\} \tag{4}$$

where $\rho_P(z)$ denotes the membership degree of $z \in Z$, and $\tau_P(z)$ expresses the non-membership degree of $z \in Z$. For any $z \in Z$, $\rho_P(z)$ and $\tau_P(z)$ meet the following conditions:

$$0 \le \rho_P(z) + \tau_P(z) \le 1 \tag{5}$$

For any $z \in Z$, the indeterminacy degree of the element z is:

$$\theta_P(z) = 1 - \rho_P(z) - \tau_P(z) \tag{6}$$

2.3. Tanimoto similarity measure

Definition 3 ([24]). *Suppose* $A = \{a_1, a_2, \ldots, a_n\}$ *and* $B = \{b_1, b_2, \ldots, b_n\}$ *are two probability distributions. The definition of the Tanimoto measure between A and B is depicted as:*

$$T(A,B) = \frac{\sum_{i=1}^{n} a_i b_i}{\sum_{i=1}^{n} a_i^2 + \sum_{i=1}^{n} b_i^2 - \sum_{i=1}^{n} a_i b_i} \tag{7}$$

3. Proposed similarity measures

In this section, we present several Tanimoto similarity measures and weighted Tanimoto similarity measures between two Pythagorean fuzzy sets (PFSs) using the concept of Tanimoto similarity measure. To compare with existing distance measures, we propose their distance measure forms. We verify their properties through various numerical experiments. Furthermore, we demonstrate the capability of the proposed measure to overcome the counter-intuitive situation of existing measures through several examples.

3.1. Tanimoto similarity measures for PFSs

Definition 4. *Considering a fixed set Z={$z_1, z_2, \dots, z_n$}, $F = \{\langle z_i, \rho_F(z_i), \tau_F(z_i)\rangle \mid z_i \in Z\}$ and $G = \{\langle z_i, \rho_G(z_i), \tau_G(z_i)\rangle \mid z_i \in Z\}$ are two PFSs. A Tanimoto similarity measure between F and G is defined as:*

$$T_{PFS}(F,G) = \frac{\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}{\sum_{i=1}^{n}(\rho_F^4(z_i)+\tau_F^4(z_i))+\sum_{i=1}^{n}(\rho_G^4(z_i)+\tau_G^4(z_i))-\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))} \tag{8}$$

Theorem 1. *If F and G are any two PFSs, the $T_{PFS}(F,G)$ satisfies the conditions:*
1. $0 \le T_{PFS}(F,G) \le 1$
2. $T_{PFS}(F,G) = T_{PFS}(G,F)$
3. $T_{PFS}(F,G) = 1$ if F = G, i.e., $\rho_F(z_i) = \rho_G(z_i)$, $\tau_F(z_i) = \tau_G(z_i)$

Proof. 1. According to $0 \le \rho(z_i) \le 1$ and $0 \le \tau(z_i) \le 1$, we have:

$$T_{PFS}(F,G) = \frac{\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}{\sum_{i=1}^{n}(\rho_F^4(z_i)+\tau_F^4(z_i))+\sum_{i=1}^{n}(\rho_G^4(z_i)+\tau_G^4(z_i))-\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))} \ge 0$$

According to the inequality $a^2 + b^2 \ge 2ab$, we thus obtain:

$$T_{PFS}(F,G) = \frac{\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}{\sum_{i=1}^{n}(\rho_F^4(z_i)+\tau_F^4(z_i))+\sum_{i=1}^{n}(\rho_G^4(z_i)+\tau_G^4(z_i))-\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}$$

$$\le \frac{\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}{\sum_{i=1}^{n} 2\rho_F^2(z_i)\rho_G^2(z_i)+\sum_{i=1}^{n} 2\tau_F^2(z_i)\tau_G^2(z_i)-\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}$$

$$= \frac{\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}{\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}$$

$$= 1$$

Therefore, $0 \le T_{PFS}(F,G) \le 1$.

2. $$T_{PFS}(F,G) = \frac{\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}{\sum_{i=1}^{n}(\rho_F^4(z_i)+\tau_F^4(z_i))+\sum_{i=1}^{n}(\rho_G^4(z_i)+\tau_G^4(z_i))-\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}$$

$$= \frac{\sum_{i=1}^{n}(\rho_G^2(z_i)\rho_F^2(z_i)+\tau_G^2(z_i)\tau_F^2(z_i))}{\sum_{i=1}^{n}(\rho_G^4(z_i)+\tau_G^4(z_i))+\sum_{i=1}^{n}(\rho_F^4(z_i)+\tau_F^4(z_i))-\sum_{i=1}^{n}(\rho_G^2(z_i)\rho_F^2(z_i)+\tau_G^2(z_i)\tau_F^2(z_i))}$$

$$= T_{PFS}(G,F)$$

3. When $F = G$, *i.e.*, $\rho_F(z_i) = \rho_G(z_i)$, $\tau_F(z_i) = \tau_G(z_i)$, we have:

$$T_{PFS}(F,G) = \frac{\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}{\sum_{i=1}^{n}(\rho_F^4(z_i)+\tau_F^4(z_i))+\sum_{i=1}^{n}(\rho_G^4(z_i)+\tau_G^4(z_i))-\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}$$

$$= \frac{\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}{\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i))}$$

$$= 1$$

When considering the degree of indeterminacy, we can define a new similarity measure as follows:

Definition 5. For $z_i \in Z$, take the degrees of indeterminacy $\theta_F(z_i)$ and $\theta_G(z_i)$. The Tanimoto similarity measure $T_{PFS}^{\theta}(F,G)$ is described as:

$$T_{PFS}^{\theta}(F,G) = \frac{\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i)+\theta_F^2(z_i)\theta_G^2(z_i))}{\sum_{i=1}^{n}(\rho_F^4(z_i)+\tau_F^4(z_i)+\theta_F^4(z_i))+\sum_{i=1}^{n}(\rho_G^4(z_i)+\tau_G^4(z_i)+\theta_G^4(z_i))-\sum_{i=1}^{n}(\rho_F^2(z_i)\rho_G^2(z_i)+\tau_F^2(z_i)\tau_G^2(z_i)+\theta_F^2(z_i)\theta_G^2(z_i))} \tag{9}$$

Theorem 2. *If F and G are any two PFSs, the* $T_{PFS}(F,G)$ *satisfies the conditions:*

1. $0 \le T_{PFS}^{\theta}(F,G) \le 1$

2. $T_{PFS}^{\theta}(F,G) = T_{PFS}^{\theta}(G,F)$

3. $T_{PFS}^{\theta}(F,G) = 1$ *if* $F = G$, *i.e.*, $\rho_F(z_i) = \rho_G(z_i)$, $\tau_F(z_i) = \tau_G(z_i)$, $\theta_F(z_i) = \theta_G(z_i)$

Proof. The properties of $T_{PFS}^{\theta}(F,G)$ are similar to **Theorem 1**.

3.2. Numerical examples

Example 1. *Let* F_1, F_2, F_3 *be three PFSs in* $Z = \{z_1, z_2\}$, *where*

$$\begin{aligned} F_1 &= \{\langle z_1, 0.4, 0.3\rangle, \langle z_2, 0.5, 0.2\rangle\}, \\ F_2 &= \{\langle z_1, 0.4, 0.3\rangle, \langle z_2, 0.5, 0.2\rangle\}, \\ F_3 &= \{\langle z_1, 0.7, 0.5\rangle, \langle z_2, 0.3, 0.4\rangle\}. \end{aligned} \tag{10}$$

Table 1. The results of two similarity measures

Measures	F_1F_2	F_2F_1	F_1F_3	F_3F_1	F_2F_3	F_3F_2
T_{PFS}	1.0000	1.0000	0.4266	0.4266	0.4266	0.4266
T_{PFS}^{θ}	1.0000	1.0000	0.6732	0.6732	0.6732	0.6732

Table 1 shows the Tanimoto similarity measures between two PFSs, we can find that when F1 = F2, the Tanimoto similarity measures between F1 and F2 , T_{PFS}(F1, F2) = 1 and T_{PFS}^{θ}(F1, F2) = 1. Besides, T_{PFS}(F1, F3) = T_{PFS}(F3, F1) and T_{PFS}^{θ}(F1, F3)= T_{PFS}^{θ}(F3, F1), which satisfy the properties 2 and 3 in Theorem 1 and Theorem 2.

Example 2. Consider F1 and F2 are two PFSs in z, where

$$F_1 = \{\langle z, \rho, \tau\rangle\},\ F_2 = \{\langle z, \tau, \rho\rangle\}$$

The parameters ρ and τ are adopted to signify the membership and non-membership of F1 and F2, correspondingly. The values of the membership degree ρ and non-membership degree τ lie in the [0, 1] interval, while satisfying condition $0 \leq \rho^2 + \tau^2 \leq 1$. In accordance with the suggested measures, figure 1 depicts the similarity between F1 and F2 across a series of parameter values for ρ and τ.

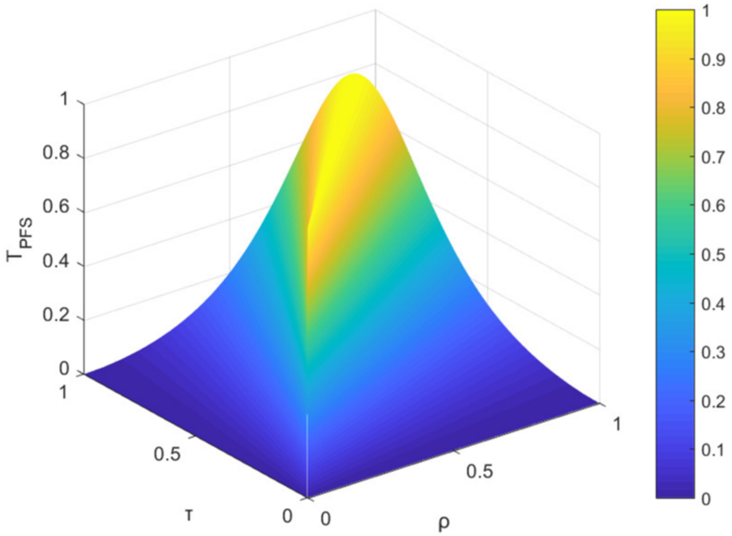

Figure 1. Tanimoto similarity measure between F_1 and F_2.

Based on the observations in figure. 1, it is clear that the similarity falls within the range of [0,1]. Notably, the maximum value of 1 is attained by the similarity measure between F1 and F2 when ρ equals τ, whereas the minimum value of 0 is reached when ρ equals 1 and τ equals 0 (or vice versa). With varying parameters ρ and τwithin [0,1], the similarity measure changes correspondingly within the same range. These conclusive results confirm that the Tanimoto similarity measures satisfy property 1 as defined in Theorem 1 and Theorem 2.

Example 3. Consider F, G1, G2 are three PFSs in Z = {z1, z2} and shown in table 2.

Table 2. PFSs F, G_1 and G_2

	z_1	z_2
F	$\langle 0.667, 0.342\rangle$	$\langle 0.503, 0.096\rangle$
G_1	$\langle 0.472, 0.152\rangle$	$\langle 0.077, 0.696\rangle$
G_2	$\langle 0.294, 0.490\rangle$	$\langle 0.684, 0.581\rangle$

Table 2 clearly shows that G1 ⊭ G2, leading to the inference that the similarity between two PFSs. Table 3 compares the outcomes of Tanimoto similarity measures with those of exponential similarity measures proposed by Zhang [21] (indicated as SM^1_{PFS} and SM^2_{PFS}). Specifically, the Tanimoto similarity measure yield accurate outcomes that align with intuition. However, the exponential similarity measures produce counter-intuitive results, and may fail to precisely distinguish differences between two PFSs in practical applications. This highlights the superior precision of the Tanimoto similarity measures and the limitations of the exponential similarity measure. Based on this example, it can be inferred that Tanimoto similarity measure exhibit higher effectiveness and superiority over the exponential similarity measure.

Table 3. The results of different similarity measures

Measures	FG_1	FG_2
T_{PFS}	0.238	0.387
T_{PFS}^{θ}	0.621	0.492
SM_{PFS}^{1}	0.577	0.577
SM_{PFS}^{2}	0.675	0.675

4. Application

In this section, we use an application with pattern recognition to demonstrate the effectiveness and superiority of the proposed similarity measures.

Example 4. This example concerns the pattern recognition of unrecognized samples, where PFSs are used to describe three samples of confirmed categories F = {F1, F2, F3} and one unrecognized sample S. Furthermore, the scope of discourse encompasses their respective attributes zi (i = 1, 2, 3, 4, 5), as illustrated intable 4.

Table 4. Known PFSs and a sample S

	z_1	z_2	z_3	z_4	z_5
F_1	⟨0.095, 0.267⟩	⟨0.601, 0.401⟩	⟨0.577, 0.215⟩	⟨0.848, 0.512⟩	⟨0.431, 0.433⟩
F_2	⟨0.521, 0.468⟩	⟨0.875, 0.472⟩	⟨0.532, 0.701⟩	⟨0.832, 0.120⟩	⟨0.549, 0.070⟩
F_3	⟨0.207, 0.258⟩	⟨0.689, 0.060⟩	⟨0.081, 0.411⟩	⟨0.468, 0.736⟩	⟨0.485, 0.056⟩
S	⟨0.693, 0.587⟩	⟨0.694, 0.232⟩	⟨0.392, 0.913⟩	⟨0.746, 0.198⟩	⟨0.747, 0.179⟩

To determine the category of the unidentified sample S, we have utilized the suggested Tanimoto measure. Further, we have conducted a comparative analysis by examining the outcomes obtained through the Dice similarity measure (DPFS) [22], two variations of exponential similarity measures ($\mathrm{SM}_{\mathrm{PFS}}^{1}$ and $\mathrm{SM}_{\mathrm{PFS}}^{2}$) [21], geomeasure similarity(GPFS) [25] and cosine similarity measures ($\mathrm{C}_{\mathrm{PFS}}^{1}$ and $\mathrm{C}_{\mathrm{PFS}}^{2}$) [20]. The obtained results are duly presented in Table 5. Leveraging the data presented in table 5 and applying the principle of maximum similarity, we have determined that the sample S and F2 exhibit the greatest similarity. This outcome has been consistently observed across various other similarity measures, affirming the efficacy of the Tanimoto similarity measures. Notably, the results obtained by the Tanimoto similarity measures vary greatly from each other. We calculate the differences between the highest and other similarity scores between different PFSs and S obtained through various measures, as shown in Table 6. It is apparent that the results of Tanimoto similarity measures show the largest differences between the highest and other similarity scores, which means that the proposed similarity measures have greater potential in complex applications.

Table 5. The results of different similarity measures

Measures	F_1	F_2	F_3
T_{PFS}	0.403	0.804	0.364
T^{θ}_{PFS}	0.387	0.713	0.373
D_{PFS}	0.560	0.886	0.555
SM^1_{PFS}	0.540	0.689	0.536
SM^2_{PFS}	0.648	0.779	0.659
G_{PFS}	0.798	0.878	0.807
C^1_{PFS}	0.719	0.859	0.707
C^2_{PFS}	0.926	0.964	0.922

Table 6. The differences between the highest and other similarity scores of various similarity measures

T_{PFS}	T^{θ}_{PFS}	D_{PFS}	SM^1_{PFS}	SM^2_{PFS}	G_{PFS}	C^1_{PFS}	C^2_{PFS}
0.841	0.666	0.657	0.302	0.251	0.151	0.292	0.080

5. Conclusion

In this paper, we propose two new similarity measures for PFSs based on the Tanimoto similarity concept. Numerical experiments demonstrate that the proposed measures not only avoid the counter-intuitive situations of the existing measures but also obtain more confident results in distinguishing PFSs. We also apply our proposed measures to pattern recognition task in Pythagorean fuzzy environment, and obtain effective and reasonable results. In the future research, we plan to explore the full potential of the proposed simiarity measures by applying it to various problems in Pythagorean fuzzy environments. By doing so, we aim to enhance its impact and validate its effectiveness in real-life scenarios.

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