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A Bio-Inspired Mathematical Approach to Design for Additive Manufacturing

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Abstract. It is well acknowledged that DfAM requires a comprehensive understanding of materials, processes and parameters, and the associated geometric opportunities and limitations. The holistic knowledge required for efficient DfAM poses a major challenge to the progression of industrial applications of additive manufacturing (AM). Whilst AM offers enhanced geometric freedom during the design process, the psychological inertia of long-standing subtractive approaches is retained in the design thinking of the engineering community and inherent in computer aided design (CAD). To create an axisymmetric form about a curved axis defined in 3D-space, the traditional method enabled by CAD is to define a centreline and/or a series of cross-sections. However, this process is constrained by the planar nature of the sketch function and can be highly time-consuming. This paper proposes a novel approach, using a mathematical framework that has proved useful in the modelling of living tissue, to enable the parametric design of axisymmetric forms. The mathematical methodologies will be presented as follows: a length-polar-projection description of the centreline and specification of the axisymmetric cross-sections. This transdisciplinary approach was developed between the disciplines of mathematics, biology and engineering. As such, it offers a completely novel, more efficient and insightful process than current commercial approaches. The results of this study offer two contributions to research knowledge: time-efficient, parametric generation of complex axisymmetric geometries defined in 3D and a process by which to upskill knowledge of the design engineer.

Keywords. Additive Manufacturing, Digital Design, Transdisciplinary Engineering

1. Introduction

Additive manufacturing (AM) is key to the realisation of Industry 4.0 [1, 2], and is highlighted as a promising technology to reach global sustainability goals [3, 4]. Yet there are some well-acknowledged barriers to releasing the potential socioeconomic impact. Of these, design for additive manufacturing (DfAM), remains a persistent and ever-evolving challenge. Engineering design is a very broad discipline; it encompasses everything involved in the lifecycle of a product, from mechanical validation and materials through to the disposal. AM, as opposed to traditional manufacturing technologies such as machining, introduces even more parameters to an already complex design process. AM creates the material during manufacturing, and as such the geometric constraints during manufacture and mechanical properties are dependent on the AM technique, platform, material and the processing parameters. Combined with the rapid development of AM technologies, DfAM poses a significant and continual problem in the increased adoption of AM in industry. This issue is extensively reviewed in academic

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literature, where many challenges have been identified or approached: fragmented software, psychological inertia in design thinking, propagation of the knowledge of capabilities and limitations, designing for specific advances such as multiple materials, and complex, computationally heavy geometry such as lattices. Essentially, whilst manufacturing technology proceeds at a rapid pace, the required advancements in theory, frameworks and software to support their implementation in industry lag behind.

Computer-aided design (CAD) is the lynchpin of mechanical design. The software, which has been commercialised by several different companies, is very well-established in industry and engineering education. The approach is founded on the concept of extruding a form and then subtracting from it; synonymous with subtracting material from a stock of material. The functionalities enabled by CAD entrenches the concept of extruding and subtracting material into the digital interface which is central to the design process. AM, the process of fusing material into a 3D object point by point or layer by layer, is completely opposite to machining technologies. AM optimisation techniques (discussed in section 2.1) enable the design of organic shapes, which are associated with structures seen in nature. Further to this, Thomas-Seale *et al.* (2019) propose that the process of AM is analogous to growth [5]. This study seeks to implement the analogy, and outlines a novel method to define curve-based geometry, the manufacturing of which is enabled by AM.

To fundamentally change how a form is created through a digital interface requires considering how the data of the geometry is represented. CAD represents geometry in a parametric manner, which means that it can be changed with respect to a parameter such as the length of an edge. The parametric representation of geometry is approached through mathematical equations. A transdisciplinary perspective was taken to implement the analogy between AM and growth, by fusing methodologies found in mathematical biology and engineering design. This research presents an adaptation of the mathematical equations of plant growth to represent axisymmetric forms in Euclidean space, to enable a more efficient, intuitive and user-informed approach to DfAM.

2. Background

2.1. Design for Additive Manufacturing

The discipline of DfAM is so all encompassing that there are research studies aimed at categorising this knowledge. Wiberg *et al.* propose the overhead categories of system, part and process design [6]. The system focusses on the context of the application, the part encompasses geometry including optimisation, constraints and validation, and the process is the interface with physical manufacturing [6]. Studies that have analysed DfAM from an industrial perspective conclude that whilst AM presents opportunities, knowledge is derived from experience, with a focus on “printability” [7], which leads to fragmented knowledge [8]. Taborda *et al.*, through analysis of review-based literature, also conclude a lack of exhaustivity in methodologies, and innovation in the products developed with them [9].

Focussing on approaches that have developed through to end-use software; some academic endeavours have targeted very specific opportunities enabled by AM. Notable examples include Foundry [10] and voxel-based design [11], which utilise the multi-material capabilities enabled by AM. Topology optimisation, a generalised term which describes the optimisation of a structure to a function(s) (e.g., weight or stiffness), underpins various commercial software. Another prominent commercial technique is Generative Design (Autodesk Inc., San Francisco, CA, USA) [12], which uses artificial intelligence to create a broad range of solutions to satisfy a defined mechanical system [13]. Whilst these commercial software techniques offer increased efficiency, the increase in design space or creativity is enabled through automated methodologies (either through optimisation to a function(s), or artificial intelligence), the user's involvement remains isolated to defining the inputs of the mechanical system. The creativity of the engineer is not utilised and so the knowledge of the engineer is not upskilled in "how" to DfAM. This constrains the efficient and creative application of DfAM to the accessibility of such automated software, which poses a socioeconomic barrier in terms of cost.

2.2. A Transdisciplinary Approach

The term "transdisciplinary", whilst becoming more prevalent in academic literature, is still evolving in terms of definition, when compared to terms such as "interdisciplinary". Research aimed at defining the term has focussed on presentations given at conferences [14, 15], concluding that the transcendence, integration and fusion of disciplines are key characteristics of transdisciplinary approaches. More specifically, this transcendence of disciplines is viewed as crucial to solving complex engineering problems, particularly in the context of digital manufacturing [16].

To overcome the mirage of barriers that exist in progressing the efficiency and efficacy of DfAM, one of the biggest challenges is to change how engineers approach design and how they "think". This psychological inertia, to think or problem-solve in the same way that you have been trained to [17], is compounded further by the nature of traditional CAD. To overcome such a multi-faceted problem, which bridges human perception and processing with respect to digital representation, a radically different approach is required.

Bio-inspired design, whilst well-established from an engineering perspective for product design and materials [18, 19], is far less prevalent in manufacturing. Literature has highlighted the potential offered by mirroring biological processes in manufacturing [20], particularly in the context of the accelerated technologies of Industry 4.0. Yet, bio-inspired manufacturing is an emerging area, which has seen relatively few applications. When considering replicating a biological process or function, there is a significant increase in complexity of implementation, when compared to replicating the geometry of a biological structure.

This study utilises a transdisciplinary approach to transcend the barrier of implementation between a biological analogy and manufacturing. AM is defined as a process of joining material, usually layer upon layer [21], as such, it is analogous to the cell-by-cell process of growth [5]. Prior to any digital considerations, the representation of geometry is founded by the discipline of mathematics. To unlock the potential offered by this biological analogy, implementation was sought through the fundamental equations which define geometry. When modelling the growth of axisymmetric plant organs such as the root, a coordinate system known as body-fitted coordinates is far more useful than conventional systems such as the Cartesian or polar [22, 23]. Body-fitted coordinates generalise cylindrical polar coordinates: instead of a straight z-axis extending vertically, there is a curved s-axis fitted to the centreline of a solid body. At any particular value of s, the plane perpendicular to the centreline contains two more coordinates, similar to the radial and angular coordinates in the cylindrical polar system.

The approach presented in this study will adopt the same framework, to address some of the shortcomings of traditional CAD as discussed in Section 1. To define an axisymmetric structure in 3D, one first specifies a centreline which is generally curved, then any axisymmetric structure can be mathematically described using body-fitted coordinates. This approach synthesises mathematical biology and engineering design, highlighting the analogy between AM and plant growth; it also simplifies the geometric specification and analysis of parts while offering the engineer greater control, thereby representing an opportunity for upskilling.

3. Mathematical Methods

3.1. Length-Polar-Projection

In this section, we define curves in 3D space as generic mathematical objects and describe the geometrical properties of the class of curves suitable to be centrelines of AM objects. We demonstrate a novel method for specifying such centrelines using mathematical functions, requiring one fewer functional inputs than traditional CAD methods.

Every point on a curve has three spatial coordinates, x, y and z, in some unit of length such as mm. At the same time, to every point on the curve we may associate a dimensionless arclength parameter, s, normalised to the interval between 0 and 1. This means that $s = 0$ and $s = 1$ label the two end-points of the curve respectively, and as the label varies continuously from 0 to 1, the corresponding point moves along the curve. For any given curve with an arclength parameter, we may write the spatial coordinates of any point on the curve as $x(s)$, $y(s)$ and $z(s)$. The notation $x(s)$ denotes that x is a function of s: it takes an s value between 0 and 1 as an input, and outputs the x coordinate of the point on the curve with label s. Conversely, any three given (continuous) functions $x(s)$, $y(s)$ and $z(s)$, where s takes values between 0 and 1, determine a curve, by specifying the spatial coordinates of every point along it. Thus, specifying a curve in 3D space amounts to nothing more or less than specifying three functions of the unit interval between 0 and 1. For example, the triplet of functions,

$$\big(x(s), y(s), z(s)\big) = \left(s - 0.5, 0, 0.25 - (s - 0.5)^2\right), \tag{1}$$

specifies a finite section of a parabola in the (x, z) plane, starting at the point $(-0.5,0,0)$ and ending at $(0.5,0,0)$. Figure 1 shows the curve in the (x, z) plane.

For our purposes, a centreline suitable for construction is any curve in 3D space whose tangent at every point makes an acute angle from the positive z-axis. The standard CAD description of a such a curve involves three functions, which may be the Cartesian $x(s), y(s), z(s)$ as described above, $\rho(s), \phi(s), z(s)$ in cylindrical coordinates, $r(s), \theta(s), \phi(s)$ in spherical polar coordinates, or another triplet of functions in some other coordinate system. We describe here a simpler description of the curve which involves only two functions and a scalar quantity.

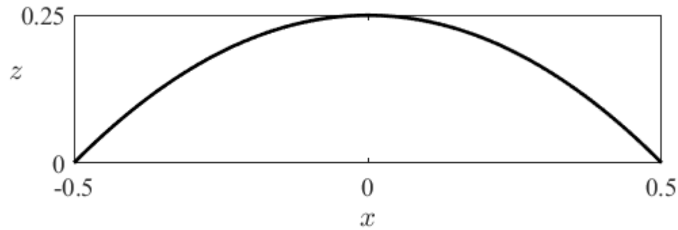

Figure 1. The finite curve described mathematically by Equation 1.

We denote by $\mathbf{e}_x, \mathbf{e}_y$ and $\mathbf{e}_z$ the unit vectors pointing along the positive x, y and z axes, respectively, and let $\mathbf{T}(s)$ be the unit tangent vector to the curve at s, measured along increasing s. Let $\theta_z(s)$ be the angle from $\mathbf{e}_z$ to $\mathbf{T}(s)$, so that

$$0 \leq \theta_z(s) < \pi/2. \tag{2}$$

Let $\theta_x(s)$ be the angle from $\mathbf{e}_x$ to the x-y projection of $\mathbf{T}(s)$, unless $\theta_z(s) = 0$ for some s, in which case we define $\theta_x(s) = 0$. Thus,

$$0 \leq \theta_x(s) < 2\pi. \tag{3}$$

We call $\theta_z(s)$ and $\theta_x(s)$ the *polar angle* and *projection angle*, respectively, of the curve at s. By definition of the angles, the tangent vector $\mathbf{T}(s)$ has the following components in the $\mathbf{e}_x, \mathbf{e}_y$ and $\mathbf{e}_z$ directions.

$$\mathbf{T}(s) = \sin\theta_z(s)\cos\theta_x(s)\,\mathbf{e}_x + \sin\theta_z(s)\sin\theta_x(s)\,\mathbf{e}_y + \cos\theta_z(s)\,\mathbf{e}_z. \tag{4}$$

From Equation (4), one may verify that $\mathbf{T}(s)$ is the *unit* tangent vector, meaning it has length 1. It is a central theorem in Differential Geometry that any curve parametrised by arclength has a unit tangent vector given by the derivative of the curve,

$$\mathbf{T}(s) = \frac{1}{L}\left(\frac{\mathrm{d}x}{\mathrm{d}s}\mathbf{e}_x + \frac{\mathrm{d}y}{\mathrm{d}s}\mathbf{e}_y + \frac{\mathrm{d}z}{\mathrm{d}s}\mathbf{e}_z\right), \tag{5}$$

where L is the length of the curve. Equating coefficients in Equations (4) and (5), and assuming that the curve begins at $(x, y, z) = (0,0,0)$, yields the following formulae for converting the functions θ_z and θ_x into the standard Cartesian coordinates of points on the curve.

$$x(s) = L\int_0^s \sin\theta_z(t)\cos\theta_x(t)\,\mathrm{d}t, \tag{6}$$

$$y(s) = L\int_0^s \sin\theta_z(t)\sin\theta_x(t)\,\mathrm{d}t, \tag{7}$$

$$z(s) = L\int_0^s \cos\theta_z(t)\,\mathrm{d}t. \tag{8}$$

Therefore, to describe any centreline, one needs to specify exactly the following: the total length L of the curve, the polar angle $\theta_z(s)$ and projection angle $\theta_x(s)$ where $0 \leq s \leq 1$. We shall call this the "length-polar-projection" (LPP) description of the centreline. In

LPP, the number of functions one needs to specify is one fewer than in the Cartesian description, making LPP a simpler description.

Another useful property of the LPP description of centrelines is that the local curvature can be expressed by a simple formula. By definition, the curvature at s is defined as

$$\kappa(s) = \left|\frac{\mathrm{d}\mathbf{T}(s)}{\mathrm{d}s}\right|. \tag{9}$$

Using Equation (4) to calculate the derivative of $\mathbf{T}(s)$ one finds, after some lengthy algebra,

$$\kappa(s) = \sqrt{\left(\frac{\mathrm{d}\theta_z(s)}{\mathrm{d}s}\right)^2 + \left(\frac{\mathrm{d}\theta_x(s)}{\mathrm{d}s}\right)^2 \sin^2\theta_z(s)}. \tag{10}$$

Note that $\kappa(s) = 0$ if and only if either

$$\theta_z(s) = 0 \text{ and } \frac{\mathrm{d}\theta_z(s)}{\mathrm{d}s} = 0, \tag{11}$$

or,

$$\theta_z(s) > 0 \text{ and } \frac{\mathrm{d}\theta_z(s)}{\mathrm{d}s} = \frac{\mathrm{d}\theta_x(s)}{\mathrm{d}s} = 0. \tag{12}$$

In particular, Equation (11) represents any point where the curve is instantaneously vertical, and Equation (12) represents any point where the curve is non-vertical and changing from concave to convex or vice versa.

3.2. Axisymmetric Cross-Sections

Once a centreline has been determined, one can mathematically describe any axisymmetric structure around the line. To do so, one chooses a number of equidistant construction points along the line and with each point as the centre, defines a circle of radius R using the equations $X = R\cos\phi$ and $Y = R\sin\phi$, where X and Y are coordinates in the plane perpendicular to the centreline and ϕ is the polar angle in that plane. All the circles can then be joined together point by point, using lines relative to the centreline, to form the surface of an axisymmetric structure.

Figure 2 shows an example of a centreline easily specified using the LPP system:

$$L = 10 \text{ cm}, \tag{13}$$

$$\theta_z(s) = \frac{\pi s}{6} \text{ radians}, \tag{14}$$

$$\theta_x(s) = 2\pi s \text{ radians}, \tag{15}$$

where s varies from 0 to 1. The same curve in the standard Cartesian system has a much more complex mathematical description, which is obtainable using Equations 6-8:

$$x(s) = \frac{10}{143\pi}\left(39\cos\frac{11\pi s}{6} - 33\cos\frac{13\pi s}{6} - 6\right)\text{ cm,} \tag{16}$$

$$y(s) = \frac{10}{143\pi}\left(39\sin\frac{11\pi s}{6} - 33\sin\frac{13\pi s}{6}\right)\text{ cm,} \tag{17}$$

$$z(s) = \frac{60}{\pi}\sin\left(\frac{\pi s}{6}\right)\text{ cm.} \tag{18}$$

This example illustrates the advantage of the LPP system over conventional descriptions of curves such as the Cartesian. Figure 3 shows an axisymmetric structure defined around the centreline, featuring circles of decreasing radii (from bottom to top), each centred at a point on the centreline and perpendicular to the line.

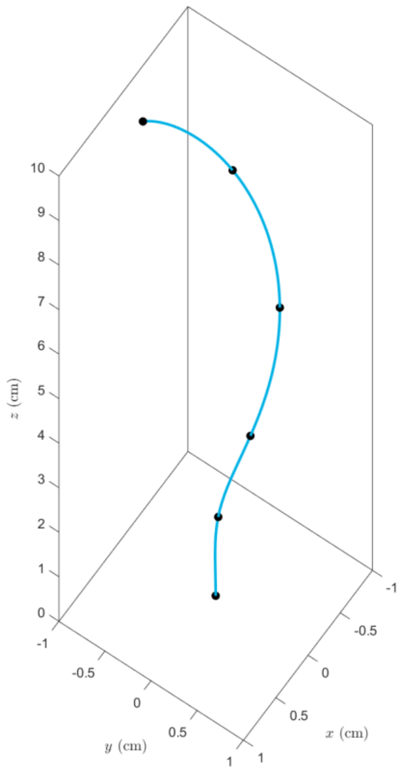

Figure 2. The centreline described mathematically by the LPP system using Equations 13-15. The curve twists around the z-axis by one full revolution while increasingly bending towards the (x, y) plane. Six equally spaced construction points are chosen along the line, to be used in defining an axisymmetric structure around the line. The LPP description is significantly simpler than the Cartesian description of the same curve, which are Equations 16-18.

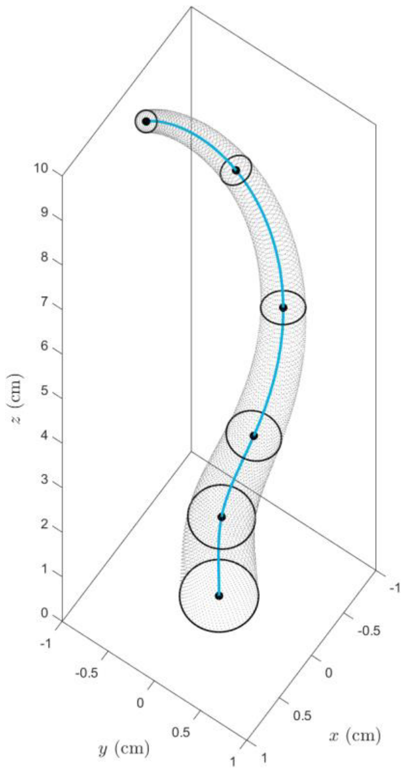

Figure 3. Six equally spaced construction points are chosen along the centreline from Figure 2. Circles of radii 0.35, 0.3, 0.25, 0.2, 0.15 and 0.1cm (from bottom to top) are drawn, centred at those construction points. Each circle is perpendicular to the centreline. An axisymmetric structure is formed by joining the circles together.

4. Discussion

In mathematical modelling, finding the most convenient coordinate system with which to describe the geometry of the problem is often half the battle. In creating a new paradigm for DfAM which improves upon traditional CAD, the "battle" is to build a framework that allows full control over the shape of a designed part, within the mathematical capacity/knowledge of the engineer. This study has demonstrated that when the part is axisymmetric with a curved centreline, the novel length-polar-projection (LPP) system for describing the centreline is significantly simple compared to conventional descriptions such as Cartesian. A centreline that requires three complicated functions to specify in Cartesian, for example Equations 16-18, can be specified using one number and two simple functions in LPP, as per Equations 13-15. Even if the centreline is geometrically simple, for example a 10cm-long straight-line segment slanted at angle α radians from the x axis, with a simple Cartesian description:

$$x(s) = 10s\cos\alpha \text{ cm}, \quad y(s) = 0 \text{ cm}, \quad z(s) = 10s\sin\alpha \text{ cm}, \tag{19}$$

the corresponding LPP description is simpler still, featuring a number L and two constant functions θ_z and θ_x:

$$L = 10 \text{ cm}, \quad \theta_z(s) = \frac{\pi}{2} - \alpha \text{ radians}, \quad \theta_x(s) = 0 \text{ radians}. \tag{20}$$

This relative simplicity ultimately stems from the fact that LPP closely follows an intuitive perspective of curves in space, contrary to Cartesian and other conventional systems. When visualising a curve and verbally describing it, language is used in terms of the curve's length and angles relative to certain reference directions. For the slanted line segment example, the description, "10cm-long, slanted at angle α", would be used, which directly translates into the LPP system (Equation 20). The verbal description of the form is not, "A curve whose x coordinate is linear in s with coefficient $10\cos\alpha$ and whose y coordinate is linear in s with coefficient $10\sin\alpha$", because such a description has no groundings in human intuition.

For the more complicated example in Figure 2, a verbal description would be, "The curve twists around the z-axis by one full revolution while increasingly bending towards the (x, y) plane." Here, "twisting around the z-axis by one full revolution" translates directly into Equation 15 while "increasingly bending towards the (x, y) plane" becomes Equation 14, where bending increases from 0 to 30 degrees. It is unclear how the corresponding Cartesian description given by Equations 16-18 stems from, or gives rise to, any visual or verbal intuition. The route to using the LPP system is therefore a straightforward one: begin by visualising a curve in one's mind, then describe it verbally using natural language about the length, slant and twist of the curve, and finally translate that language directly into the three corresponding mathematical expressions. In terms of implementation, the framework can be coded through any programming language, for example MATLAB or C++. Figure 3 was generated using MATLAB 2021a (Mathworks, Natick, MA, USA).

The inspiration behind the framework presented in this paper is the mathematical modelling of plant growth, where body-fitted coordinates have been the system of choice in such modelling efforts [22, 23]. Once a centreline has been specified, for example using the novel LPP system described above, body-fitted coordinates enable design engineers to specify geometry around the centreline, in the same way that plant modellers can specify the shape of a plant organ such as the root relative to its centreline. The plant root is typically modelled as axisymmetric, meaning every cross-section perpendicular to the centreline is circular; but body-fitted coordinates are a generic system and not restricted to specifying circular cross-sections. As such, the next step will be to generalise the framework presented here, allowing non-circular cross-sectional shapes to be designed.

This study has demonstrated the level of innovation that may be attained using a transdisciplinary synthesis. We have presented a novel approach to DfAM that transcends and integrates the disciplines of mathematics, biology and engineering. As such, it supplements and improves traditional CAD and has the potential to shift the paradigm of DfAM. This transdisciplinary research was conceptualised and the implemented across disciplines, during which significant resources were invested to upskill the knowledge of the associated researchers. To develop this approach into a transdiscipline would require significant investment to cross-train mathematician and engineer researchers, in the context of bioinspired manufacturing.

5. Conclusions

Exploiting the analogy between AM and growth, the new framework adopts the mathematical principles behind plant growth modelling, particularly with regard to describing the geometry of axisymmetric parts. An intuitive and simple system was introduced, called length-polar-projection (LPP), for defining curves in 3D to serve as centrelines for design parts, and the superiority of this system compared to conventional ones such as Cartesian was demonstrated through examples. Building around a user-defined centreline, the new framework then allows for the custom definition of any axisymmetric part using body-fitted coordinates, a method popular with mathematical modellers. This framework improves the connection between a design engineer's workflow and their intuition for AM, whilst also representing an opportunity to upskill the user.

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