Material Strength and Applied Mechanics
A. Khotsianovsky (Ed.)

doi:10.3233/ATDE230413

A Developed Probabilistic Elasto-Plastic Bi-Directional Structural Optimization Framework

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Abstract. This work introduces a new approach for probabilistic elastoplastic topology optimization based on the improved bidirectional structural optimization (BESO) technique. To consider uncertainties, the volume fraction and the material properties are considered randomly. Thus, the reliability-based design is integrated into the deterministic design by applying a reliability constraint to the optimization problem. Furthermore, using limit analysis, a bound is applied to the plastic limit load multipliers to govern the plastic behavior of the problems. Results from a 2D benchmark problem are used to illustrate how adequate the approach that has been provided is. Also, a 2D elastoplastic numerical example is shown to illustrate the proposed method's capability of identifying the best topology for elastoplastic models in the context of reliability-based design. The results indicate that the reliability constraints work effectively as a bound that reduces the yielding states.

Keywords: Topology optimization, elasto-plastic, reliability-based design, structural optimization.

1. Introduction

The objective of topology optimization of structures is to produce superior structural performance while satisfying various restrictions. Compared to other methods of structural optimization, topology optimization may be a far more flexible tool for helping engineers create innovative and extremely productive structures. According to several academic studies and investigations, it is one of the most lucrative business strategies [1]. In addition to building large-scale structures, it may be used to create nanoscale materials. Zhou and Rozvany [2] used continuum-based optimality criteria to identify the optimal topologies of various problems. Also, an approach for optimizing the topology of structures under diverse loading conditions was developed by Li et al.[3]. Topology optimization is regarded as a dynamic study and development field. Numerous algorithms have been created. The bidirectional structural optimization (BESO) method is among the most advanced techniques in this area [4,5]. BESO operates by concurrently removing and materials adding from the least efficient sections to the most efficient parts, such that the resultant topology is optimal. A parallel topology optimization technique for dynamic and static properties of multiphase materials was suggested by Gan and

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Wang [6] using the BESO method. Wang et al. [7] proposed a multi-resolution topology optimization algorithm by developing BESO method.

There are still a lot of uncertainties in the topology optimization, such as varied applied load conditions, material properties, and other important factors [8,9]. Therefore, various optimization methods and frameworks have been developed to successfully deal with uncertainty in structural design such as reliability-based topology optimization and robust topology optimization algorithms [10–12]. A novel technique of integrating reliability-based design into thermoelastic structural topology optimization was proposed by Habashneh and Rad [13].

When a material undergoes elasto-plastic deformation, its properties will change depending on the amount of plastic strain applied, resulting in a nonlinear connection between stress and strain. This will have an effect on the strain energy, which is significant since it is often the focus of structural engineering challenges that aim to reduce energy consumption. Using a single constraint on the accumulation of similar plastic strains, Amir [14] demonstrates a way for creating stress-constrained optimum topology designs. After using the basic theory of plasticity, Tauzowski et al. [8] came up with a new way for optimizing the topology of elasto-plastic structures from the perspective of reliability.

The proposed work in this paper is a continuation of the research leading to the improvement of the BESO method [15,16] where a novel algorithm of the BESO method is proposed by adopting reliability-based topology optimization of elastoplastic analysis in the face of uncertainties in which the volume fraction and material properties are assumed to be random variables.

2. Problem Statement

2.1. Reliability-Based Analysis

By assuming two independent random variables (X_R and X_S), where X_R stands for the non-negative limit of X_S. Therefore, $X_R \leq X_S$ defined the failure problem. Considering that the PDFs for X_R and X_S are $f_R\ (X_R)$ and $f_S\ (X_S)$, respectively. according to this basic concept of reliability analysis, the probability of failure (P_f) then is constructed as[17]:

$$P_f = P[X_R \leq X_S] = \iint_{X_R \leq X_S} f_R\ (X_R)\, f_S\ (X_S) dX_R dX_S. \tag{1}$$

An alternative concept in terms of the limit state function is formulated as:

$$g(X_R, X_S) = X_R - X_S. \tag{2}$$

where $g\ \leq\ 0$ measures the failure domain (D_f). Accordingly, we can calculate P_f by:

$$P_f = F_g(0). \tag{3}$$

Also, P_f can be calculated as:

$$P_f = \int_{g(X_R, X_S)\leq 0} f(X)dX = \int_{D_f} f(X)dX. \quad (4)$$

In the accessible literature, one can find a detailed description and the formulations of Monte-Carlo sampling (MCS) method which is used to determine the P_f. Therefore, here in this work we only focus on how the reliability constraint could be utilized to form the probabilistic problem. To establish the reliability constraint, the reliability index may be used as [13]:

$$\beta_{target} - \beta_{calc} \leq 0 \quad (5)$$

Considering that Equation (5) shows the associated reliability condition with the volume fraction. Furthermore, the following expressions are used to obtain β_{target} and β_{calc}:

$$\beta_{target} = -\Phi^{-1}(P_{f,target}); \quad (6)$$

$$\beta_{calc} = -\Phi^{-1}(P_{f,calc}). \quad (7)$$

2.2. Elastoplastic Limit Analysis

The basic concept of limit analysis can be illustrated as follows: Consider an elasto-plastic body that is always being exposed to an increasing force F_i. For a formal expression of the proportionate loading, one may use the following:

The limit analysis problem is exemplified as follows: Consider an elastoplastic body exposed to a certain force F_i and increasing this force continuously. The proportionate loading may be expressed as follows:

$$F_i = m\, F_0 \quad (8)$$

m is a scalar quantity that increases monotonically and is known as the load multiplier. F_0 represents the initial specified externally applied forces. As m continues to increase, the regions of the body which contain plastic state gradually expand, till reaching a certain intensity (m_p), a free plastic flow is attained to the point where a rise in plastic deformations are achievable for the first time under constant external stresses acting during the loading process. Therefore, the ultimate plastic load of the body is $F_p = m_p F_0$.

2.3. Developed Optimization Algorithm

A brief description of the objective and various constraints is presented here, while the full description of the BESO method including the formulations and updated techniques can be found in the earlier work of the authors [15,16]. Using the aforementioned steps, one may determine the uniqueness of the extended portion of the technique. Hence, the probabilistic optimization problem considering the plastic ultimate load constraint is constructed as:

$$Minimize: C = u^T K u \tag{9}$$

$$Subject\ to:\ V^* - \sum_{i=1}^{N} V_i x_i = 0 \tag{10}$$

$$\beta_{target} - \beta_{calc} \leq 0 \tag{11}$$

$$x_i \in \{0,1\} \tag{12}$$

$$m_s - m_p \leq 0. \tag{13}$$

where the objective function of this optimization problem is to minimize C which is the mean compliance, K is global stiffness matrix and u represents the nodal displacement vector. Besides, V^*, N, V_i, x_i are total volume, total number of elements, elemental volume and design variable, respectively. Equation (11) shows the reliability constraint and Equation (13) indicates the constraint for plastic ultimate load multiplier, which states that, according to the static principle, m_s must be less than or equal m_p.

3. Numerical Examples

A reliability-based geometrical nonlinear elastic numerical example of L-shaped beam is considered as the first example and the results of this problem are compared with a benchmark results which was performed by Movahedi et al. [16] to approve the validity of the proposed method. Also, another example is considered in the case of elasto-plastic material which is a L-shaped beam. It is worth mentioning that the MCS method is adopted to perform probabilistic evaluations. V_f , and the material properties are considered randomly to represent uncertainties.

3.1. Two-Dimensional Elastic Problem

The design domain of the considered L-shaped beam is produced by fixing the L-shaped beam at the top as it is shown in Figure 1. The considered applied load F is $12kN$. Material properties are $70\ GPa$, and 0.3 for Young's modulus and Poisson's ratio values, respectively. The BESO parameters are assumed as the following: $AR_{max} = 1\%$, $ER = 1\%$, $\tau = 1\%, and\ r_{min} = 18\ mm$. To indicate uncertainties, V_f , and the material properties as random variables. Also, for MCS purposes, Z, which is the number of sampling points, is considered 3.0×10^6.

The effectiveness of the suggested technique was shown by comparing the optimized forms in the deterministic case to the results of a benchmark issue previously solved by Movahedi et al. [16] of linear and geometrically nonlinear designs. Table 1 displays the results of the calculations performed to determine the optimum topology, complementary work (W^C), and maximum Huber-Mises-Hencky stress (σ_{HMH}^{max}) of both linear and geometrically nonlinear designs. As is readily apparent, geometrically nonlinear designs have a lower complementary work value than their linear counterparts.

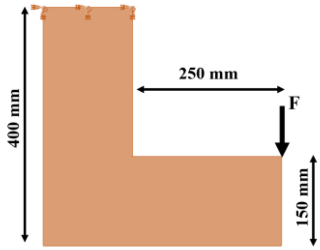


Figure 1. 2D L-shaped example4.

Table 1. Obtained deterministic results.

Design	Optimized shape	$W^C(kJ)$	$\sigma_{HMH}^{max}(MPa)$
Linear		5.31	314
Geometrically nonlinear		5.26	310

The results of the proposed algorithm by considering V_f, and the material properties as random variables are presented in Table 2 in which it shows the final optimized shapes, maximal Huber-Mises-Hencky stress, and the complementary work in the case of probabilistic designs. Taking into account that the results show two different values of β_{target}. Probabilistic designs have been shown to have less complementary work compared to deterministic designs, both for linear and geometrically nonlinear. Considering geometrically nonlinear results, the complementary work has been effectively reduced by 16.16% from 5.26 kJ in the case of deterministic design to 4.41 kJ in the case of the probabilistic design when $\beta_{target} = 3.77$. In the case of linear results, the complementary work is declined by 9.04% from 5.31 kJ in the case of deterministic design to 4.83 kJ in the case of probabilistic approach by considering $\beta_{target} = 3.77$. Furthermore, the maximal Huber-Mises-Hencky stress produced by probabilistic designs is less than that produced by deterministic designs in both linear and geometrically nonlinear designs. Both the maximum stress and the complementary work values for linear and geometrically nonlinear systems rise with decreasing β_{target} in the probabilistic situation.

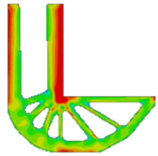
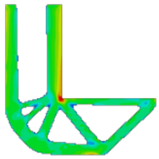
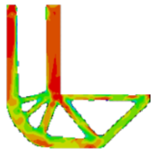
Table 2. Obtained probabilistic results.

Design	Optimized shape	$W^C(kJ)$	$\sigma_{HMH}^{max}(MPa)$
Linear			
$\beta_{target} = 4.44$		4.54	299
$\beta_{target} = 3.77$		4.83	302
Geometrically nonlinear			
$\beta_{target} = 4.44$		4.32	292
$\beta_{target} = 3.77$		4.41	295

3.2. Two-Dimensional Elastoplastic Problem

A probabilistic elastoplastic design of a 2D L-shaped beam problem is considered the second example. As was previously indicated, MCS is adapted to the process of evaluating degrees of uncertainty. With the material properties and V_f modeled as independent random variables, the probabilistic design is presented. Note that this example is identical to the one in Section (3.1) in terms of design domain, material characteristics, BESO parameters, and the number of MCS simulations. Elasto-plastic modeling assumes a yield stress of $110\ MPa$ in the beam at its initial, predetermined load of $F_0 = 4\ kN$. For the whole design space, the plastic limit load multiplier $m_p = 4.25$ is used. There are three load examples presented: $F_1 = 0.5\, F_0$, $F_2 = 2\, F_0$, and $F_3 = 3\, F_0$, all of which serve to illustrate the impact of load multiplier. The results of considering the effect of plastic-limit load multiplier in the case of probabilistic designs considering different values of β_{target} are presented in Table 3. As expected, the absence of plastic zones is most apparent at the lightest load condition. In the second scenario, however, we see plastic areas. Large plastic areas are produced in the third scenario. Also, it can be noted that the percentage of the yielded elements within the resulted shape increases as β_{target} decreases for each load case. For instance, in the case of $F_3 = 3\, F_0$, the percentage of yielded elements increased by 33.99% from 20.65% when $\beta_{target} = 4.90$ to 33.85% when $\beta_{target} = 3.79$.

Table 3. Resulted topological shapes and $\boldsymbol{HMH}$ stresses according to the different load cases.

β_{target}	$F_1 = 0.5\,F_0$	Ratio of yielded elements (%)	$F_2 = 2\,F_0$	Ratio of yielded elements (%)	$F_3 = 3\,F_0$	Ratio of yielded elements (%)
4.9		0		0.22		20.65
3.79		0		0.38		33.85

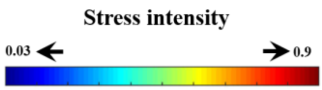
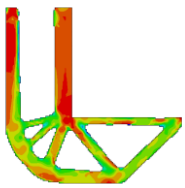
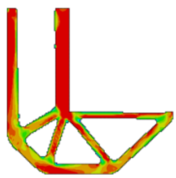
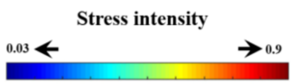


Table 4 compares yielding state discrepancies between deterministic and probabilistic designs. Probabilistic yielding states have fewer components than deterministic ones. By considering $\beta_{target} = 3.79$ and load case ($F_3 = 3\,F_0$), probabilistic design with provided 5.25% fewer model components than deterministic design.

Table 4. Applied load and yielding state.

Ratio of the applied load	Probabilistic design ($\beta_{target} = 3.79$)	Deterministic design
$F_3 = 3\,F_0$		



4. Conclusions

This work uses the expanded BESO approach to conduct reliability-based design for elastic and elasto-plastic topology optimization of structures with uncertainties. Optimization considers volume fraction and material properties randomly owing to uncertainty. Thus, reliability theory and topology optimization are used to identify the best structural topology that meets reliability criteria. Monte-Carlo simulation is also used to calculate reliability index.

This study's key findings:

- Since failure factors are taken into account in the reliability-based design, geometrically nonlinear systems have complementary work less than deterministic designs.
- Considering the volume fraction and material properties as random variables have influenced the resulting shapes.
- In probabilistic elastic designs, geometrically nonlinear designs have lower values of maximum Huber-Mises-Hencky stress than deterministic linear stiffness designs. Also, the maximum stress rises when the reliability index decreases.
- Since failure parameters are addressed in elastoplastic design, the probabilistic yielding state has fewer elements than the deterministic yielding state.

References

[1] Sigmund O, Maute K. Topology optimization approaches. Struct Multidiscip. O. 2013; 48:1031–55.

[2] Zhou M, Rozvany GIN. The COC algorithm, Part II: Topological, geometrical and generalized shape optimization. Comput. Methods Appl Mech Eng. 1991; 89:309–36.

[3] Li J, Guan Y, Wang G, Wang G, Zhang H, Lin J. A meshless method for topology optimization of structures under multiple load cases. Structures 2020; 25:173–9.

[4] Xie YM, Steven GP. A simple evolutionary procedure for structural optimization. Comput Struct. 1993; 49:885–96.

[5] Yang XY, Xie YM, Steven GP, Querin OM. Bidirectional evolutionary method for stiffness optimization. AIAA Journal 1999; 37:1483–8.

[6] Gan N, Wang Q. Topology optimization of multiphase materials with dynamic and static characteristics by BESO method. Adv Eng Softw. 2021; 151:102928.

[7] Wang H, Liu J, Wen G. An efficient evolutionary structural optimization method for multi-resolution designs. Struct Multidiscip O. 2020; 62:787–803.

[8] Tauzowski P, Blachowski B, Lógó J. Topology optimization of elasto-plastic structures under reliability constraints: A first order approach. Comput Struct. 2021; 243:106406.

[9] Kim C, Wang S, Rae K, Moon H, Choi KK. Reliability-based topology optimization with uncertainties. J Mech Sci Technol. 2006; 20:494.

[10] Lógó J. SIMP type topology optimization procedure considering uncertain load position. Period Poly Tech-CIV 2012; 56:213–9.

[11] Csébfalvi A, Lógó J. Critical examination of volume-constrained topology optimization for uncertain load magnitude and direction. Civil-Comp Proc. 2015; 108.

[12] Meng Z, Wu Y, Wang X, Ren S, Yu B. Robust topology optimization methodology for continuum structures under probabilistic and fuzzy uncertainties. Int J Numer Methods Eng. 2021; 122:2095–111.

[13] Habashneh M, Rad MM. Reliability based topology optimization of thermoelastic structures using bi-directional evolutionary structural optimization method. Int J Mech Mater Des. 2023; 1–16.

[14] Amir O. Stress-constrained continuum topology optimization: a new approach based on elasto-plasticity. Struct Multidiscip O. 2017; 55:1797–818.

[15] Habashneh M, Movahedi Rad M. Reliability based geometrically nonlinear bi-directional evolutionary structural optimization of elasto-plastic material. Sci Rep. 2022; 12:1–22.

[16] Movahedi Rad M, Habashneh M, Lógó J. Elasto-Plastic limit analysis of reliability based geometrically nonlinear bi-directional evolutionary topology optimization. Structures 2021; 34:1720–33.

[17] Haldar A, Mahadevan S. Probability, reliability, and statistical methods in engineering design. John Wiley & Sons Incorporated. 2000.