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# A Most Unfavorable Active Earth Pressure Method Considering Stability Tests for Retaining Wall

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**Abstract.** Earth pressure is a critical foundation for retaining wall design and verification, but in the earth pressure calculation, the geometrical parameters of the retaining wall itself are not taken into account in the tests of the stability of the retaining wall. In order to obtain a more complete method for calculating retaining walls, this paper analyses the forces between the soil behind the wall and the retaining wall and shows that when calculating the Coulomb earth pressure, the earth pressure behind the wall only needs to stay within the limit state. The direction of the earth pressure can therefore only vary within a defined range of angles and there exists a direction of earth pressure which is the most unfavourable for the retaining wall out of many. this paper proposes a mechanical model for the most unfavourable active earth pressure in retaining wall design and derives a calculation formula; the calculation example proves that the calculation results are significantly different from Coulomb's theory and the resulting earth pressure makes the retaining wall less stable during the verification.

**Keywords.** Coulomb earth pressure; stability tests; most unfavorable condition; active earth pressure; retaining wall;

## 1. Introduction

A Retaining wall is a structure that prevents earth collapse or intercepts earth slope extensions and is widely used in a variety of engineering activities. The basic steps in retaining wall design are:

Select a suitable size of retaining wall section using calculations, and determine the structural form of the retaining wall. Then,Using suitable formulas for earth pressure calculations on retaining walls. Finally, tests of the stability of retaining walls against slipping and overturning.

Slip stability is an essential indicator of the security of a retaining wall and is expressed in $K_s$. The larger the ratio, the better the security and vice versa.

The analysis shows that the earth pressure need not be maximized to achieve the smallest slip stability factor $K_s$, but rather to search for the most unfavorable force on the retaining wall, thus obtaining a smaller slip stability factor $K_s$ than the other methods. On this basis, a method of calculating the most unfavorable active earth pressure is proposed in this paper.

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Coulomb[1-3] and Rankine[4-5] are common methods for calculating earth pressure. The Coulomb is widely used in many retaining wall design projects at home and abroad due to its simple calculation principle, wide applications, and the calculation results being close to the actual earth pressure.

It has been studied extensively by scientists for years. Tang Y[6] proposed an active earth pressure calculation method for unit finite soils in order to be more in line with engineering practice; Ma P[7] a method for calculating earth pressure in finite soils. Professor Mao Y S raised several questions on Coulomb[8].The questioning analysis shows that when active damage occurs in the soil behind the wall, it is only necessary that the shear forces on the sliding surface of the soil behind the wall wedge reach the ultimate shear strength, while the shear forces on the soil wedge at the wall surface do not necessarily reach the ultimate shear strength. If the active damage defined in the active earth pressure solution is as stated in the previous sentence, then the earth pressure solution needs to take more factors into account.

As the back thrust of the wall can be in a non-critical state, the direction of action of the earth pressure cannot be determined, but its direction of action can only be taken within a limited range, i.e. not exceeding the limit state[9]. The direction of earth pressure therefore only has to be satisfied within the permissible angle, which suggests that there may be a situation where one of the many directions of action makes the retaining wall most likely to slide. Hopefully, the work in this paper will lead to a more rational answer to the question.

## 2. Most Unfavorable Active Earth Pressure Model

Assumptions: The soil behind the retaining wall is homogeneous sandy soil, and the sliding soil wedge *ABC* is rigid; the sliding surface *BC* is a flat sliding surface.

The analytical model in this paper is similar to Coulomb's theory, as shown in Figure 1, *H* is the height of the retaining wall; α is the angle between the back of the wall *AB* and the vertical line; *ω* is the angle between the bottom of the wall and the horizontal plane; *β* is the angle between the fill surface *AC* and the horizontal plane; *δ* is the friction angle between the back of the wall and the fill, *φ* is the friction angle of the fill. *θ* is the angle between the sliding surface *BC* and the horizontal plane. Considering the static equilibrium condition of the soil wedge *ABC* and its forces are as follows:

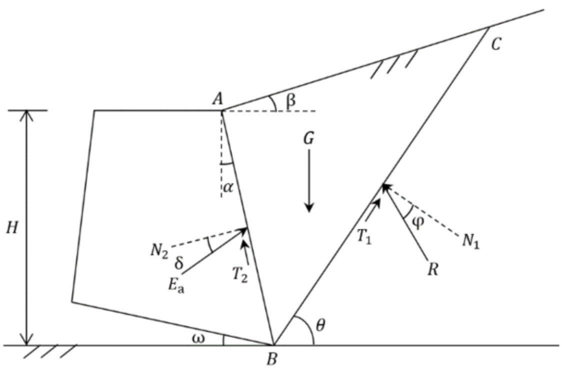


**Figure 1.** Model of Coulomb's active earth pressure

1) The gravity of the soil wedge *ABC* is *G*. If the value of $\theta$ is known, then the magnitude, direction, and action point of *G* are known.

2) The reaction force *R* acting on the sliding surface *BC* by the soil wedge *ABC* is the combined force of the frictional force $T_1$ on the *BC* surface and the normal reaction force $N_1$, with the angle normal to the *BC* surface equal to the angle of internal friction of the soil $\varphi$. The direction of action of *R* is known, and its magnitude is unknown.

3) The reaction force $E_a$ of the retaining wall against the earth wedge, its magnitude and direction are unknown, but the direction of $E_a$ should be in the range $\delta$ above and below the normal $N_2$.

## 3. Most Unfavorable Active Earth Pressure Solution

In order to solve the $E_a$, the soil wedge *ABC* force analysis is performed with the magnitude and direction of gravity *G* known, the direction of *R* known, and the range of direction change for $E_a$ known, so the force triangle *OAB* for forces *G*, *R* and $E_a$ can be drawn, as shown in Figure 2. The range of direction change for $E_a$ corresponds between the *OC* and *OD* vectors in Figure 2.

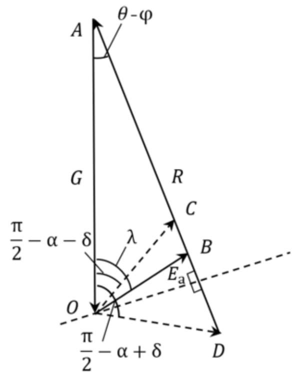


**Figure 2.** Triangle of forces for the most unfavorable active earth pressure

Analysis of the force triangle shows that:

$$\angle A = \theta - \varphi \tag{1a}$$

$$\angle OBA = \pi - \theta - \lambda + \varphi \tag{1b}$$

where, $\lambda$ = angle of the most unfavorable active earth pressure $E_a$ to the vertical line.

The range of values is: $\pi/2 - \alpha - \delta \leq \lambda \leq \pi/2 - \alpha + \delta$ .

In the triangle *OAB* by the sine theorem, it is shown that:

$$\frac{G}{\sin\angle OBA} = \frac{E_a}{\sin\angle A} \tag{2}$$

Substitution of equation (1):

$$E_a = \frac{G\sin(\theta-\varphi)}{\sin(\theta+\lambda-\varphi)} \tag{3}$$

The gravity *G* of the soil wedge *ABC* can be solved from Figure 2：

$$G = \frac{1}{2}\gamma H^2 \frac{\cos(\alpha-\theta)\cos(\beta-\alpha)}{\cos^2\alpha\sin(\theta-\beta)} \tag{4}$$

The expression for $E_a$ versus $\lambda$ is derived by analyzing the force triangle, and the previous article gives a range of values for $\lambda$. The problem of solving for $E_a$ translates into finding the earth pressure direction most likely to produce slip against the retaining wall in a range of reasonable angles.

$E_a$ can be decomposed into tangential and normal components along the lower face of the retaining wall. The tangential part of the force increases the slip force of the bottom surface. Depending on its compression or tension at the bottom surface, the normal component of the force will increase or decrease the shear strength of the bottom surface. If $E_a$ is in a direction of the action, as the $\lambda$ direction of Figure 2, the total increase or decrease and anti-slip force at the base of the retaining wall *F* is maximum, it can be considered that $E_a$ at this time is the most unfavorable active earth pressure.

where F is:

$$F = E_a\sin(\lambda-\omega) - \mu E_a\cos(\lambda-\omega) \tag{5}$$

where, $\mu$ = coefficient of substrate friction for retaining walls, 0.45 is often taken in engineering.

Substitution of equation (4) :

$$F = \frac{G\sin(\theta-\varphi)\left[\sin(\lambda-\omega)-\mu\cos(\lambda-\omega)\right]}{\sin(\theta+\lambda-\varphi)} \tag{6}$$

First order derivative of the function $F(\lambda)$：

$$F'(\lambda) = \frac{G\sin(\theta-\varphi)\left\{\sin(\theta+\lambda-\varphi)\left[\cos(\lambda-\omega)+\mu\sin(\lambda-\omega)\right]-\cos(\theta+\lambda-\varphi)\left[\sin(\lambda-\omega)-\mu\cos(\lambda-\omega)\right]\right\}}{\sin^2(\theta+\lambda-\varphi)} \tag{7}$$

Make the $\gamma = \theta - \varphi$, Simplification gives:

$$F'(\lambda) = \frac{G\sin\gamma\left[\sin(\gamma+\omega)+\mu\cos(\gamma+\omega)\right]}{\sin^2(\lambda+\gamma)} \tag{8}$$

The analysis of the derivative function $F'(\lambda)$ shows that the denominator $\sin^2(\lambda + \gamma) > 0$ is constant. The numerator $G \sin \gamma\, [\sin(\gamma + \omega) + \mu \cos(\gamma + \omega)]$ is known, so the relationship between $F'(\lambda)$ and 0 can be found after bringing in.

When $F'(\lambda) > 0$, $F(\lambda)$ increases monotonically and is maximised at $\lambda = \pi / 2 - \alpha + \delta$; when $F'(\lambda) < 0$, $F(\lambda)$ decreases monotonically and is maximised at $\lambda = \pi / 2 - \alpha - \delta$.

The above calculation gives the two directions of $E_a$. Substitution of equation (4) into equation (3), the active earth pressure is solved as follows:

$$E_a = \frac{1}{2}\gamma H^2 \frac{\cos(\alpha-\theta)\cos(\beta-\alpha)\sin(\theta-\varphi)}{\cos^2\alpha\sin(\theta-\beta)\sin(\theta+\lambda-\varphi)} \tag{9}$$

where, $\gamma$, $H$, $\alpha$, $\beta$, $\varphi$, and $c$ all are constants.

Make the $k = \cot(\theta - \beta)$, Substitution equation (9) into equation (10).

$$E_a = \frac{\gamma H^2\cos^2(\alpha-\beta)\sin(\varphi-\beta)\left[k+\tan(\alpha-\beta)\right]\left[\cot(\varphi-\beta)-k\right]}{2\cos^2\alpha\sin(\lambda-\varphi+\beta)\left[k+\cot(\lambda-\varphi+\beta)\right]} \tag{10}$$

*Ea* is only a single-valued function of the rupture angle $\theta$, that is, $E_a = f(\theta)$, from $dE_a/dk = 0$ can be obtained from the quadratic equation of $k$, thus further finding out the most unfavorable active earth pressure to take the maximum corresponding rupture angle $\theta$.

The solution is:

$$k = \cot(\theta-\beta) = -\cot(\lambda-\varphi+\beta) + M \tag{11}$$

where $M = \sqrt{\left[\cot(\lambda-\varphi+\beta)-\tan(\alpha-\beta)\right]\left[\cot(\lambda-\varphi+\beta)+\cot(\varphi-\beta)\right]}$

Substitution of equation (11) into equation (10) leads to the solution equation for the final most adverse active earth pressure $E_a$.

It is clear from the previous analysis that the most unfavorable active earth pressure solution depends primarily on $\lambda$. By analysing equation (8), $\lambda$ is taken in two cases as follows:

When $F'(\lambda) > 0$, $\lambda = \pi/2 - \alpha + \delta$;when $F'(\lambda) < 0$, $\lambda = \pi/2 - \alpha - \delta$.

The final most unfavorable active earth pressure is determined by the relationship between $F'(\lambda)$ and 0, and the corresponding rupture angle $\theta$ is found according to equation (11).

Particularly, an improved method of calculating active Coulomb soil pressure, solving for the distribution of soil pressure, the magnitude of the combined force, and the point of action according to the original Coulomb theory.

## 4. Example Analysis

In this paper, two examples of calculations are selected for analysis. The self-weight of the retaining wall is taken as $G = 200kN$, and the coefficient of friction of the substrate is taken as $\mu = 0.45$.

Example 1(Back of wall vertical): selecting example 3-3 in chapter 3 of the literature[10], the retaining wall is shown in Figure 3. The specific parameters are as follows: $H = 6m$, $\alpha = 0°$, $\beta = 10°$, $\varphi = 30°$, $\delta = 15°$, $\gamma = 16.5kN/m^3$.Table 1 shows the final calculation results.

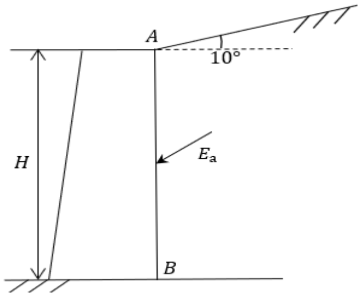


**Figure 3.** Example 1

**Table 1.** Calculation of results

| Most unfavorable active earth pressure | | | |
|---|---|---|---|
| $E_a$ ($kN/m$) | Direction | $\theta$ (°) | $K_s$ |
| 135.42 | ↖ | 62.1 | 0.57 |
| Coulomb active earth pressure | | | |
| $E_a$ ($kN/m$) | Direction | $\theta$ (°) | $K_s$ |
| 101.93 | ↙ | 53.9 | 1.04 |

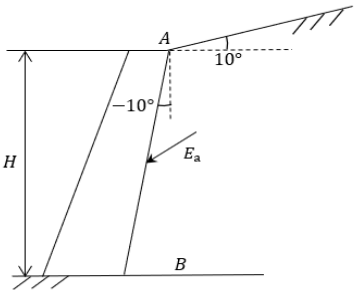


**Figure 4.** Example 2

**Table 2.** Calculation of results

| Most unfavorable active earth pressure | | |
|---|---|---|
| Direction | $\theta$ (°) | $K_s$ |
| ↖ | 62.1 | 0.57 |
| Coulomb active earth pressure | | |
| Direction | $\theta$ (°) | $K_s$ |
| ↙ | 53.9 | 1.04 |

Example 2(Back slope of the wall): $\alpha = -10°$, other parameters are the same as in Example 2, the retaining wall is shown in Figure 4, and table 2 shows the final calculation results.

The above example shows that the most unfavorable active earth pressure is typically taken as the vector *OD* in Figure 1. On the other hand, the Coulomb earth pressure is the vector *OC*. Thus there is an obvious difference between the method of this paper and the Coulomb theory. In calculating the stability factor $K_s$, the $K_s$ calculated by the method is smaller than the Coulomb theory.

## 5. Conclusions

This paper presents the solution based on Coulomb's theory, finding the most unfavorable force states that may exist in retaining walls. The conclusions obtained are as follows:

(1) A method for solving the most unfavorable active earth pressure is proposed, and specific calculation steps are given. The method puts the forces on the retaining wall structure in the most dangerous state. It solves the most unfavorable active earth pressure, a significant reference value for ensuring engineering safety.

(2) The calculation example proves that the method in this paper obtains values significantly different from Coulomb's theory. When using the most unfavorable active earth pressure in the anti-slip stability verification, it obtains $K_s$ smaller than Coulomb's theory, indicating that the retaining wall may be unstable under this method calculation, which has important reference significance for the safety part of retaining wall design.

(3) In this paper, only the anti-slip stability test is considered, and the anti-overturning stability test is not considered. Still, the basic principle of the analysis is similar. For passive earth pressure, no further work is carried out in this paper, but the research idea is the same as this paper.

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